Cosmological and astrophysical tests of Ultralight vector dark matter

Tomás Ferreira Chase

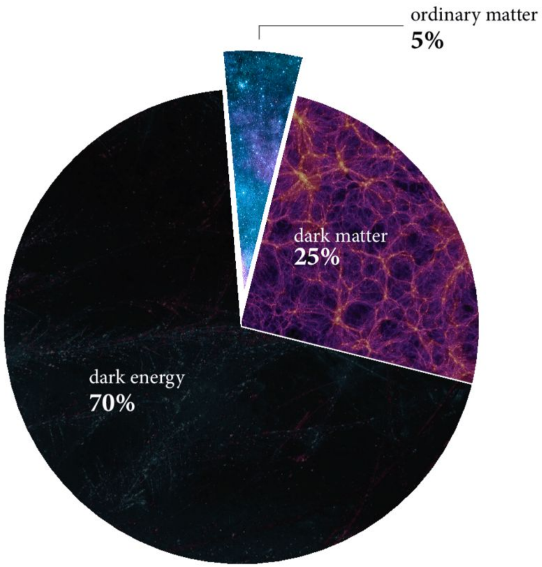
In colab with Diana López Nacir

Universidad de Buenos Aires

Motivation - Dark Matter

ΛCDM

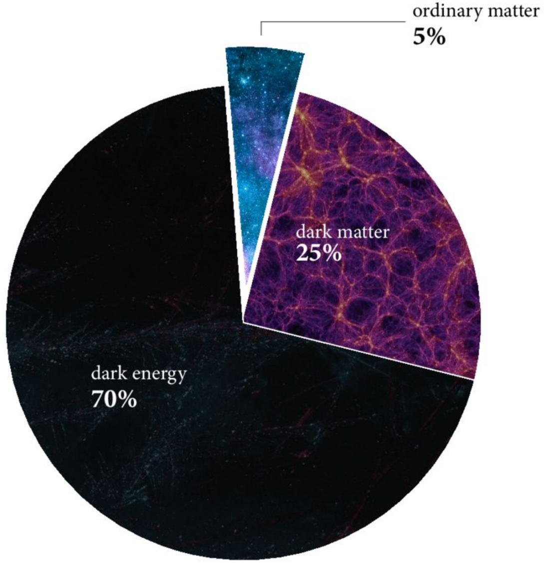
- **Cosmological constant**
- **DM** → Pressureless and non-interacting fluid.
- **Isotropy+Homogeneity**



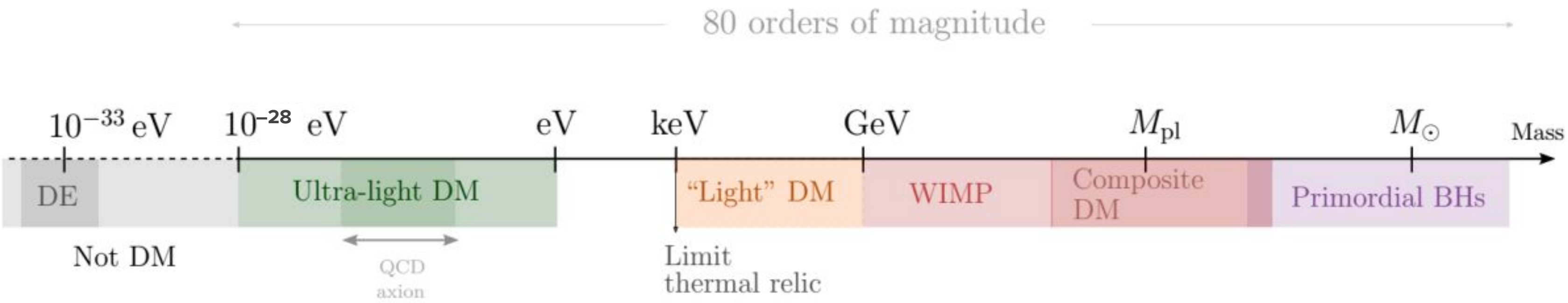
Motivation - Dark Matter

ΛCDM

- **Cosmological constant**
- **DM** → Pressureless and non-interacting fluid.
- **Isotropy+Homogeneity**



DM candidates



Motivation - Ultralight Dark Matter

m

10^{-28}eV — 1eV

- **Classical (free) field approach** $\longrightarrow$ Large occupation number

$$n = \frac{\rho_{dm}}{m} \gg 1 \qquad \rho_{dm} \sim 0.4\frac{\text{GeV}}{\text{cm}^3} \sim 0.01\frac{\text{M}_\odot}{\text{pc}^3}$$

Motivation - Ultralight Dark Matter

m

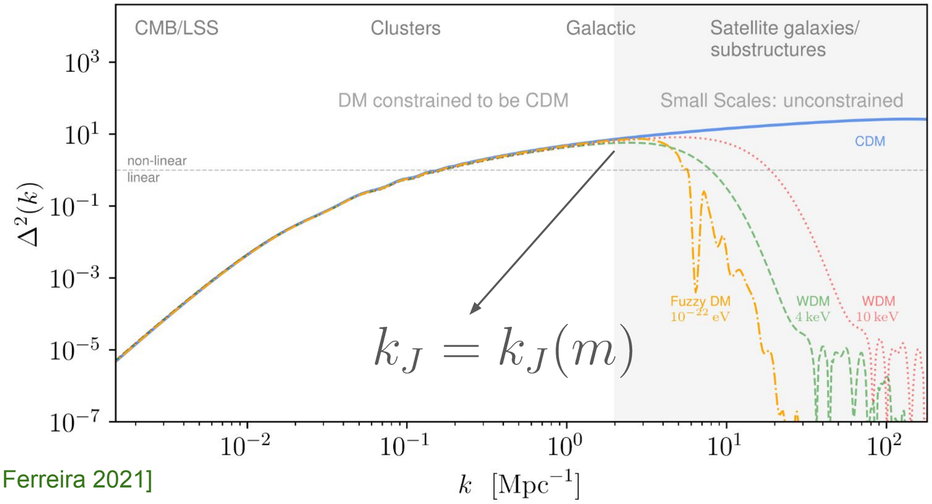
$10^{-28}\mathrm{eV}$ — $1\mathrm{eV}$

- **Classical (free) field approach** $\longrightarrow$ Large occupation number

$$n = \frac{\rho_{dm}}{m} \gg 1 \qquad \rho_{dm} \sim 0.4\frac{\mathrm{GeV}}{\mathrm{cm}^3} \sim 0.01\frac{\mathrm{M}_\odot}{\mathrm{pc}^3}$$

- **Observables highly dependant on boson mass**

- **Characteristic small scale predictions**



[Elisa Ferreira 2021]

Ultralight Dark Matter - Setup

$$S = S_{\mathrm{EH}} + S_{\mathrm{Mat}} + S_{\mathrm{DM}}$$

Spin 0

$$\boxed{S_{\mathrm{DM}} = \int dt\, dx^3 \sqrt{-g}\left[\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m^2}{2}\phi^2\right]}$$

Spin 1

$$\boxed{S_{\mathrm{DM}} = -\int dt\, dx^3 \sqrt{-g}\left[\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{m^2}{2}A^\mu A_\mu\right]}$$

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

Ultralight Dark Matter - Setup

$$S = S_{\mathrm{EH}} + S_{\mathrm{Mat}} + S_{\mathrm{DM}}$$

Spin 0

$$\boxed{S_{\mathrm{DM}} = \int dt\, dx^3 \sqrt{-g}\left[\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m^2}{2}\phi^2\right]}$$

- $\phi(\tau,\vec{x}) = \tilde{\phi}(\tau) + \delta\phi(\tau,\vec{x}) \qquad \delta\phi \ll \tilde{\phi}$

Spin 1

$$\boxed{S_{\mathrm{DM}} = -\int dt\, dx^3 \sqrt{-g}\left[\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{m^2}{2}A^\mu A_\mu\right]}$$

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

- $A^\mu(\tau,\vec{x}) = \tilde{A}^\mu(\tau) + \delta A^\mu(\tau,\vec{x}) \qquad \begin{cases} \tilde{A}^i = A(\tau)\overbrace{\hat{A}}^{\equiv\, const} \\ \delta A^\mu \ll \tilde{A}^\mu \end{cases}$

Ultralight Dark Matter - Setup

$$S = S_{\mathrm{EH}} + S_{\mathrm{Mat}} + S_{\mathrm{DM}}$$

Spin 0

$$\boxed{S_{\mathrm{DM}} = \int dt\, dx^3 \sqrt{-g} \left[\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m^2}{2}\phi^2\right]}$$

- $\phi(\tau,\vec{x}) = \tilde{\phi}(\tau) + \delta\phi(\tau,\vec{x}) \qquad \delta\phi \ll \tilde{\phi}$
- **Isotropic** background: FLRW

$$ds^2 = a^2(-d\tau^2 + \delta_{ij}\, dx^i dx^j)$$

Spin 1

$$\boxed{S_{\mathrm{DM}} = -\int dt\, dx^3 \sqrt{-g} \left[\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{m^2}{2}A^\mu A_\mu\right]}$$

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

- $A^\mu(\tau,\vec{x}) = \tilde{A}^\mu(\tau) + \delta A^\mu(\tau,\vec{x}) \qquad \begin{cases} \tilde{A}^i = A(\tau)\underbrace{\hat{A}}_{\equiv\, const} \\ \delta A^\mu \ll \tilde{A}^\mu \end{cases}$
- **Anisotropic** background: Bianchi I

$$ds^2 = a^2(-d\tau^2 + \gamma_{ij}\, dx^i dx^j) \longrightarrow \boxed{\sigma_{ij} \equiv \frac{1}{2}(\gamma_{ij})'}$$

Shear Tensor

Ultralight Dark Matter - Setup

$$S = S_{\rm EH} + S_{\rm Mat} + S_{\rm DM}$$

Spin 0

$$\boxed{S_{\rm DM} = \int dt\, dx^3 \sqrt{-g} \left[\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m^2}{2}\phi^2\right]}$$

- $\phi(\tau,\vec{x}) = \tilde{\phi}(\tau) + \delta\phi(\tau,\vec{x}) \qquad \delta\phi \ll \tilde{\phi}$
- **Isotropic** background: FLRW

$$ds^2 = a^2(-d\tau^2 + \delta_{ij}\, dx^i dx^j)$$

Spin 1

$$\boxed{S_{\rm DM} = -\int dt\, dx^3 \sqrt{-g} \left[\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{m^2}{2}A^\mu A_\mu\right]}$$

