Non-Lorentzian Gravity and Torsion

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Outline

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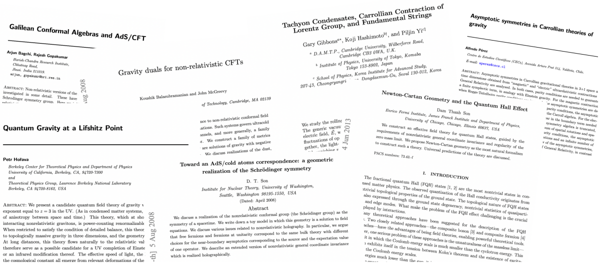
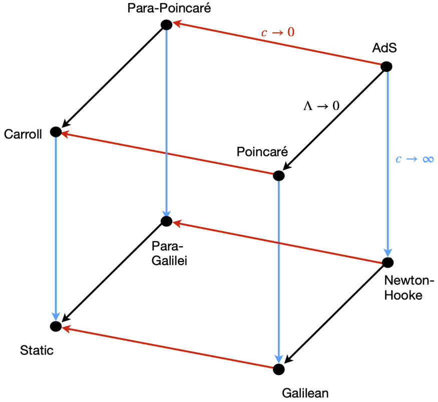
Introduction

- Non- and ultra-relativistic (**Non-Lorentzian**) symmetries have received a growing interest due to their utilities in diverse physical theories.
- Non-relativistic (NR) theories have been useful to approach strongly coupled condensed matter systems as well as NR effective field theories.
- Ultra-relativistic (UR) symmetries have found recent applications in the study of tachyon condensation, warped conformal field theories, tensionless (super)strings, asymptotic symmetries, flat holography, and black hole horizon.



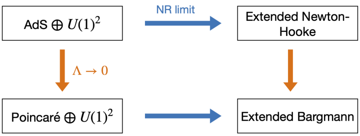
Introduction

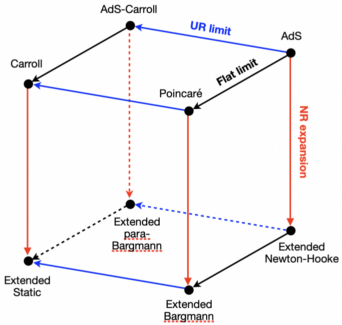
- A Non-Lorentzian theory can be obtained by a suitable **limiting process** from a relativistic theory.
- In particular, through this talk, the NR and UR limit corresponds to the limit in which $c \to \infty$ and $c \to 0$, respectively.



Introduction

- In $D = 2+1$, the Chern-Simons (CS) formalism allows us to construct NR gravity actions whose underlying symmetry can be obtained as a NR limit of a relativistic algebra.
- In the limit $c \to \infty$ there might appear infinities and degeneracy in the contraction of the original Lagrangian.





[P. Concha, D. Pino, L. Ravera, E. Rodríguez, JHEP 01 (2024) 040]

Introduction

- The inclusion of a **non-vanishing torsion** in a NR setting requires a more subtle treatment. Indeed, Torsional Newton-Cartan gravity emerges from gauging a conformal extension of the Bargmann algebra, known as the Schrödinger algebra. Within this framework, the time-component of the torsion is non zero.
- To our knowledge, in the Carrollian case, there is no existing formulation in the literature that simultaneously allows for both torsion and non-metricity within a unified geometric structure.
- Here, we propose a unified approach to introduce torsion consistently in both NR and UR regimes by exploring the non-Lorentzian limits of the three-dimensional **Mielke-Baekler gravity** model formulated in the Chern-Simons framework.

[E.A. Bergshoeff, J. Hartong, J. Rosseel, Class. Quant. Grav 32 (2015) 135017]
[P. Concha, N. Merino, E. Rodríguez, Eur. Phys. J. C 84 (2024) 407]
[P. Concha, N. Merino, L. Ravera, E. Rodríguez, coming soon]

Outline

The Mielke-Baekler gravity formalism

A three-dimensional gravity model that is characterized by a non-vanishing torsion was proposed by Mielke and Baekler (MB). The MB Lagrangian is the most general three-form constructed with the dreibein one-form E^A and the spin connection one-form W^A.

