Cosmic slowing down of acceleration with the Chaplygin-Jacobi gas as a dark fluid?

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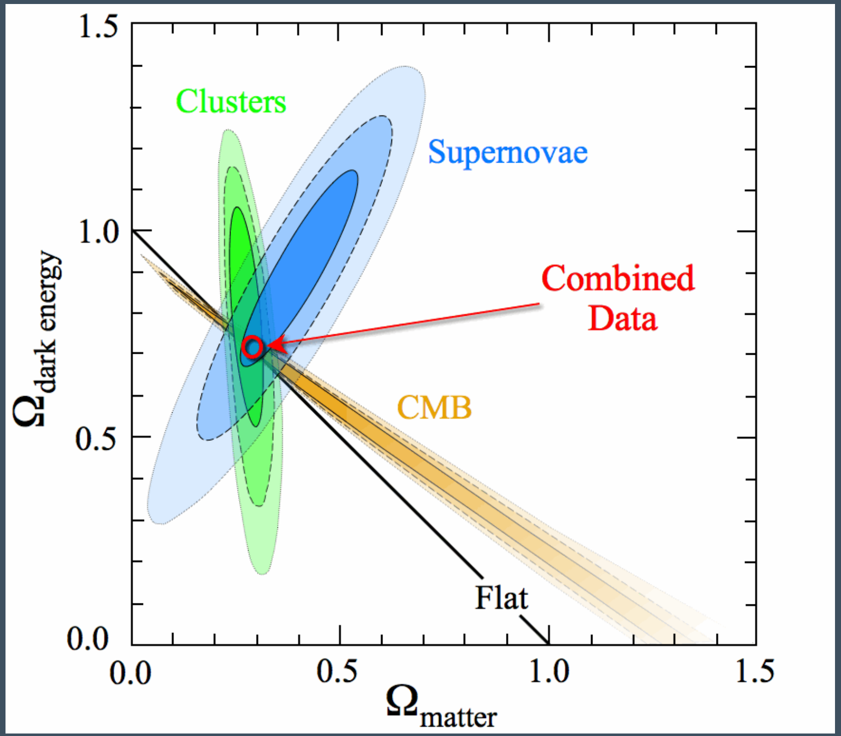
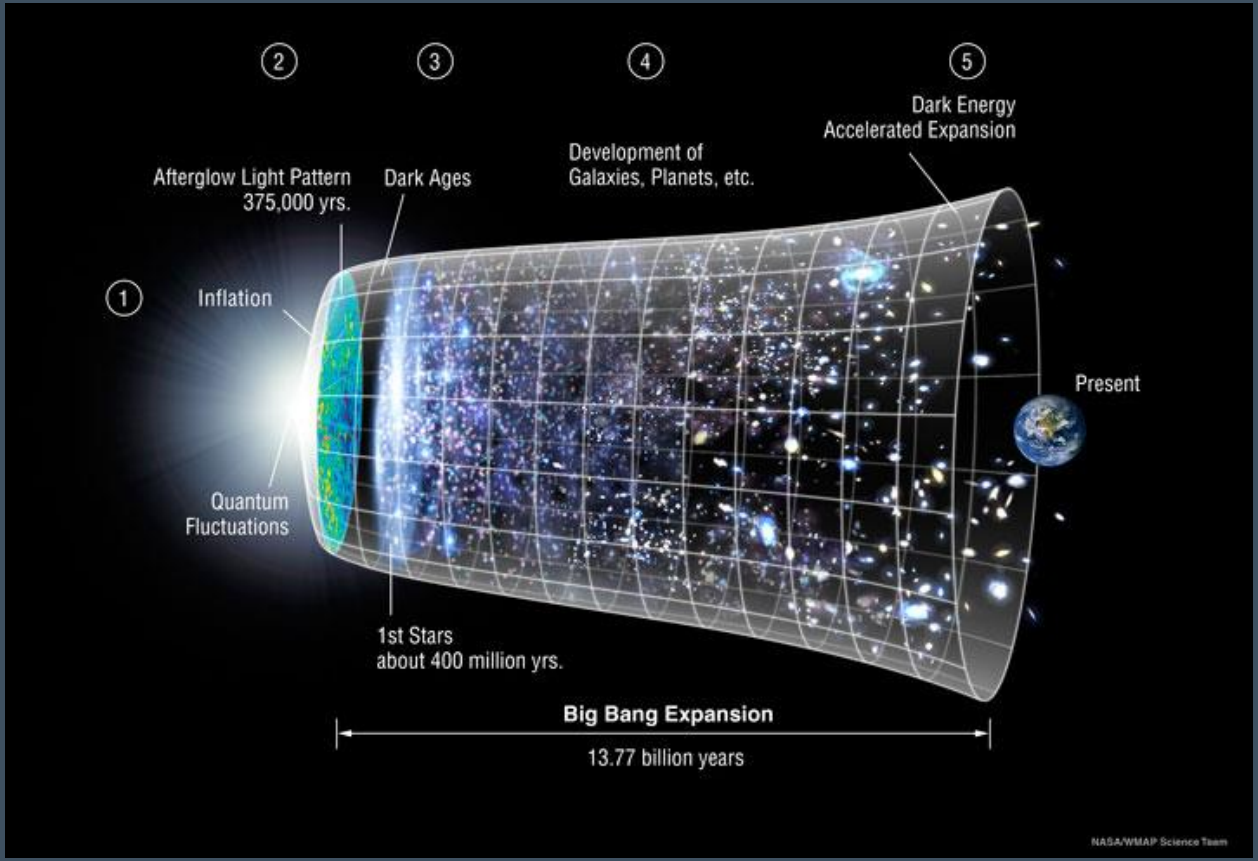
Outline