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

- $A^\mu(\tau,\vec{x}) = \tilde{A}^\mu(\tau) + \delta A^\mu(\tau,\vec{x}) \quad \begin{cases} \tilde{A}^i = A(\tau)\underbrace{\hat{A}}_{\equiv\, const} \\ \delta A^\mu \ll \tilde{A}^\mu \end{cases}$
- **Anisotropic** background: Bianchi I

$$ds^2 = a^2(-d\tau^2 + \gamma_{ij}\, dx^i dx^j) \longrightarrow \boxed{\sigma_{ij} \equiv \frac{1}{2}(\gamma_{ij})'}$$

Shear Tensor

- Small anisotropies assumption $\longrightarrow$ $\underbrace{G^\mu{}_\nu}_{|\sigma_{ij}| = \mathcal{O}(1)} = 8\pi G\, \underbrace{T^\mu{}_\nu}_{|\sigma_{ij}| = 0}$

Ultralight Dark Matter - Setup

$$S = S_{\mathrm{EH}} + S_{\mathrm{Mat}} + S_{\mathrm{DM}}$$

Spin 0

$$\boxed{S_{\mathrm{DM}} = \int dt\, dx^3 \sqrt{-g}\left[\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m^2}{2}\phi^2\right]}$$

- $\phi(\tau,\vec{x}) = \tilde{\phi}(\tau) + \delta\phi(\tau,\vec{x}) \qquad \delta\phi \ll \tilde{\phi}$

- **Isotropic** background: FLRW

$$ds^2 = a^2(-d\tau^2 + \delta_{ij}\, dx^i dx^j)$$

- Cosmological constraints

CMB+LSS $\longrightarrow$ $m > 10^{-24}\mathrm{eV}$ [Hložek et al. 2018]

Lyman α $\longrightarrow$ $m > 2\times 10^{-20}\mathrm{eV}$ [Rogers & Peiris 2020]

Spin 1

$$\boxed{S_{\mathrm{DM}} = -\int dt\, dx^3 \sqrt{-g}\left[\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{m^2}{2}A^\mu A_\mu\right]}$$

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

- $A^\mu(\tau,\vec{x}) = \tilde{A}^\mu(\tau) + \delta A^\mu(\tau,\vec{x})$ $\quad \begin{cases} \tilde{A}^i = A(\tau)\hat{A} & \hat{A} \equiv const \\ \delta A^\mu \ll \tilde{A}^\mu \end{cases}$

- **Anisotropic** background: Bianchi I

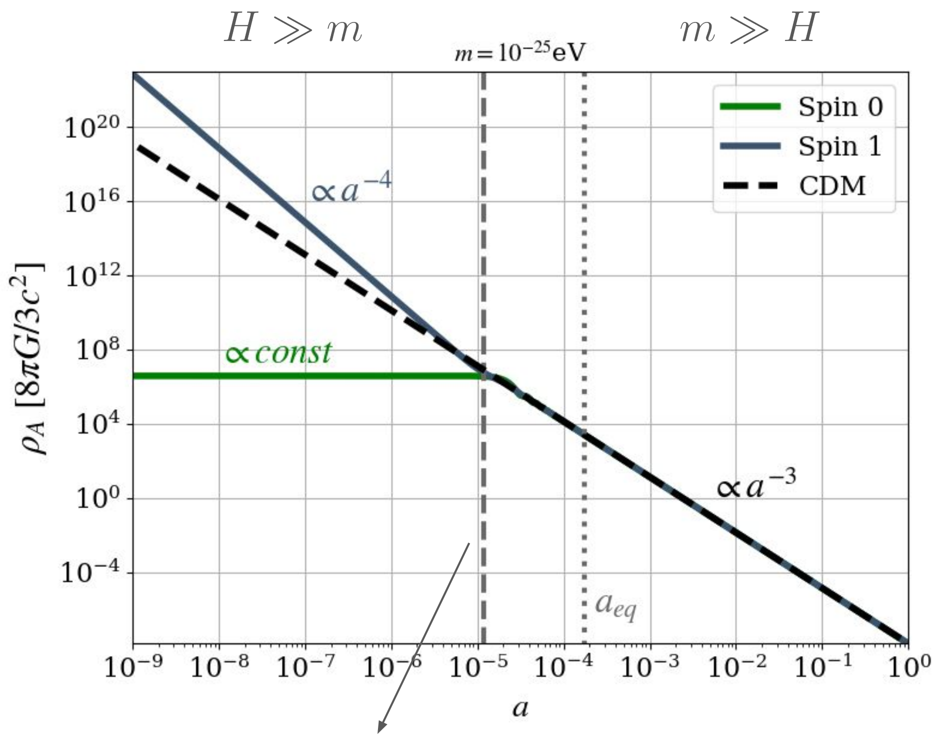
$$ds^2 = a^2(-d\tau^2 + \gamma_{ij}\, dx^i dx^j) \longrightarrow \boxed{\sigma_{ij} \equiv \frac{1}{2}(\gamma_{ij})'}$$

Shear Tensor

- Small anisotropies assumption $\longrightarrow$ $\underbrace{G^\mu{}_\nu}_{|\sigma_{ij}| = \mathcal{O}(1)} = 8\pi G \underbrace{T^\mu{}_\nu}_{|\sigma_{ij}| = 0}$

Background evolution - Fluid variables

$$T^0{}_0 = -\rho$$
$$T^i{}_j = P\delta^i{}_j + \Sigma^i{}_j$$



$H(a_{osc}) = m$

Background evolution - Fluid variables

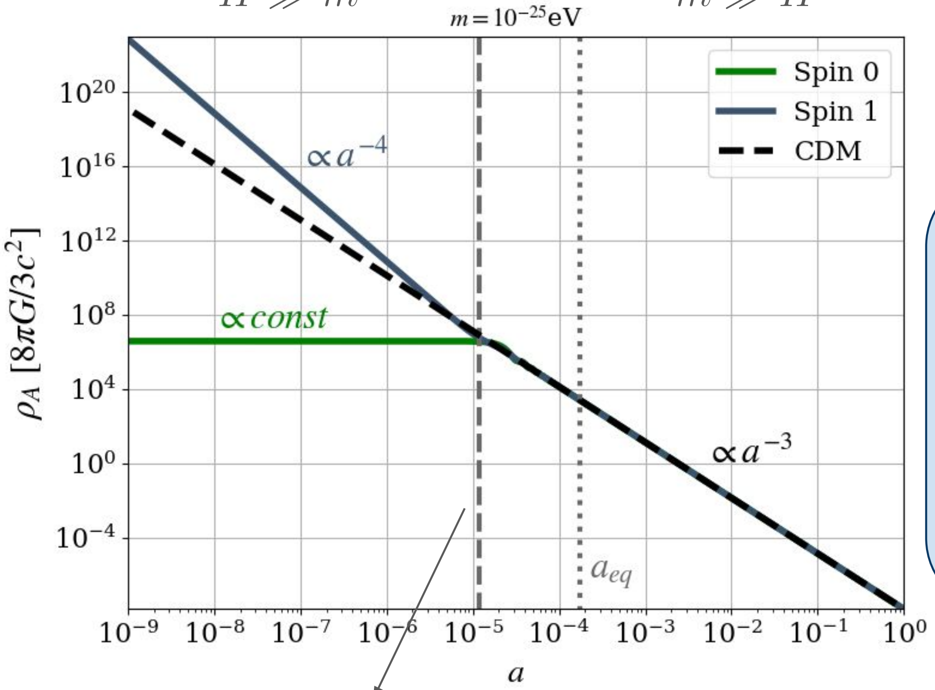
$$T^0{}_0 = -\rho$$
$$T^i{}_j = P\delta^i{}_j + \Sigma^i{}_j$$

Spin 0

$$P_\phi = \begin{cases} -\rho_\phi & a \ll a_{osc} \\ 0 & a \gg a_{osc} \end{cases}$$

$$\Sigma^\phi_{ij} = 0$$

$H \gg m$ $\quad$ $m \gg H$



$H(a_{osc}) = m$

Spin 1

$$P_A = \begin{cases} \frac{\rho_A}{3} & a \ll a_{osc} \\ 0 & a \gg a_{osc} \end{cases}$$

$$\Sigma^A_{ij} = -6P_A(\hat{A}_i\hat{A}_j - \gamma_{ij})$$

Background evolution - Einstein equations for Spin 1

Friedmann Equation

Anisotropies add an **extra component**:

$$H^2 - \frac{1}{6}\frac{\sigma^2}{a^2} = \frac{1}{3m_P^2}\rho$$

$$\boxed{\Omega_\sigma = \frac{\sigma^2}{6\mathcal{H}^2}}$$

Anisotropy abundance

Background evolution - Einstein equations for Spin 1

Friedmann Equation

Anisotropies add an **extra component**:

$$H^2 - \frac{1}{6}\frac{\sigma^2}{a^2} = \frac{1}{3m_P^2}\rho$$

$$\boxed{\Omega_\sigma = \frac{\sigma^2}{6\mathcal{H}^2}}$$

Anisotropy abundance

Einstein Traceless Equation

The Spin 1 field is the only source:

$$(\sigma_{ij})' + 2\mathcal{H}\sigma_{ij} = \frac{a^2}{m_P^2}\Sigma^A_{ij}$$

Formal solution

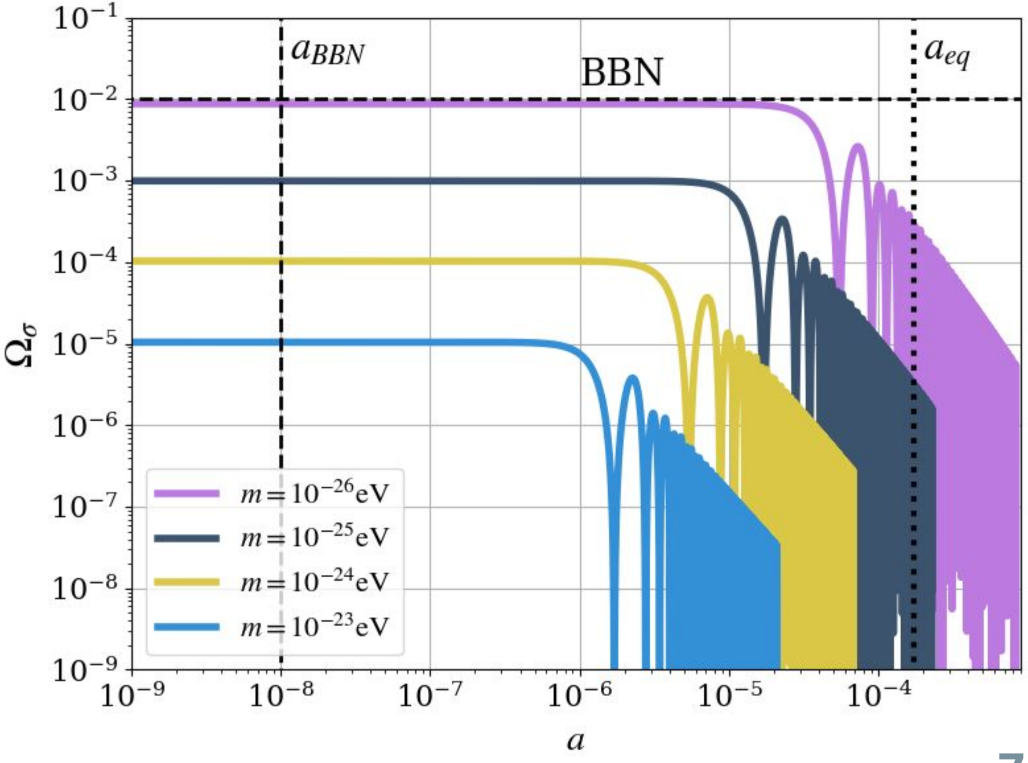
$$\sigma^i{}_j = \frac{1}{a^2}\int_{a_{\rm ini}}^{a} s^3 \frac{\Sigma^i_{Aj}}{\mathcal{H}m_P^2}ds + \text{decaying modes}$$