$$L_{\text{MB}}[E^A, W^A] = \sigma_0 L_0[E^A] + \sigma_1 L_1[E^A, W^A] + \sigma_2 L_2[W^A] + \sigma_3 L_3[E^A, W^A]\,,$$

where σ_i are independent constants and

$$\begin{aligned} L_0[E^A] &= \frac{1}{3}\epsilon_{ABC}E^A E^B E^C\,, \\ L_1[E^A, W^A] &= 2E_A R^A\,, \\ L_2[W^A] &= W^A dW_A + \frac{1}{3}\epsilon^{ABC}W_A W_B W_C\,, \\ L_3[E^A, W^A] &= E_A T^A\,. \end{aligned}$$

Here

$$\begin{aligned} R^A &= dW^A + \frac{1}{2}\epsilon^{ABC}W_B W_C\,, \\ T^A &= dE^A + \epsilon^{ABC}W_B E_C\,. \end{aligned}$$

[E.W. Mielke, P. Baekler, Phys. Lett. A 156 (1991) 399]

The Mielke-Baekler gravity formalism

The equations of motion obtained from the MB Lagrangian are given by

$$2\sigma_1 R^A + \sigma_0 \epsilon^{ABC} E_B E_C + 2\sigma_3 T^A = 0\,,$$
$$2\sigma_1 T^A + 2\sigma_2 R^A + \sigma_3 \epsilon^{ABC} E_B E_C = 0\,.$$

Then, assuming $\sigma_1^2 - \sigma_2\sigma_3 \neq 0$, the field equations can be rewritten as

$$2T^A + q\epsilon^{ABC} E_B E_C = 0\,, \qquad 2R^A + p\epsilon^{ABC} E_B E_C = 0\,,$$

where

$$q := \frac{\sigma_1\sigma_3 - \sigma_0\sigma_2}{\sigma_1^2 - \sigma_2\sigma_3}\,, \qquad p := \frac{\sigma_0\sigma_1 - \sigma_3^2}{\sigma_1^2 - \sigma_2\sigma_3}\,.$$

In this way, the field configurations are characterized by constant curvature and constant torsion. Let us note that the Riemann-Cartan curvature R^A can be expressed in terms of its Riemannian part $\tilde{R}^A$ and the contorsion one-form K^A. Indeed, decomposing the spin-connection as $W^A = \tilde{W}^A + K^A$, we find

$$2\tilde{R}^A = \Lambda\epsilon^{ABC} E_B E_C\,, \qquad 2\tilde{T}^A = 0\,,$$

where

$$\Lambda := -\left(p + \frac{q^2}{4}\right)\,.$$

Mielke-Baekler gravity à la Chern-Simons

The MB gravity model can be written as a CS theory considering the following algebra

MB algebra

$$
\begin{aligned}
[J_A, J_B] &= \epsilon_{ABC} J^C \,, \\
[J_A, P_B] &= \epsilon_{ABC} P^C \,, \\
[P_A, P_B] &= \epsilon_{ABC} \left(p J^C + q P^C \right) \,.
\end{aligned}
$$

Here the indices $A, B = 0, 1, 2$ are raised and lowered with the Minkowski metric η_{AB} with the mostly plus signature convention. ϵ_{ABC} is the three-dimensional Levi Civita tensor which satisfies $\epsilon_{012} = -\epsilon^{012} = 1$. On the other hand, (p, q) are arbitrary constants which can be fixed to recover known relativistic Lie algebras

Relativistic algebra	p	q
Teleparallel algebra	0	$-2/\ell$
$\mathfrak{so}(2,2)$ algebra	$1/\ell^2$	0
$\mathfrak{iso}(2,1)$ algebra	0	0

Mielke-Baekler gravity à la Chern-Simons

Let A be the gauge connection one-form for the MB algebra,

$$A = W^A J_A + E^A P_A \,.$$

Thus the curvature two-form reads

$$F = \mathcal{R}^A(W)\, J_A + \mathcal{R}^A(E)\, P_A \,,$$

where

$$\mathcal{R}^A(W) := dW^A + \frac{1}{2}\epsilon^{ABC} W_B W_C + \frac{p}{2}\epsilon^{ABC} E_B E_C \,,$$
$$\mathcal{R}^A(E) := dE^A + \epsilon^{ABC} W_B E_C + \frac{q}{2}\epsilon^{ABC} E_B E_C \,.$$

The MB algebra admits an invariant bilinear form with the following non-vanishing components

$$\langle J_A J_B \rangle = \sigma_2\, \eta_{AB} \,, \qquad \langle J_A P_B \rangle = \sigma_1\, \eta_{AB} \,, \qquad \langle P_A P_B \rangle = (p\sigma_2 + q\sigma_1)\, \eta_{AB} \,.$$