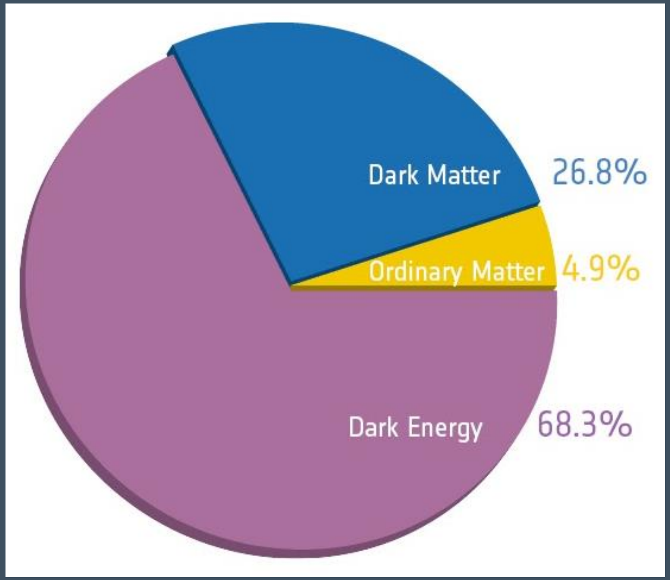
- Motivation
- Cosmology of the Chaplygin-Jacobi gas
- Data analysis and results
- Evolution of linear density fluctuations
- Conclusions

ΛCDM Model



Kowalski et al. (2008)





Planck Collaboration

$$E(z)^2 \equiv \frac{H(z)^2}{H_0^2} = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda$$

Some theoretical and observational challenges of ΛCDM

- The cosmological constant problem

- The coincidence problem

- The tension between measurements of the Hubble parameter H_0 (Planck vs. local)

- The S_8 tensión (growth rate)

Some theoretical and observational challenges of ΛCDM

- The cosmological constant pr
- The coincide
- The t een measurements of the Hubbl parameter H_0 (Planck vs. local)
- The S_8 tensión (growth rate)

Could the tensions between early and late universe measurements indicate the presence of new physics beyond the ΛCDM model?

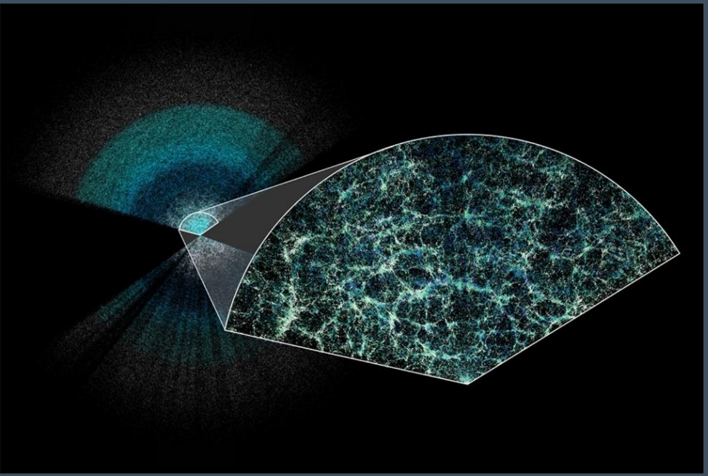
Classification of DE Models

Category	Example	Key Features
Constant w	ΛCDM	Simple, but rigid
Dynamical $w(z)$	Quintessence, phantom	Rich dynamics
Unified dark fluids	Chaplygin-type	Combine DM + DE