Background evolution - BBN constraint

Shear tensor dynamics

$$\sigma^i{}_j = \frac{1}{a^2}\int_{a_{\rm ini}}^{a} s^3 \frac{\Sigma^i_{Aj}}{\mathcal{H} m_P^2} ds$$

$$\boxed{\Omega_\sigma \propto \begin{cases} m^{-1} & a < a_{osc} \\ a^{-2} & a_{osc} < a < a_{eq} \\ a^{-3} & a_{eq} < a \end{cases}}$$



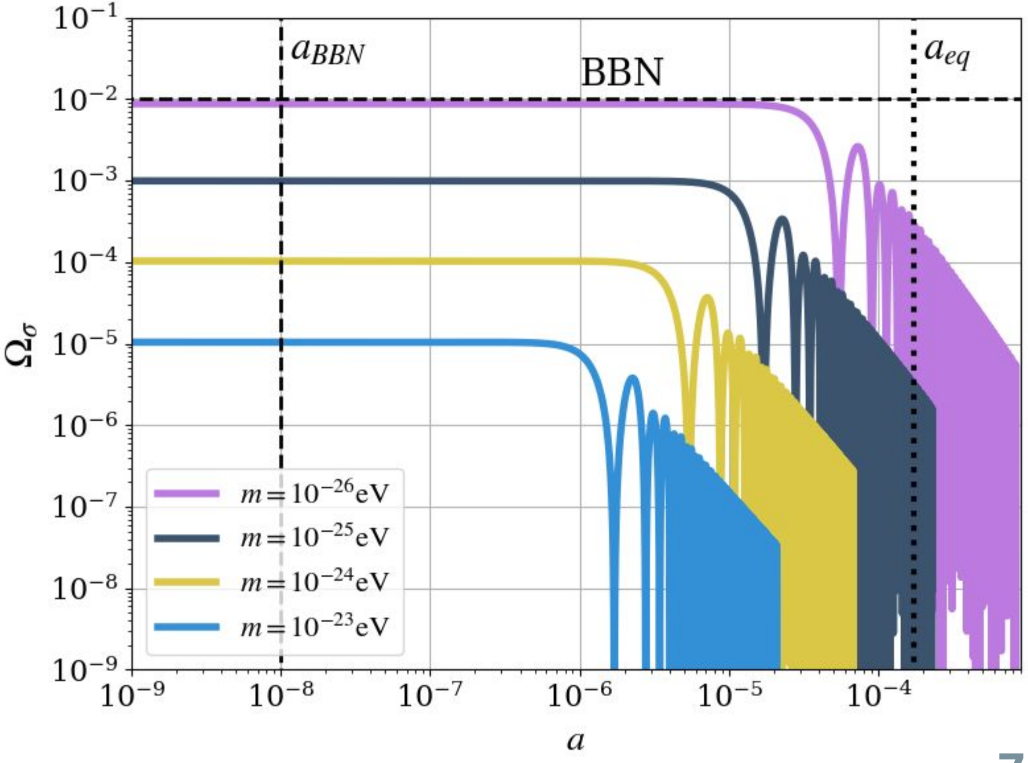
Background evolution - BBN constraint

Anisotropy abundance allowed by BBN: $\Omega_\sigma(a_{\rm BBN}) \leq 10^{-2}$

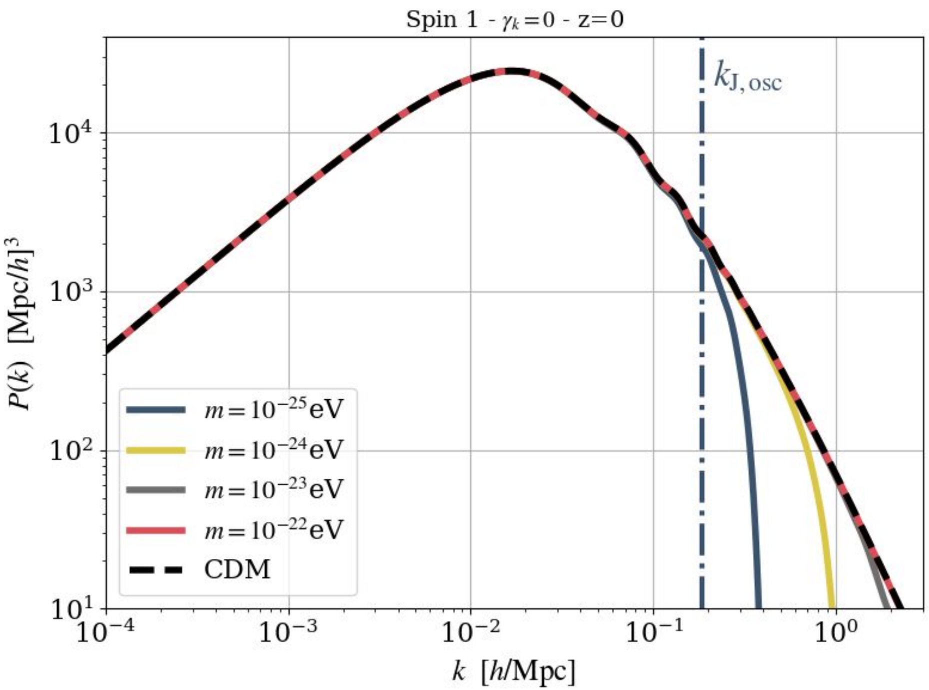
Shear tensor dynamics

$$\sigma^i{}_j = \frac{1}{a^2}\int_{a_{\rm ini}}^{a} s^3 \frac{\Sigma^i_{Aj}}{\mathcal{H} m_P^2} ds$$

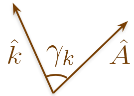
$$\Omega_\sigma \propto \begin{cases} m^{-1} & a < a_{osc} \\ a^{-2} & a_{osc} < a < a_{eq} \\ a^{-3} & a_{eq} < a \end{cases}$$



Jean suppression in ULDM

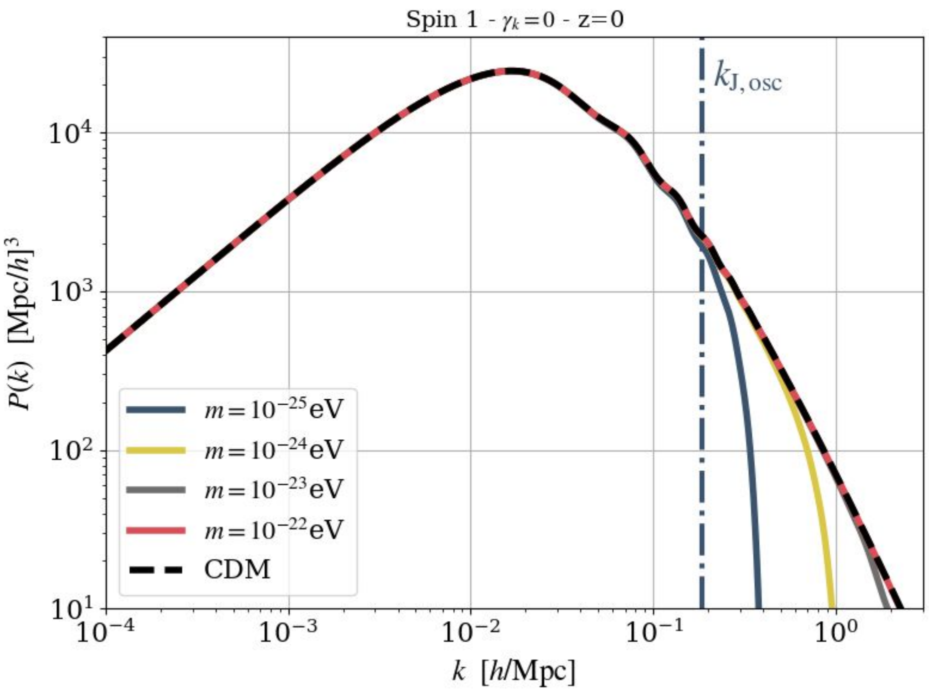


$$\langle \delta(\tau, \vec{k}) \delta^*(\tau, \vec{k}') \rangle = (2\pi)^3 \delta^3(\vec{k} - \vec{k}') \times \begin{cases} P(k) & \text{Spin 0} \\ P(k, \gamma_k) & \text{Spin 1} \end{cases}$$



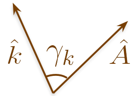
$$\delta = \frac{\delta T^0{}_0}{T^0{}_0}$$