Considering the gauge connection one-form A and the non-vanishing components of the invariant tensor in the three-dimensional CS expression, we find up boundary terms

$$L_{\text{CS}}[A] = L_{\text{MB}}[E^A, W^A] \,,$$

where we have imposed

$$\sigma_3 = p\sigma_2 + q\sigma_1 \,, \qquad \sigma_0 = p\sigma_1 + q\sigma_3 \,.$$

[M. Geiller, C. Goeller, N. Merino, JHEP 02 (2021) 120]

Outline

Non-relativistic limit of the MB algebra

The NR MB algebra can be derived from its relativistic counterpart by means of an Inönü–Wigner contraction. To perform this contraction, we first decompose the Lorentz index as $A = (0, a)$ with $a = 1, 2$, distinguishing the temporal and spatial components. This decomposition naturally induces the following splitting of the relativistic generators:

$$J_A \to \{J_0 \equiv J, J_a\}, \qquad P_A \to \{P_0 \equiv H, P_a\}.$$

Now, under the rescaling

$$J_a \to \xi G_a, \qquad P_a \to \xi P_a,$$

it is straightforward to verify that, in the $\xi \to \infty$ limit, the MB algebra reduces to its non-relativistic version:

$$\begin{aligned} [J, G_a] &= \epsilon_{ab} G_b, & [J, P_a] &= \epsilon_{ab} P_b, \\ [H, G_a] &= \epsilon_{ab} P_b, & [H, P_a] &= \epsilon_{ab} (p J_b + q P_b). \end{aligned}$$

NR algebra	p	q
NR torsional algebra	0	$-2/\ell$
Newton-Hooke algebra	$1/\ell^2$	0
Galilei algebra	0	0

Non-relativistic limit of the MB algebra

The degeneracy problem appears after the NR limit is avoided by adding two $\mathfrak{u}(1)$ generators in the relativistic MB algebra. We define the contraction process through the identification of the relativistic generators with the NR generators as

$$
\begin{aligned}
J_0 &= \frac{J}{2} + \xi^2 S\,, \qquad J_a = \xi G_a\,, \qquad Y_2 = \frac{J}{2} - \xi^2 S\,,\\
P_0 &= \frac{H}{2\xi} + \xi M\,, \qquad P_a = P_a\,, \qquad Y_1 = \frac{H}{2\xi} - \xi M\,,
\end{aligned}
$$

along with the following scaling

$$
p \to \frac{p}{\xi^2}\,, \qquad\qquad q \to \frac{q}{\xi}\,,
$$

which is required to have a well-defined limit $\xi \to \infty$. After performing the NR limit, we get a NR version of the MB algebra:

$$
\begin{aligned}
&[J, G_a] = \epsilon_{ab} G_b\,, && [J, P_a] = \epsilon_{ab} P_b\,, && [G_a, P_b] = -\epsilon_{ab} M\,,\\
&[G_a, G_a] = -\epsilon_{ab} S\,, && [P_a, P_b] = -\epsilon_{ab}\left(pS + qM\right)\,, && [H, G_a] = \epsilon_{ab} P_b\,,\\
&[H, P_a] = \epsilon_{ab}\left(pG_b + qP_b\right)\,.
\end{aligned}
$$

Let us consider the corresponding gauge connection one-form A,

$$A = \tau H + e^{a} P_{a} + \omega J + \omega^{a} G_{a} + mM + sS \,.$$

The curvature two-form $F = dA + \frac{1}{2}[A, A]$ is given by

$$F = R(\tau) H + R^{a}\left(e^{b}\right) P_{a} + R(\omega) J + R^{a}\left(\omega^{b}\right) G_{a} + R(m) M + R(s) S \,,$$

where the components are explicitly given by:

$$R(\tau) = d\tau \,, \qquad R^{a}\left(e^{b}\right) = de^{a} + \epsilon^{ac}\omega e_{c} + \epsilon^{ac}\tau\omega_{c} + q\epsilon^{ac}\tau e_{c} \,,$$

$$R(\omega) = d\omega \,, \qquad R^{a}\left(\omega^{b}\right) = d\omega^{a} + \epsilon^{ac}\omega\omega_{c} + p\epsilon^{ac}\tau e_{c} \,,$$

$$R(m) = dm + \epsilon^{ac} e_{a}\omega_{c} + \frac{q}{2}\epsilon^{ac} e_{a} e_{c} \,, \qquad R(s) = ds + \frac{1}{2}\epsilon^{ac}\omega_{a}\omega_{c} + \frac{p}{2}\epsilon^{ac} e_{a} e_{c} \,.$$