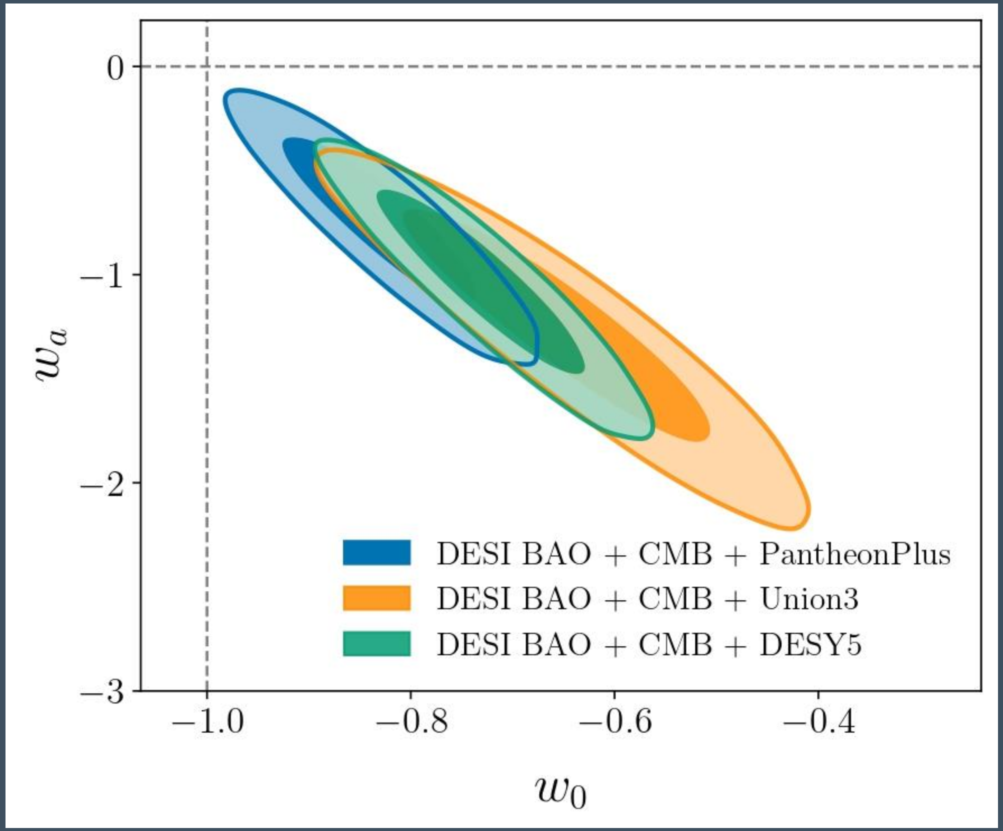
DESI Results and Hints of Dynamical DE

DESI (2024)+CMB+SnIa data suggest deviation from $w = -1$.

IMPLICATIONS: **further exploration of dynamical DE in place of a cosmological constant in the ΛCDM model.**



DESI Collaboration



$$w(a) = w_0 + w_a(1 - a)$$

[A. G. Adame et al. (DESI), (2024)]

Dynamical DE models: Chaplygin gas

- A theoretical fluid with an exotic equation of state (EoS):

$$p = -\frac{B}{\rho}$$

- Initially studied in aerodynamics, later applied for cosmology [**A. Y. Kamenshchik et al. (2001)**].

- As $\rho \to \infty$, behaves like pressureless dust (dark matter).

- As $\rho \to 0$, behaves like a cosmological constant (DE).