Jean suppression in ULDM



$$\langle \delta(\tau,\vec{k})\delta^*(\tau,\vec{k}')\rangle = (2\pi)^3\delta^3(\vec{k}-\vec{k}')\times \begin{cases} P(k) & \text{Spin 0} \\ P(k,\gamma_k) & \text{Spin 1} \end{cases}$$

$$\delta = \frac{\delta T^0{}_0}{T^0{}_0}$$



Einstein Equations + Equation of motion

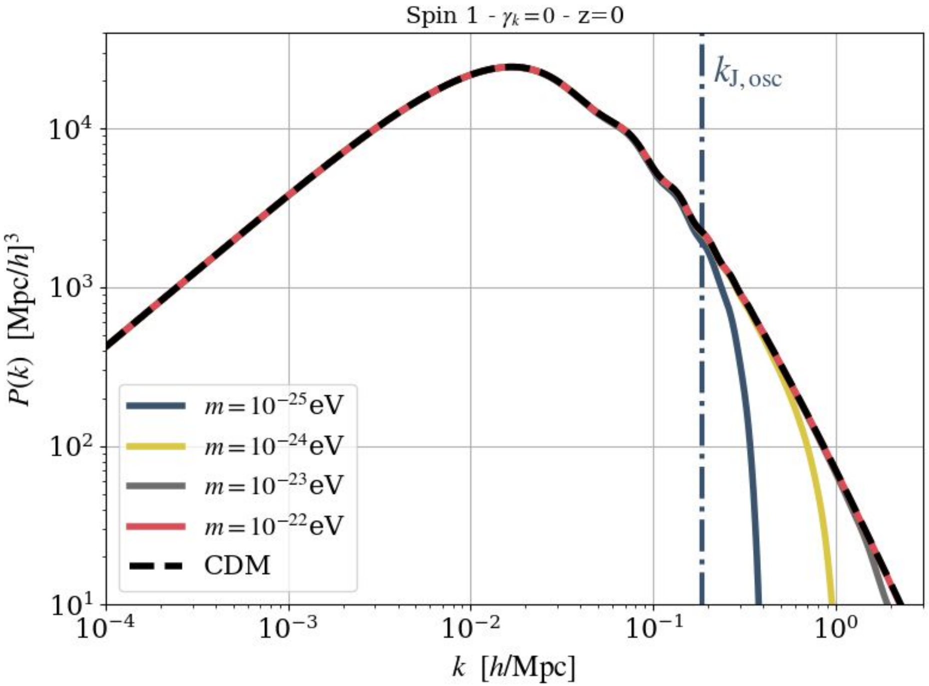
$$\delta'' + \frac{\delta'}{2} + \left(\frac{k^4}{k_J^4} - \frac{3}{2}\right)\delta = \text{Metric coupling} \qquad (\ )' = \frac{d}{d\log a} \equiv \frac{d}{dN}$$

$$k \ll k_J \ \rightarrow \delta \sim a$$

$$k \gg k_J \ \rightarrow \delta \sim oscillates$$

$$\boxed{k_J = a\sqrt{mH}}$$

Jean suppression in ULDM



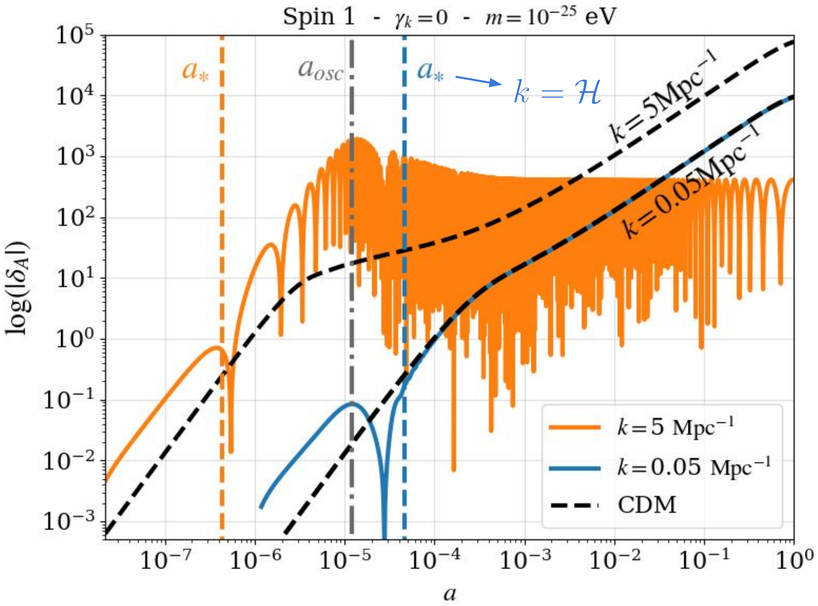
Einstein Equations + Equation of motion

$$\delta'' + \frac{\delta'}{2} + \left(\frac{k^4}{k_J^4} - \frac{3}{2}\right)\delta = \text{Metric coupling} \qquad (\ )' = \frac{d}{d\log a} \equiv \frac{d}{dN}$$

$k \ll k_J \rightarrow \delta \sim a$

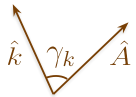
$k \gg k_J \rightarrow \delta \sim oscillates$

$$\boxed{k_J = a\sqrt{mH}}$$

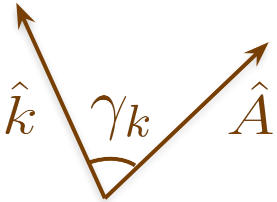


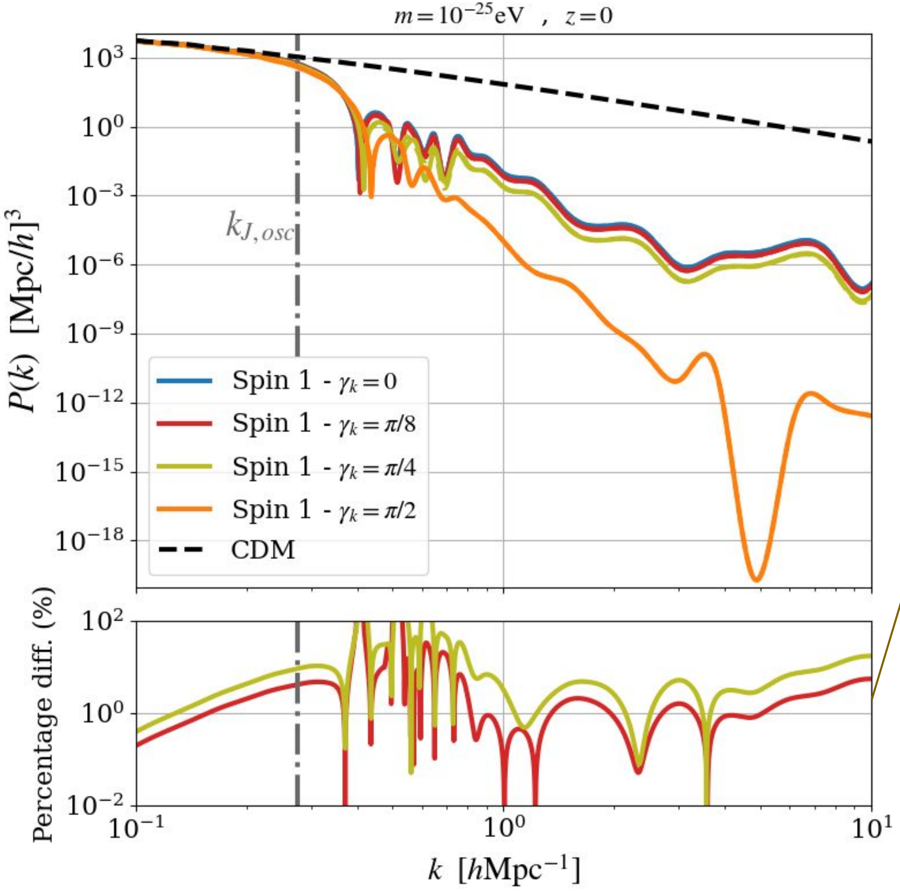
$$\langle \delta(\tau, \vec{k})\delta^*(\tau, \vec{k}')\rangle = (2\pi)^3\delta^3(\vec{k} - \vec{k}') \times \begin{cases} P(k) & \text{Spin 0} \\ P(k, \gamma_k) & \text{Spin 1} \end{cases}$$

$$\delta = \frac{\delta T^0{}_0}{T^0{}_0}$$



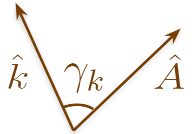
Small scales - Anisotropic imprint

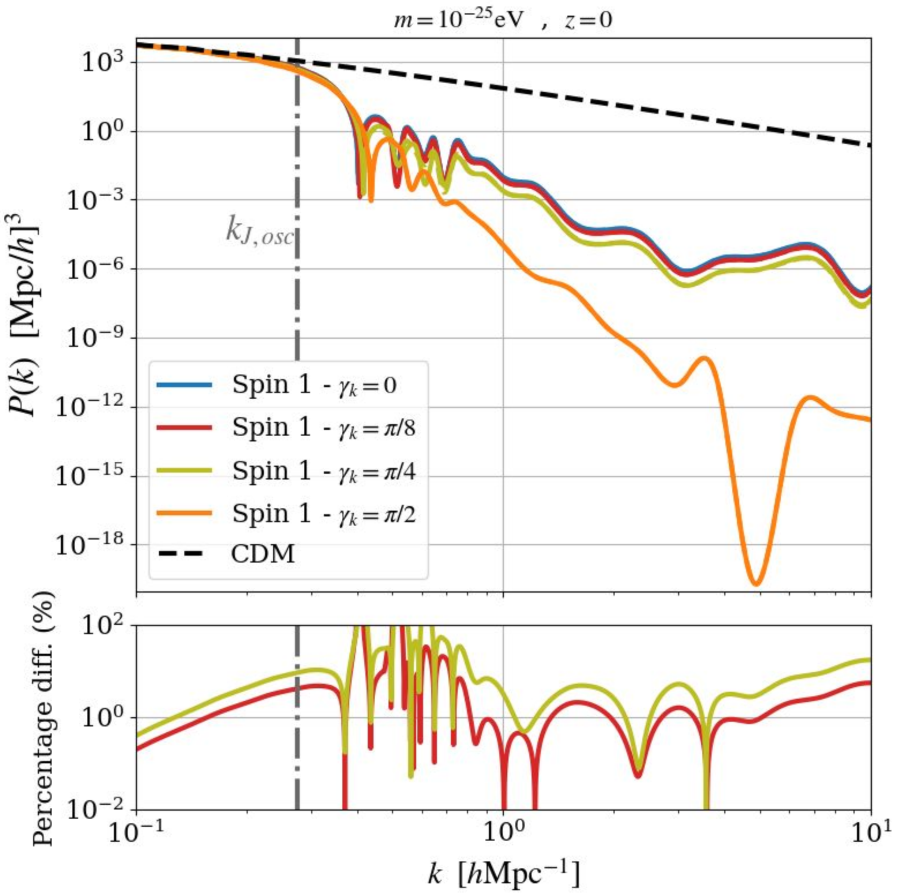




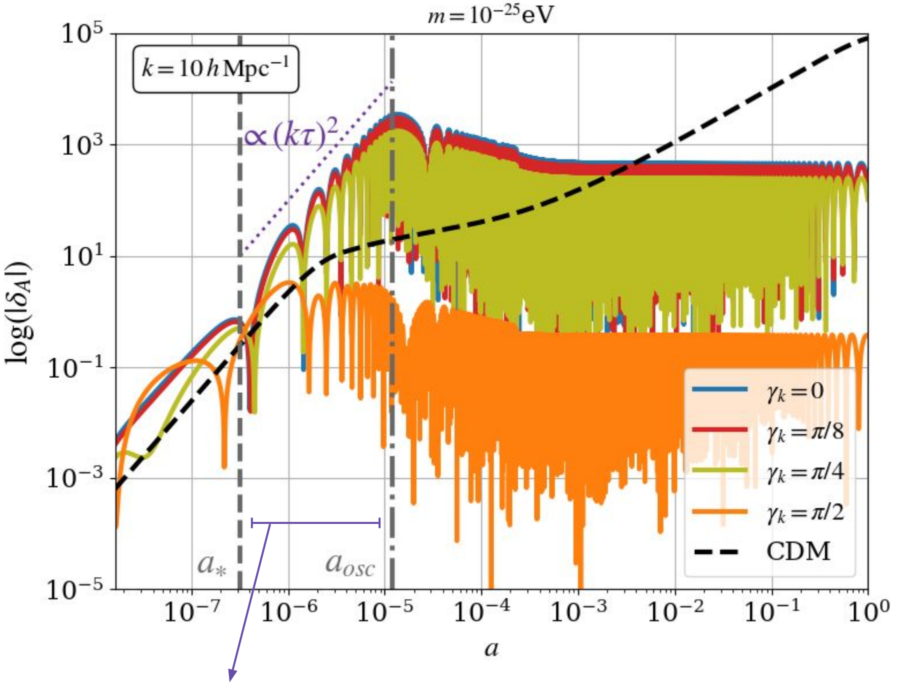
$$P(k, \gamma_k) = P(k, \frac{\pi}{2}) + \left[P(k, 0) - P(k, \frac{\pi}{2})\right] \cos(\gamma_k)^4$$