The non-vanishing components of a non-degenerate invariant tensor are obtained by applying the contraction process to the relativistic invariant tensors.

$$\langle JS \rangle = -\tilde{\sigma}_{2} \,, \qquad \langle G_{a} G_{b} \rangle = \tilde{\sigma}_{2}\delta_{ab} \,,$$

$$\langle G_{a} P_{b} \rangle = \tilde{\sigma}_{1}\delta_{ab} \,, \qquad \langle HS \rangle = \langle MJ \rangle = -\tilde{\sigma}_{1} \,,$$

$$\langle P_{a} P_{b} \rangle = \left(p\tilde{\sigma}_{2} + q\tilde{\sigma}_{1}\right)\delta_{ab} \,, \qquad \langle HM \rangle = -\left(p\tilde{\sigma}_{2} + q\tilde{\sigma}_{1}\right) \,,$$

where we have considered the following rescaling for the σ_1 and σ_2 parameters

$$\sigma_{1} = \tilde{\sigma}_{1}\xi \,, \qquad \sigma_{2} = \tilde{\sigma}_{2}\xi^{2} \,.$$

Non-relativistic MB CS gravity

The NR CS Lagrangian based on the NR extended MB algebra reads

$$
\begin{aligned}
L_{\text{NRMB}} \quad = \quad & -\tilde{\sigma}_0 \epsilon^{ac} \tau e_a e_c + \tilde{\sigma}_1 \left[e_a \hat{R}^a \left(\omega^b \right) + \omega_a \hat{R}^a \left(e^b \right) - 2mR(\omega) - 2sR(\tau) \right] \\
& + \tilde{\sigma}_2 \left[\omega_a \hat{R}^a \left(\omega^b \right) - 2sR\left(\omega \right) \right] + \tilde{\sigma}_3 \left[e_a \hat{R}^a \left(e^b \right) - mR(\tau) - \tau \hat{R}(m) \right] ,
\end{aligned}
$$

where we have defined

$$
\begin{aligned}
\hat{R}^a \left(e^b \right) &= de^a + \epsilon^{ac} \omega e_c + \epsilon^{ac} \tau \omega_c , \\
\hat{R}^a \left(\omega^b \right) &= d\omega^a + \epsilon^{ac} \omega \omega_c , \\
\hat{R}\left(m \right) &= dm + \epsilon^{ac} e_a \omega_c ,
\end{aligned}
$$

and

$$
\tilde{\sigma}_3 = p\tilde{\sigma}_2 + q\tilde{\sigma}_1 , \qquad \tilde{\sigma}_0 = p\tilde{\sigma}_1 + q\tilde{\sigma}_3 .
$$

The non-degeneracy ensures that the field equations are given by the vanishing of the curvatures. Let us note that the vanishing of $R^a \left(e^b \right) = 0$ and $R^a \left(\omega^b \right) = 0$ implies the non-vanishing of the spatial torsion $\hat{R}^a \left(e^b \right) = -q\epsilon^{ac} \tau e_c$ and the curvature $\hat{R}^a \left(\omega^b \right) = -p\epsilon^{ac} \tau e_c$.

NR gravity	p	q
NR Teleparallel gravity	0	$-2/\ell$
Extended Newton-Hooke gravity	$1/\ell^2$	0
Extended Bargmann gravity	0	0

[P. Concha, N. Merino, E. Rodríguez, Eur. Phys. J. C 84 (2024) 407]

Ultra-relativistic limit of the MB algebra

The UR limit is performed by first considering the following rescaling of the relativistic generators:

$$P_0 \to \sigma H\,, \qquad J_a \to \sigma K_a\,, \qquad q \to 1/\sigma\, q\,,$$

and then applying the $\sigma \to \infty$ limit. The obtained UR algebra, denoted as the *Carroll Mielke-Baekler algebra* (C-MB), satisfies the following non-vanishing commutators:

C-MB algebra

$$\begin{aligned} [J, K_a] &= \epsilon_{ab} K_b\,, & [K_a, P_b] &= -\epsilon_{ab} H\,, \\ [J, P_a] &= \epsilon_{ab} P_b\,, & [P_a, P_b] &= -\epsilon_{ab}\left(pJ + qH\right)\,, \\ [H, P_a] &= p\epsilon_{ab} K_b\,. & & \end{aligned}$$

Carrollian-type algebra	p	q
UR torsional algebra	0	$-2/\ell$
AdS-Carroll algebra	$1/\ell^2$	0
Carroll algebra	0	0