UNIFIED DESCRIPTION OF DARK ENERGY AND DARK MATTER

Dynamical DE models: Chaplygin gas

- A theoretical fluid with
-
- A
- As

Key issues:

1. Instabilities and oscillations in structure formation
2. Rapid growth of the sound speed of perturbations
3. Fine-tuning problem
4. Incompatibility with CMB

ON OF DARK ENERGY
AND DARK MATTER

Generalized Chaplygin Gas (GCG)

- Modified EoS [**M. C. Bento et al. (2002)**]:

$$p = -\frac{B}{\rho^{\alpha}}$$

where $0 \leq \alpha \leq 1$.

- $\alpha = 1$: Original Chaplygin gas.

- $0 < \alpha < 1$: Interpolates between dust and dark energy.

- More flexibility in describing cosmic evolution.

Generalized Chaplygin Gas (GCG)

- Modified EoS [**M. C. Bento et al. (2002)**]

Key issues:

1. Problems with structure formation
2. Rapid growth of the sound speed of perturbations
3. CMB Constraints and Tension with Observations
4. Hubble tension and expansion rate
5. Fine-tuning of parameters

- ... energy.
- M... in describing cosmic evolution.

Modified Chaplygin Gas (MCG)

- Further modification of the EoS [**H. Benaoum (2022)**]:

$$p = A\rho - \frac{B}{\rho^{\alpha}}$$

- Allows for an additional barotropic fluid term $A\rho$.

- The extra parameter A allows the model to have a more realistic transition between dark matter and dark energy behavior.

Modified Chaplygin Gas (MCG)

- Further modification of the EoS [**H. Benaoum (20**

$p = A$

a more
and dark energy

Key issues:

1. Problems with structure formation
2. Rapid growth of the sound speed of perturbations
3. Fine-tuning of parameters
4. Incompatibility with observational data

Chaplygin-Jacobi Gas (CJG)

- The Chaplygin–Jacobi Gas (CJG) generalizes these models using the Hamilton–Jacobi formalism with elliptic functions [J.R. Villanueva (2015)]:

$$p = -\frac{Bk}{\rho^{\alpha}} - k'\rho\left(2 - \frac{1}{B}\rho^{\alpha+1}\right)$$

- $k = 1, B > 0$, and $0 \leq \alpha \leq 1$: GCG
- $k = 1, B > 0$, and $\alpha=0$: ΛCDM model
- Allows extra non-linear term via elliptic modulus ($0 < k < 1$)
- Enables **transient acceleration**: current acceleration followed by future deceleration.

Physical Parameter Regions

Energy density of the CJG:

$$\rho = \rho_0 \left[B_s - \frac{B_s(1-B_s)}{(1-B_s)k' - (k'+B_s k)a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \qquad (B = {\rho_0}^{1+\alpha} B_s)$$

Physical (non-singular, positive) solutions exist in subregions of parameter space $\mathcal{M} = \{(\alpha, B_s, k)\colon\ \alpha \in \mathbb{R}, B_s \in \mathbb{R}, 0 < k < 1\}$:

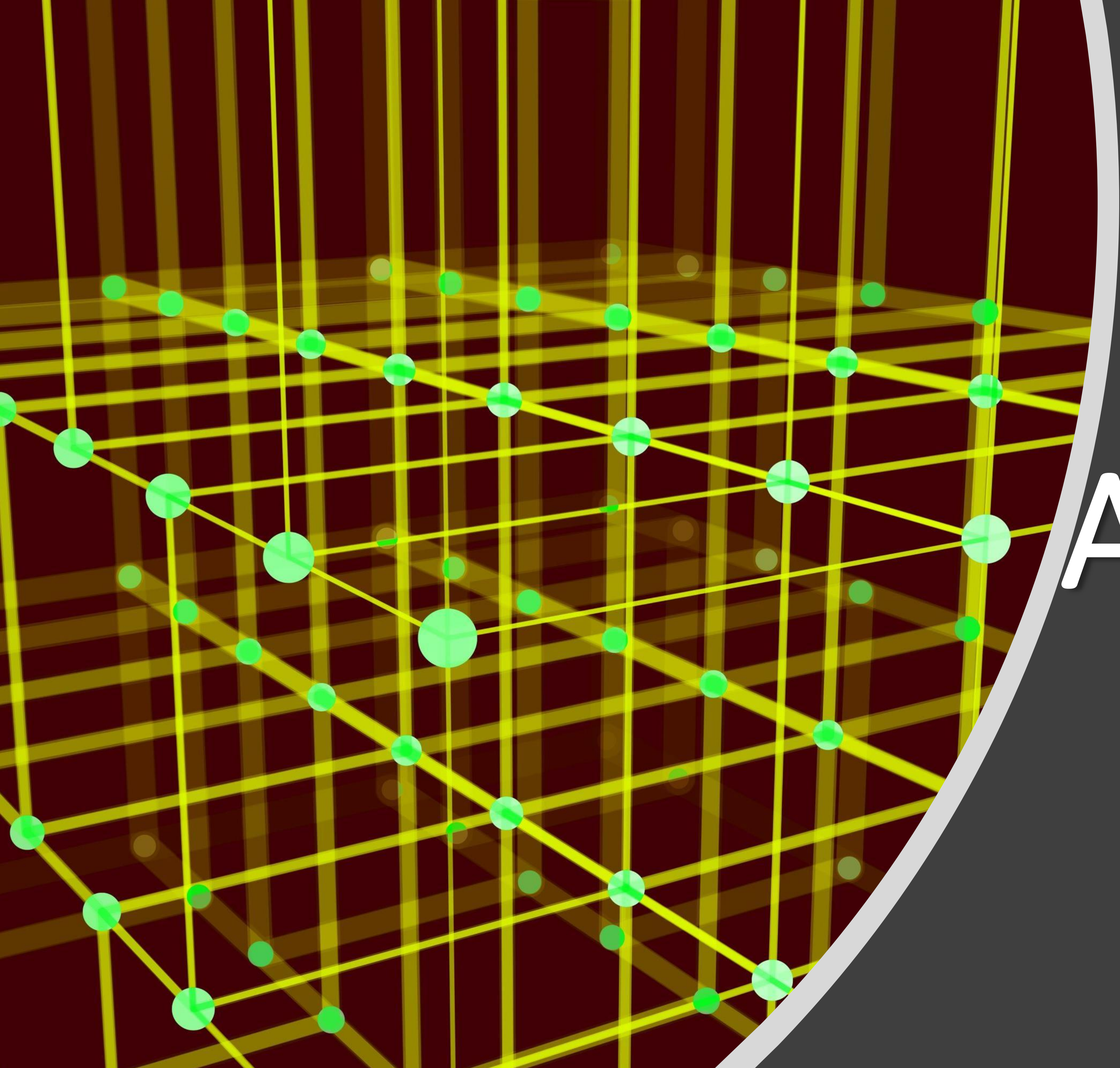
- $\mathcal{N}_1$: $\alpha < -1$, $B_s > 1$
- $\mathcal{N}_2$: $\alpha < -1$, $0 < B_s < 1$
- $\mathcal{N}_3$: $\alpha \geq -1$, $-k'/k < B_s < 1$
- $\mathcal{N}_4$: $\alpha > -1$, $B_s < -k'/k$

Cosmology of the CJG as DE

$$H^2 = \frac{8\pi G}{3}(\rho_r + \rho_m + \rho_{de}) \quad \rightarrow \quad E(z)^2 \equiv \frac{H^2(z)}{H_0^2},$$

$$\frac{H^2(z)}{H_0^2} = \Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + (1-\Omega_{m0}-\Omega_{r0})\left[B_s + \frac{B_s(1-B_s)(1+z)^{3(1+\alpha)}}{(k'+B_s k)-(1-B_s)k'(1+z)^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}$$

$$\boxed{\begin{gathered} w_{eff} = -1 + \frac{(1+z)}{3E^2}\frac{dE^2}{dz^2} \\ r = 1 + \frac{1}{2E^2}\left[(1+z)^2\frac{d^2E^2}{dz^2} - 2(1+z)\frac{dE^2}{dz^2}\right] \\ s = \frac{1}{3}\frac{(1+z)^2\frac{d^2E^2}{dz^2} - 2(1+z)\frac{dE^2}{dz}}{(1+z)\frac{dE^2}{dz^2} - 3E^2} \end{gathered}}$$



Data Analysis and Results

Markov Chain Monte Carlo (MCMC) analysis

The MCMC analysis is performed using the publicly available Python package Polychord using:

a) Cosmic Chronometers (CC) data only: 31 measurements of the Hubble rate $H(z)$ via the CC method, compiled in [**G.-J. Wang et al. (2020)**].