Small scales - Anisotropic imprint



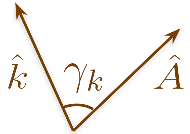


$$P(k, \gamma_k) = P(k, \frac{\pi}{2}) + \left[P(k, 0) - P(k, \frac{\pi}{2})\right] \cos(\gamma_k)^4$$

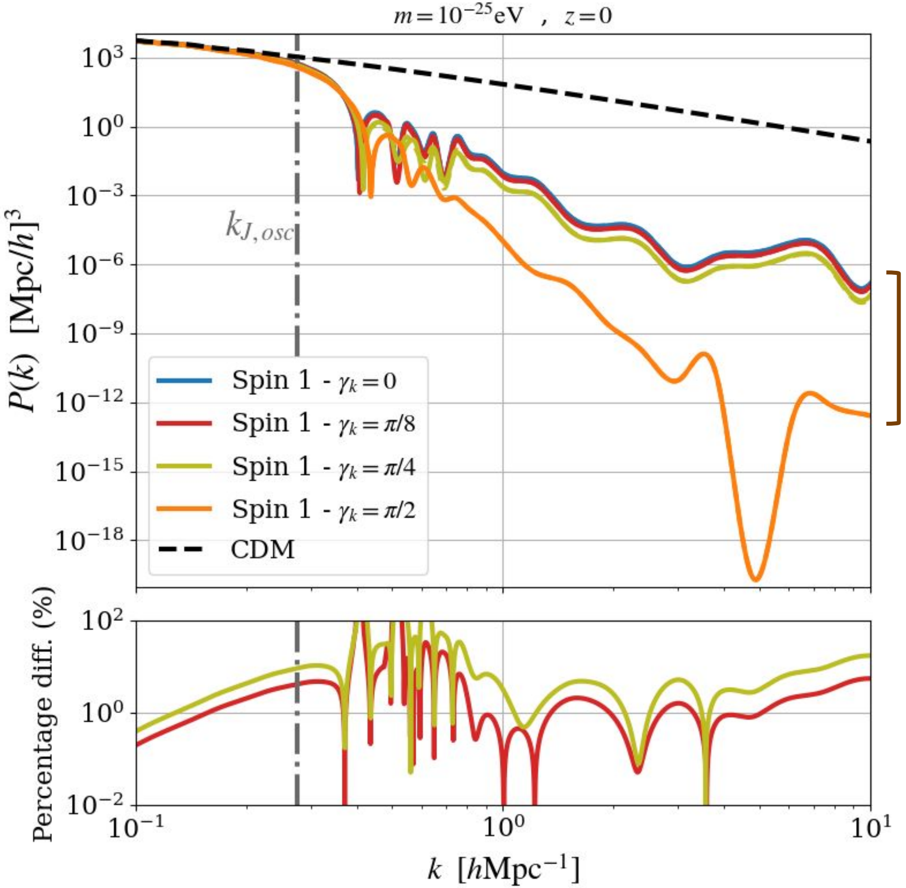


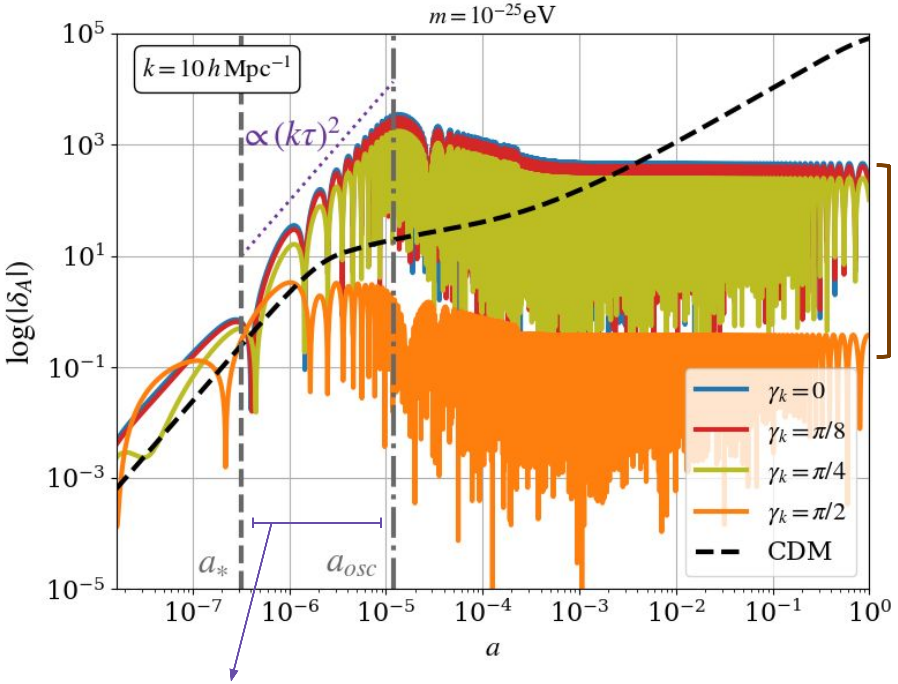
$$\delta_A = \left[28(k\tau)^2 \cos(\gamma_k)^2 - 2\sin(\gamma_k)^2\right] \eta_0$$

Small scales - Anisotropic imprint



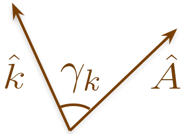
$$P(k,\gamma_k) = P(k,\frac{\pi}{2}) + \left[P(k,0) - P(k,\frac{\pi}{2})\right]\cos(\gamma_k)^4$$



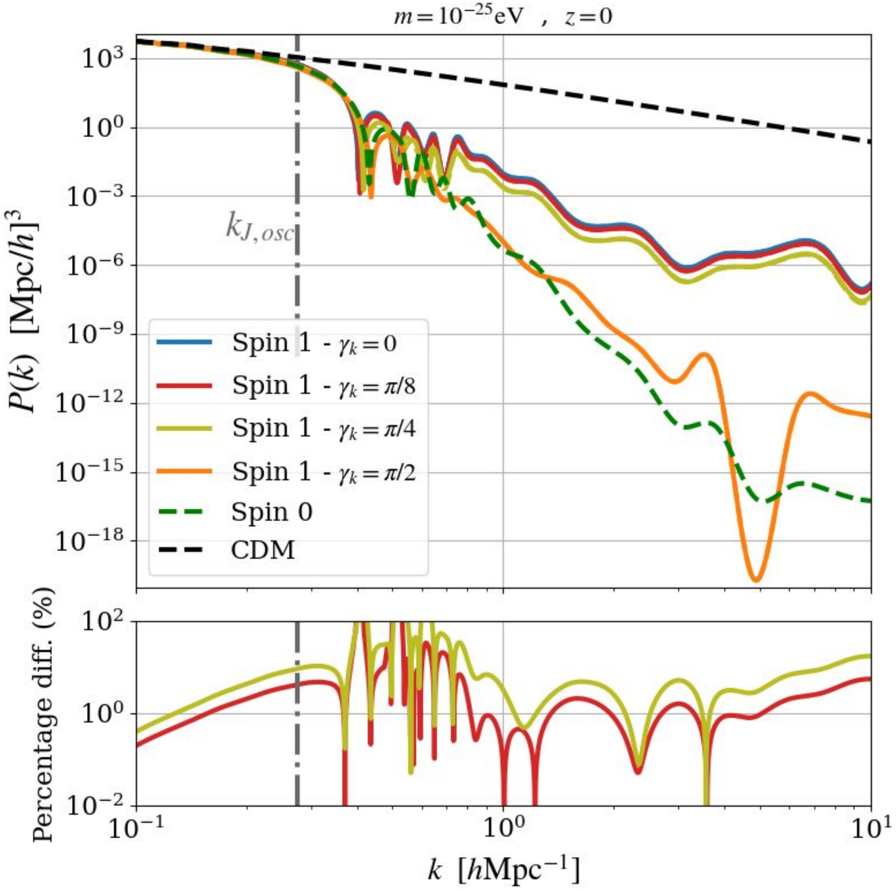


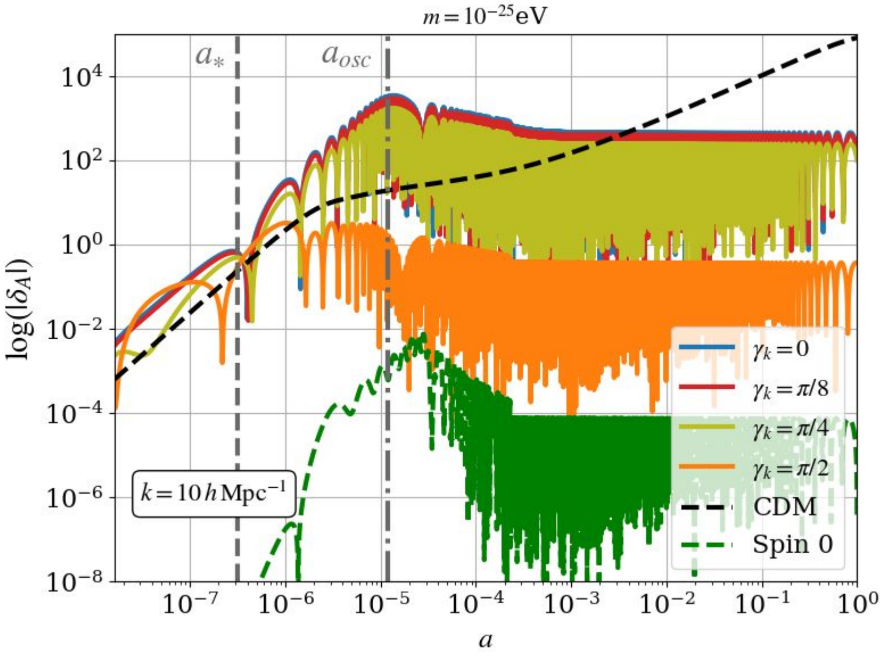
$$\delta_A = \left[28(k\tau)^2\cos(\gamma_k)^2 - 2\sin(\gamma_k)^2\right]\eta_0$$