Ultra-relativistic MB CS gravity

To explicitly construct the CS Lagrangian based on the C-MB algebra, we first consider the corresponding gauge connection 1-form

$$A = \tau H + e^a P_a + \omega J + \omega^a K_a \,,$$

together with the curvature 2-form $F = dA + \frac{1}{2}[A, A]$, that is

$$F = R(\tau) H + R^a \left(e^b\right) P_a + R(\omega) J + R^a \left(\omega^b\right) K_a \,,$$

where the component 2-forms explicitly read

$$\begin{aligned} R(\omega) &:= d\omega + \frac{p}{2}\epsilon^{ac} e_a e_c \,, \\ R^a\left(\omega^b\right) &:= d\omega^a + \epsilon^{ac}\omega\omega_c + p\epsilon^{ac}\tau e_c \,, \\ R(\tau) &:= d\tau + \epsilon^{ac} e_a \omega_c + \frac{q}{2}\epsilon^{ac} e_a e_c \,, \\ R^a\left(e^b\right) &:= de^a + \epsilon^{ac}\omega e_c \,. \end{aligned}$$

The C-MB algebra admits the following components of the invariant tensor:

$$\langle JJ\rangle = -\tilde{\sigma}_2 \,, \qquad \langle K_a P_b\rangle = \tilde{\sigma}_1 \delta_{ab} \,,$$
$$\langle JH\rangle = -\tilde{\sigma}_1 \,, \qquad \langle P_a P_b\rangle = \left(p\tilde{\sigma}_2 + q\tilde{\sigma}_1\right)\delta_{ab} \,,$$

where we have considered

$$\sigma_1 \to \sigma\tilde{\sigma}_1 \,, \qquad \sigma_2 \to \tilde{\sigma}_2 \,.$$

Ultra-relativistic MB CS gravity

The UR CS Lagrangian based on the C-MB algebra reads

$$L_{\text{C-MB}} = -\, p\tilde{\sigma}_1\epsilon^{ac}\tau e_a e_c + 2\tilde{\sigma}_1 \left[e_a \tilde{R}^a \left(\omega^b\right) - \tau \tilde{R}\left(\omega\right) \right] - \tilde{\sigma}_2 \omega \tilde{R}\left(\omega\right) + \tilde{\sigma}_3 e_a R^a \left(e^b\right) \,,$$

where we have defined

$$\tilde{R}\left(\omega\right) := d\omega \,,$$
$$\tilde{R}^a \left(\omega^b\right) := d\omega^a + \epsilon^{ac}\omega\omega_c \,.$$

For arbitrary values of p and q (and $\tilde{\sigma}_1 \neq 0$), the field equations of the theory are given by the vanishing of UR the curvature 2-forms. Of particular interest is the vanishing of $R\left(\tau\right)$, which implies the non-vanishing of the **temporal component** of the Carrollian torsion:

$$d\tau + \epsilon^{ac} e_a \omega_c = -\frac{q}{2}\epsilon^{ac} e_a e_c \,.$$

Fixing the parameters p and q allows us to reproduce different UR gravity models, with and without a non-vanishing temporal torsion curvature component. In particular, the choice $p = 0$ and $q = -2/\ell$ leads to a Carrollian model in which the cosmological constant emerges as a source of temporal Carrollian torsion. This construction represents the UR counterpart of relativistic teleparallel CS gravity and suggests a potential connection to torsional Newton-Cartan geometry.

[P. Concha, N. Merino, L. Ravera, E. Rodríguez, Work in progress]

Outline

Opening new paths toward holography, geometry, and asymptotic symmetries.

The results obtained here could provide valuable insight into the role of torsion in the Non-Lorentzian regime and may serve as a starting point for several future research directions:

- It would be interesting to investigate whether a non-vanishing torsion in a relativistic and no-Lorentzian frameworks modifies the asymptotic symmetries algebra after imposing suitable boundary conditions. In the non-Lorentzian case, such an asymptotic symmetry algebra would represent the non-Lorentzian counterpart of well-known asymptotic symmetry algebras, such as the $\mathfrak{bms}_3$ algebra.
- A promising direction is to explore holography beyond the relativistic regime, where non-Lorentzian geometries naturally arise. In this context, it would be interesting exploring potential torsional generalizations of the BMS/GCA and BMS/CCA correspondences.
- One could study the thermodynamics of three-dimensional gravity solutions based on the MB symmetry, considering both the torsionless and torsional sectors.
- Another natural step would be to explore the posibility of constructing supersymmetric or higher-spin extensions of our non-Lorentzian MB model.

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