Small scales - DM spin comparison



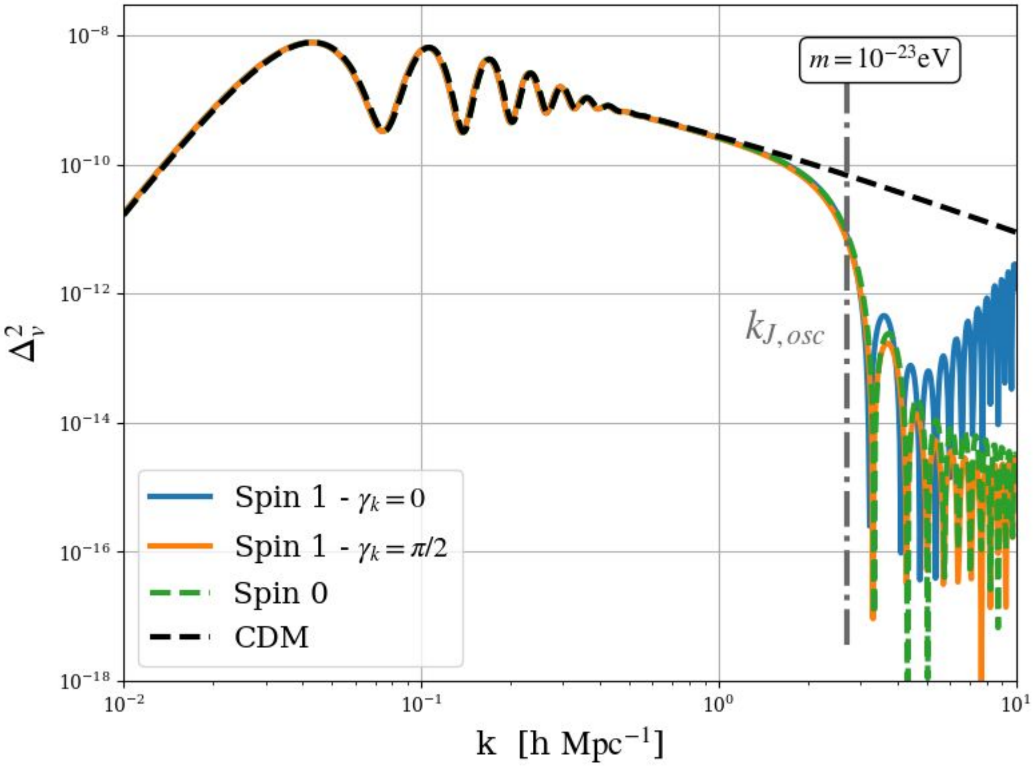
$$P(k,\gamma_k) = P(k,\frac{\pi}{2}) + \left[P(k,0) - P(k,\frac{\pi}{2})\right]\cos(\gamma_k)^4$$





Velocity Invariant Power Spectrum

$$(\rho + P)\theta = \hat{k}^i \delta T^0{}_i$$



Velocity Power Spectrum:

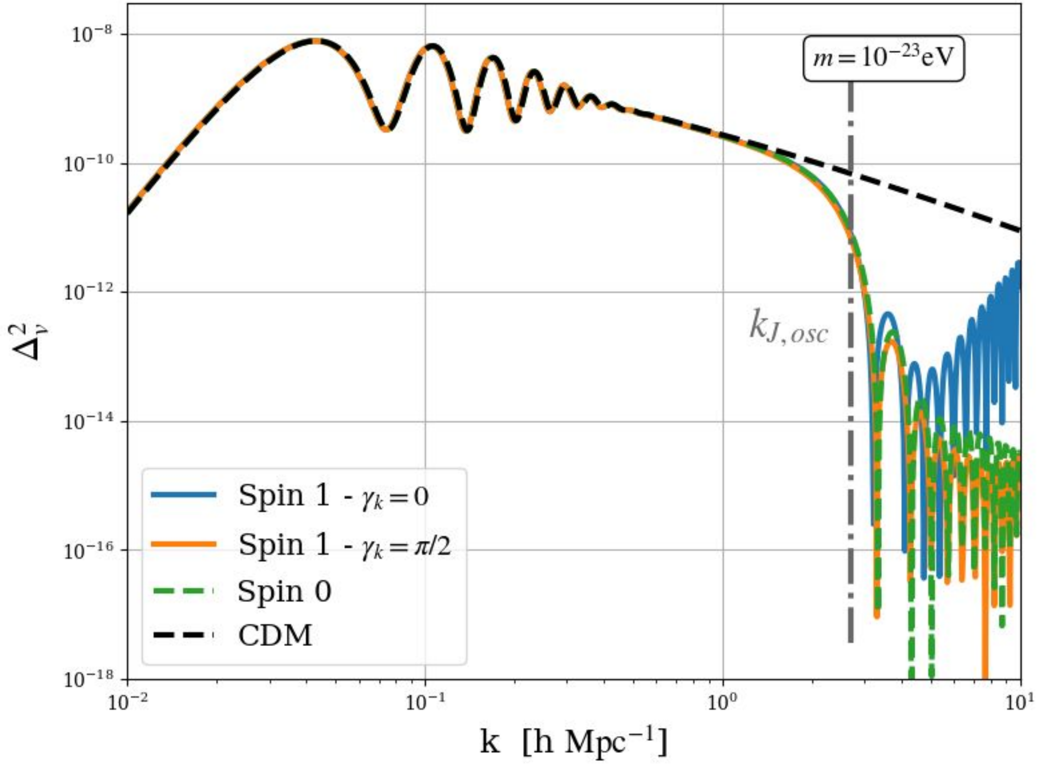
$$\langle \vec{v}_{bd}^{\,2} \rangle = \int \frac{dk}{k} \Delta_v^2(k)$$

Relative Velocity
(gauge invariant)

$$\boxed{\Delta_v^2(k) = \Delta_\zeta^2(k) \left(\frac{\theta_b - \theta_d}{k} \right)^2}$$

Velocity Invariant Power Spectrum

$$(\rho + P)\theta = \hat{k}^i \delta T^0{}_i$$



Velocity Power Spectrum:

$$\langle \vec{v}_{bd}^{\,2} \rangle = \int \frac{dk}{k} \Delta_v^2(k)$$

Relative Velocity
(gauge invariant)

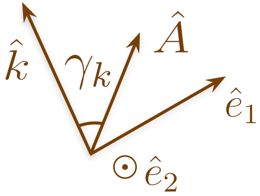
$$\Delta_v^2(k) = \Delta_\zeta^2(k) \left(\frac{\theta_b - \theta_d}{k} \right)^2$$

Relevant for 21-cm

Initial conditions for 21cm simulations → For spin 0 could probe $m < 10^{-18}$eV

[Marsh PRD 2015, Hotinli et al PRD 2022]

SVT decomposition



Isotropic background

$$\delta G^{\mu}{}_{\nu} = \frac{1}{m_P^2}\delta T^{\mu}{}_{\nu}$$

*degrees of freedom

Scalar (4*)

Decoupled

Vector (4)

Tensor (2)

Anisotropic background

$$\delta G^{\mu}{}_{\nu} = \frac{1}{m_P^2}\delta T^{\mu}{}_{\nu}$$

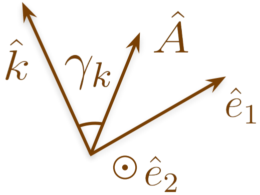
Production of GW

Scalar (4)

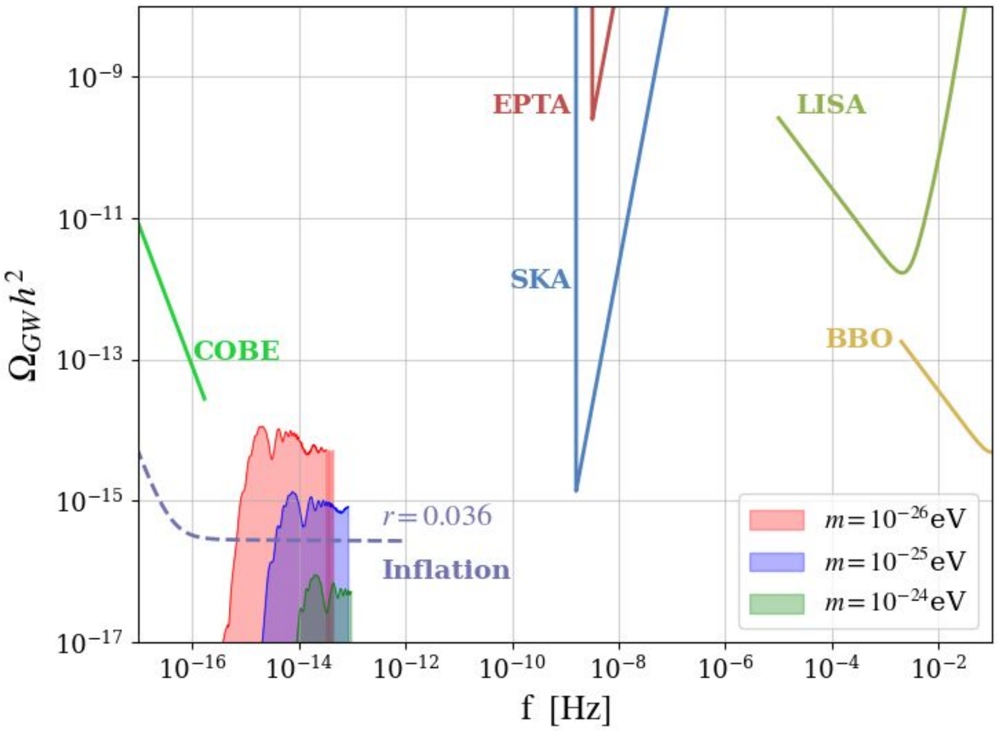
Coupled $\begin{cases} \sigma_{ij} \\ A^{\mu} \end{cases}$

Vector (4) $\rightleftarrows$ **Tensor (2)**

Gravitational wave abundance - Spin 1



Work in progress

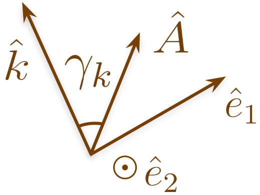


Tensor perturbations

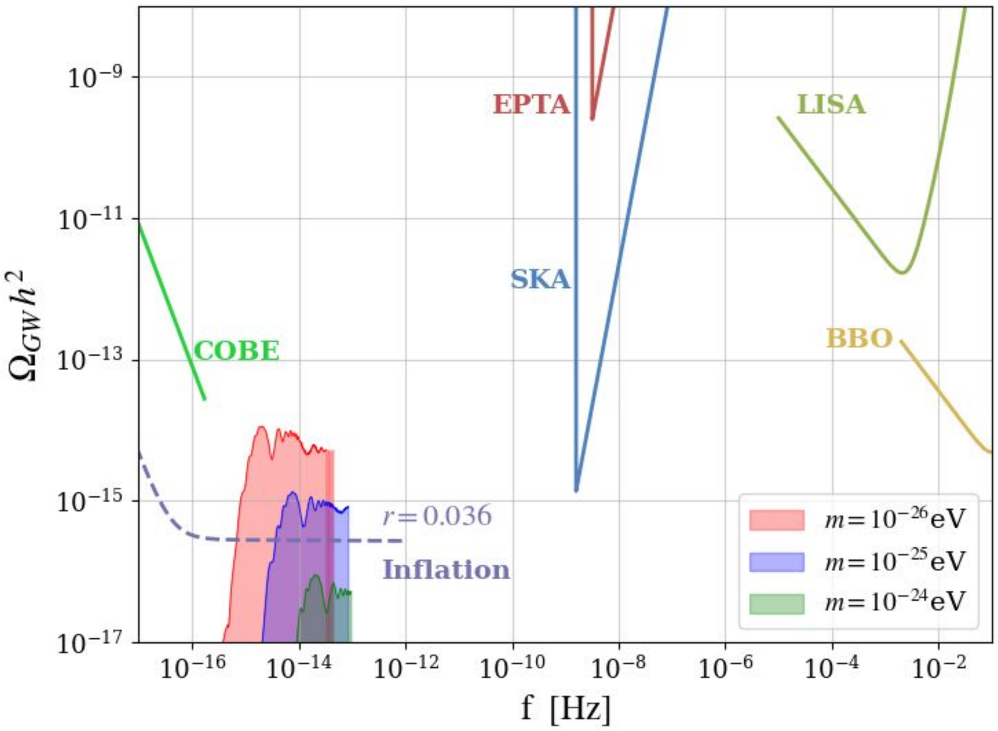
$$\delta g_{ij} = a^2[\hat{k}_i \hat{k}_j (h + 6\eta) - 2\frac{\sigma_{ij}}{\mathcal{H}}\eta + 2E_{ij}]$$

$$E_{ij} = (\hat{e}^1_i\, \hat{e}^1_j - \hat{e}^2_i\, \hat{e}^2_j)\, h_+ + (\hat{e}^1_i\, \hat{e}^2_j + \hat{e}^2_i\, \hat{e}^1_j)\, h_\times$$

Gravitational wave abundance - Spin 1



Work in progress



Tensor perturbations

$$\delta g_{ij} = a^2[\hat{k}_i \hat{k}_j (h + 6\eta) - 2\frac{\sigma_{ij}}{\mathcal{H}}\eta + 2E_{ij}]$$

$$E_{ij} = (\hat{e}^1_i\, \hat{e}^1_j - \hat{e}^2_i\, \hat{e}^2_j)\, h_+ + (\hat{e}^1_i\, \hat{e}^2_j + \hat{e}^2_i\, \hat{e}^1_j)\, h_\times$$

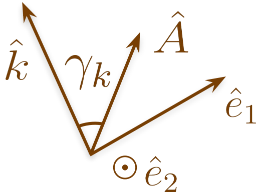
Einstein tensor equation

$$h''_+ + 2\mathcal{H}h'_+ + \left[k^2 - \frac{6a^2}{m_P^2}\sin(\gamma_k)P_A\right] h_+ = S(\delta A^\mu, \eta, h)$$

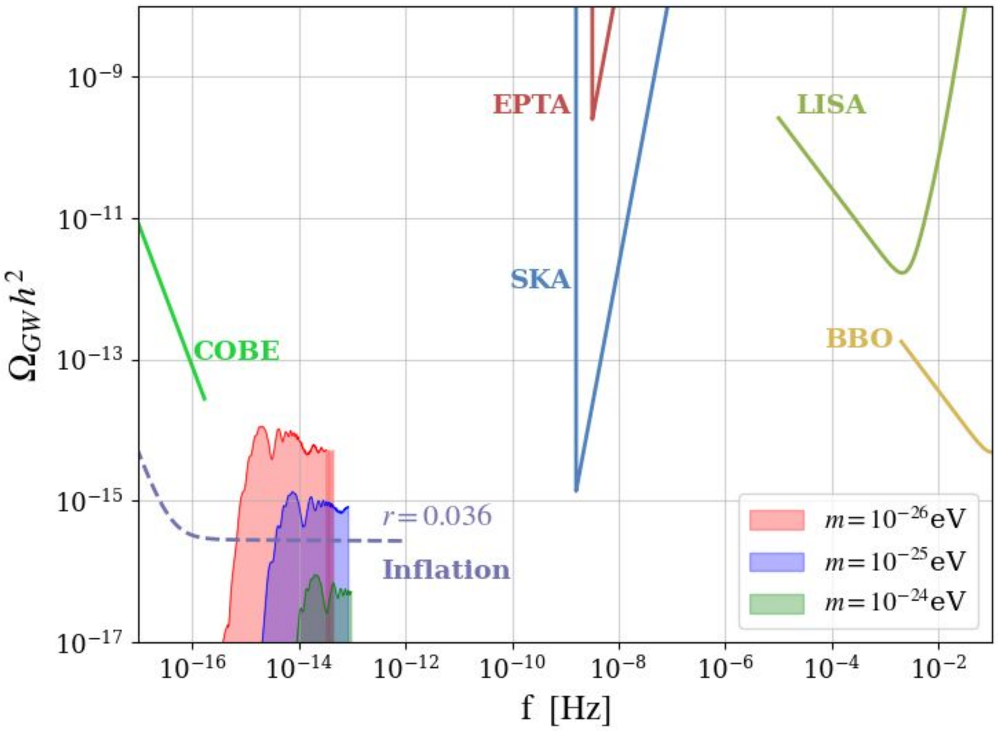
linear in δA^μ

$$h_+(\tau_{ini}) = h'_+(\tau_{ini}) = 0$$

Gravitational wave abundance - Spin 1



Work in progress



Tensor perturbations

$$\delta g_{ij} = a^2[\hat{k}_i\hat{k}_j(h+6\eta) - 2\frac{\sigma_{ij}}{\mathcal{H}}\eta + 2E_{ij}]$$

$$E_{ij} = (\hat{e}^1_i\,\hat{e}^1_j - \hat{e}^2_i\,\hat{e}^2_j)\,h_+ + (\hat{e}^1_i\,\hat{e}^2_j + \hat{e}^2_i\,\hat{e}^1_j)\,h_\times$$

Einstein tensor equation

$$h''_+ + 2\mathcal{H}h'_+ + \left[k^2 - \frac{6a^2}{m_P^2}\sin(\gamma_k)P_A\right]h_+ = S(\delta A^\mu, \eta, h)$$

linear in δA^μ

$$h_+(\tau_{ini}) = h'_+(\tau_{ini}) = 0$$

Abundance of cosmological GW

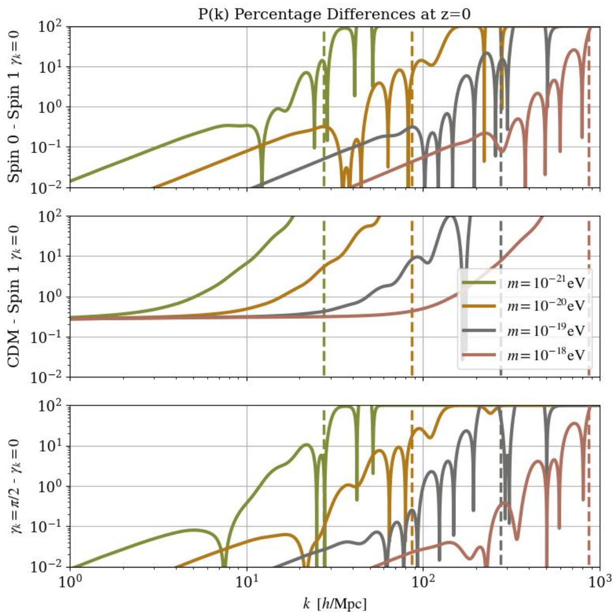
$$\Omega_{GW}(k) = \frac{1}{\rho_c}\frac{d^2\rho_{GW}}{d\Omega\,d\log(k)} = \frac{k^3\,h'^2_+}{48\pi^3 H_0^2}$$

Conclusions - Cosmological tests

- Different predictions at **small scales** $\longrightarrow k_J$

- **Spin 1** has anisotropic suppression in the $P(k, \gamma_k)$

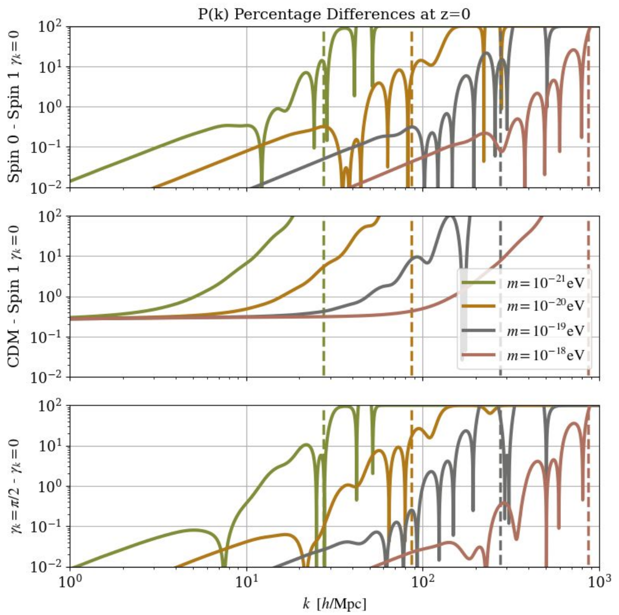
- Production of $\Omega_{GW}(k)$ in spin 1 model



References

- T. F. Chase, D. L. Nacir - "Ultralight vector dark matter, anisotropies, and cosmological adiabatic modes" - *Phys. Rev. D* (2024)
- T. F. Chase, et. al. - "Cosmological perturbations with ultralight vector dark matter fields: Numerical implementation in class" - *Phys. Rev. D* (2025)

Conclusions - Cosmological tests



- Different predictions at **small scales** $\longrightarrow k_J$
- **Spin 1** has anisotropic suppression in the $P(k, \gamma_k)$
- Production of $\Omega_{GW}(k)$ in spin 1 model

Future work

- Compute $\mathcal{C}_\ell \rightarrow$ extend the code to account for anisotropies
- Simulations with **anisotropic** $P(k, \gamma_k)$

References

- T. F. Chase, D. L. Nacir - “Ultralight vector dark matter, anisotropies, and cosmological adiabatic modes” - *Phys. Rev. D* (2024)
- T. F. Chase, et. al. - “Cosmological perturbations with ultralight vector dark matter fields: Numerical implementation in class” - *Phys. Rev. D* (2025)

Extra slides

Linear Regime - Scalar Sector

Cosmological perturbation theory: $$Q(t,\vec{x}) = Q(t) + \delta Q(t,\vec{x})$$

$$\boxed{\delta G^{\mu}{}_{\nu} = 8\pi G\,\delta T^{\mu}{}_{\nu}}$$

$$\left.\begin{aligned} \delta T^{\mu}{}_{\nu} &= \delta T^{\mu}{}_{\nu}(\delta\rho, \delta P, \delta\Sigma^{i}{}_{j}) \\ \delta G^{\mu}{}_{\nu} &= \delta G^{\mu}{}_{\nu}(\delta g_{ij}) \end{aligned}\right\}$$

For calculating cosmological observables such as CMB, P(k), etc.

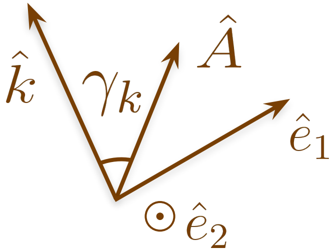
Synchronous gauge:

$$\delta g_{ij} = a^2\left[-2\overbrace{\left(\gamma_{ij} + \frac{\sigma_{ij}}{\mathcal{H}}\right)}^{\delta_{ij}\text{ for spin 0}}\eta + \hat{k}_i\hat{k}_j(h+6\eta)\right]$$

$$\left\{\begin{matrix} h \\ \eta \end{matrix}\right. \longrightarrow \text{Scalar metric perturbations}$$

[Pereira, Pitrou & Uzan 2007]

Spin 1 perturbations: $$\{\hat{e}_1, \hat{e}_2, \hat{k}\}$$



Initial Conditions - Outside the horizon + Early times

Initial conditions
ΛCDM

[Ma-Bertschinger 1995]

$\longrightarrow$

Radiation era
+
Outside horizon
$k\tau \ll 1$

$\longrightarrow$

Radiation (γ)
+
Metric

$$\delta G^0{}_0 = -8\pi G\, \delta\rho$$
$$\delta G^i{}_j = 8\pi G\, (\delta P \delta^i{}_j + \delta\Sigma^i{}_j)$$

+

Photon Fluid equation

$$\xrightarrow[\text{Radiation domination}]{k\tau \ll 1}$$

$$\begin{cases} \eta = \eta_0 + \mathcal{O}(k^2\tau^2) \\ h = \mathcal{O}(k^2\tau^2) \\ \delta\rho_\gamma = \mathcal{O}(k^2\tau^2) \\ \hat{k}^i T^0_{\gamma\, i} = \mathcal{O}(k^4\tau^3) \end{cases}$$

$$\hat{k}^i \delta G^0{}_i = 8\pi G\, \hat{k}^i \delta T^0{}_i \longrightarrow$$

σ_{ij} important **outside the horizon**

Initial Conditions - Outside the horizon + Early times

$$\boxed{\hat{k}^i \delta G^0{}_i = 8\pi G\, \hat{k}^i \delta T^0{}_i} \rightarrow \underbrace{\cancel{k^2 \eta'}}_{\mathcal{O}(k^4)} + \underbrace{\frac{3}{2} \cancel{k^2 \sigma_\parallel} \eta}_{\mathcal{O}(k^2)} = \frac{a^2}{2m_P^2} \hat{k}^i \left[\underbrace{\cancel{\delta T_\gamma{}^0{}_i}}_{\mathcal{O}(k^4)} + \underbrace{\cancel{\delta T_A{}^0{}_i}}_{\mathcal{O}(k^2)} \right]$$

$$\eta = \eta_0 + \mathcal{O}(k^2 \tau^2) \longleftarrow \mathcal{O}(k^4)$$

$$\boxed{\sigma_\parallel = \hat{k}^i \hat{k}^j \sigma_{ij}}$$

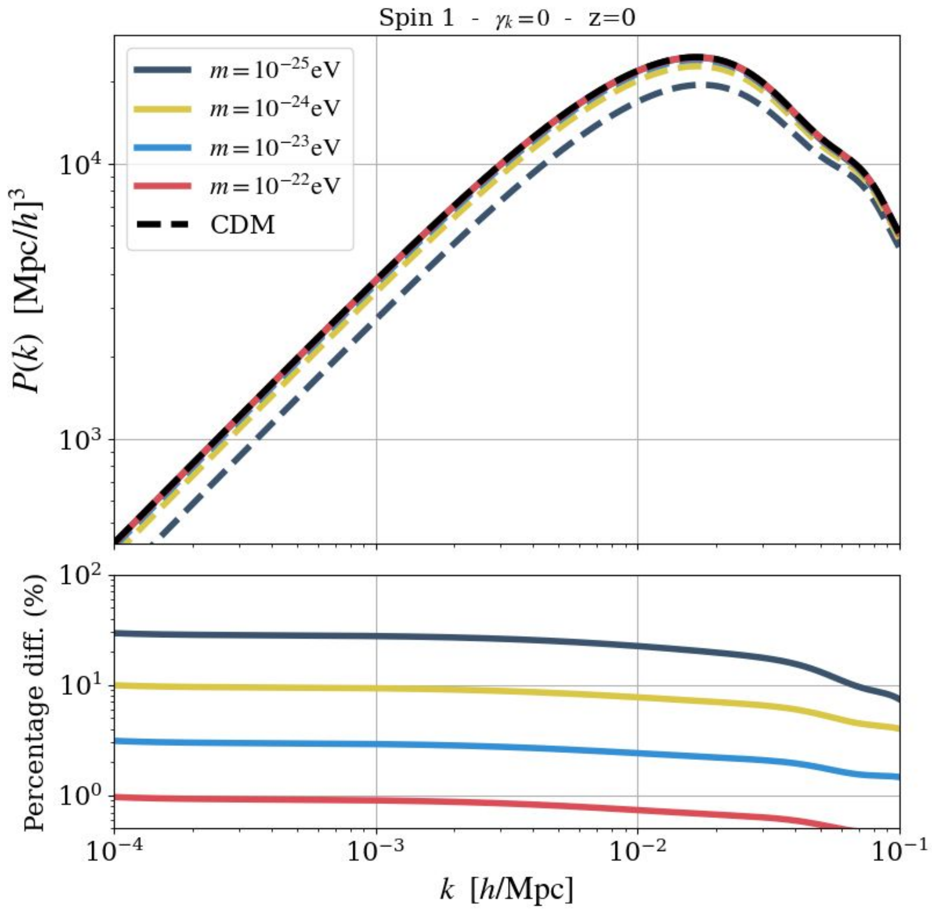
Dominates when $m < 10^{-22}\mathrm{eV}\,(0.1/(\mathrm{k}\tau_\mathrm{i}))^4$

Attractor Solution + Adiabatic I-C $\longrightarrow$ $\frac{a^2}{2m_P^2} \hat{k}^i \delta T_A{}^0{}_i = \frac{3}{2} k^2 \sigma_\parallel \eta_0$

Exactly cancels the shear's contribution

Summary: If the model is evolved in FRW, one would have **big infrared contributions** from the vector field at early times.

Big scales - Metric **shear** impact - Spin 1



FRW

$$k^2\eta' = \frac{a^2}{2m_P^2}\hat{k}^i\delta T^0{}_i$$

Bianchi I

$$k^2\eta' - \frac{3}{2}k^2\sigma_\parallel\eta = \frac{a^2}{2m_P^2}\hat{k}^i\delta T^0{}_i$$

Numerical implementation in CLASS - Background

$$A_i'' + m^2 a^2 A_i = 0 \longrightarrow \begin{cases} \theta' = \sin(\theta) + 2\dfrac{m}{H} \\ \Omega_A' = 3(w_T - w_A)\Omega_A \end{cases}$$

with

$$w_T = P_{tot}/\rho_{tot}$$

$$w_A = P_A/\rho_A$$

$$\boxed{\begin{aligned} \vec{A} &= \sqrt{\rho_A}\, m^{-1} \sin(\theta(\tau)/2)\hat{A} \\ \vec{A}' &= \sqrt{\rho_A} \cos(\theta(\tau)/2)\hat{A} \end{aligned}}$$

Averaging procedure

$$\sin(\theta) \to \frac{1}{2}(1 - \tanh(\theta^2 - \theta_0^2))\sin(\theta)$$

$$\cos(\theta) \to \frac{1}{2}(1 - \tanh(\theta^2 - \theta_0^2))\cos(\theta)$$

Fluid variables

$$\begin{cases} \rho_A \longrightarrow \text{Dynamical variable} \\ P_A = \dfrac{1}{3}\cos(\theta)\rho_A \end{cases}$$

Numerical implementation in CLASS - Perturbations

$$\delta\vec{A} = \delta A_L \hat{k} + \delta A_T \hat{e}_1$$

$$\begin{cases} \delta A_L = \sqrt{\rho_A}\, m^{-1} \left[\sin(\theta/2)\delta_{L,0} + \cos(\theta/2)\delta_{L,1}\right] \\ \delta A_L' = \sqrt{\rho_A} \left[\cos(\theta/2)\delta_{L,0} - \sin(\theta/2)\delta_{L,1}\right] \end{cases}$$

$\{\delta_{L,0}, \delta_{L,1}\}$

$\downarrow$

Oscillations in k and growing solutions

$$\begin{cases} \delta A_T = \sqrt{\rho_A}\, m^{-1} \left[\sin(\theta/2)\delta_{T,0} + \cos(\theta/2)\delta_{T,1}\right] \\ \delta A_T' = \sqrt{\rho_A}\sqrt{1 + \frac{k^2}{m^2 a^2}} \left[\cos(\theta/2)\delta_{T,0} - \sin(\theta/2)\delta_{T,1}\right] \end{cases}$$

$\{\delta_{T,0}, \delta_{T,1}\}$

$\downarrow$

Oscillations in k