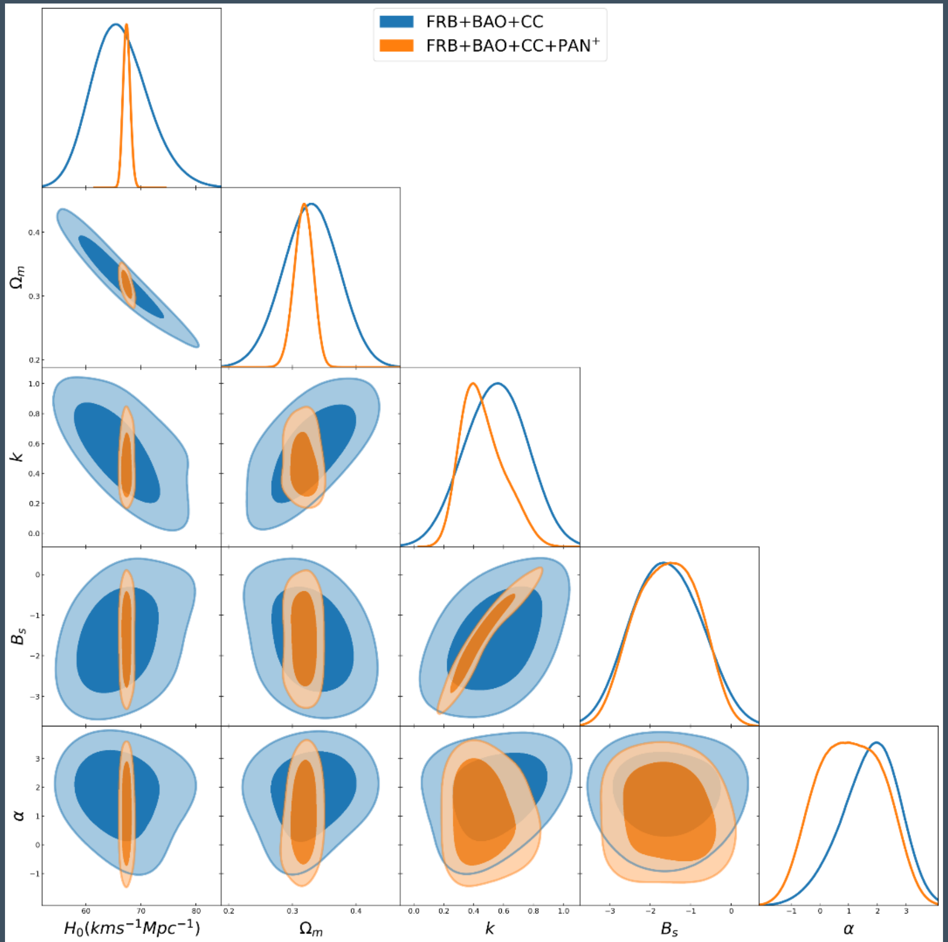
b) Type Ia Supernovae (SnIa) data only: Pantheon + dataset, which comprises 1701 data points [**A. G. Riess et al. (2022)**].

c) Fast Radio Bursts (FRBs) data only: subset of 23 FRBs, compiled by [**K. Yang et al. (2022)**], which contain cosmological information on the dispersion measure DM.

d) Baryon Acoustic Oscillations (BAO): DESI 2024+SDSS/eBOSS

Markov Chain Monte Carlo (MCMC) analysis

Model	Data	H_0	Ω_m	B_s	k	α	M_B	$\chi^2_{\min}$	χ^2_ν	AIC	BIC
CJG	(a)	$66.10^{+4.20}_{-5.40}$	0.329 ± 0.041	-1.60 ± 0.79	0.55 ± 0.20	$1.73^{+1.10}_{-0.75}$	–	55.11	1.04	67.11	79.57
	(b)	67.41 ± 0.62	0.318 ± 0.014	-1.57 ± 0.75	$0.46^{+0.08}_{-0.18}$	1.1 ± 1.1	$-19.423^{+0.018}_{-0.020}$	1623.85	0.97	1637.85	1675.83
ΛCDM	(a)	68.66 ± 0.89	0.306 ± 0.016	–	–	–	–	48.67	0.87	60.67	60.90
	(b)	67.56 ± 0.57	0.3296 ± 0.0097	–	–	–	-19.429 ± 0.016	1595.39	0.95	1607.39	1625.09



(a) FRB+BAO+CC

(b) FRB+BAO+CC+PAN$^+$

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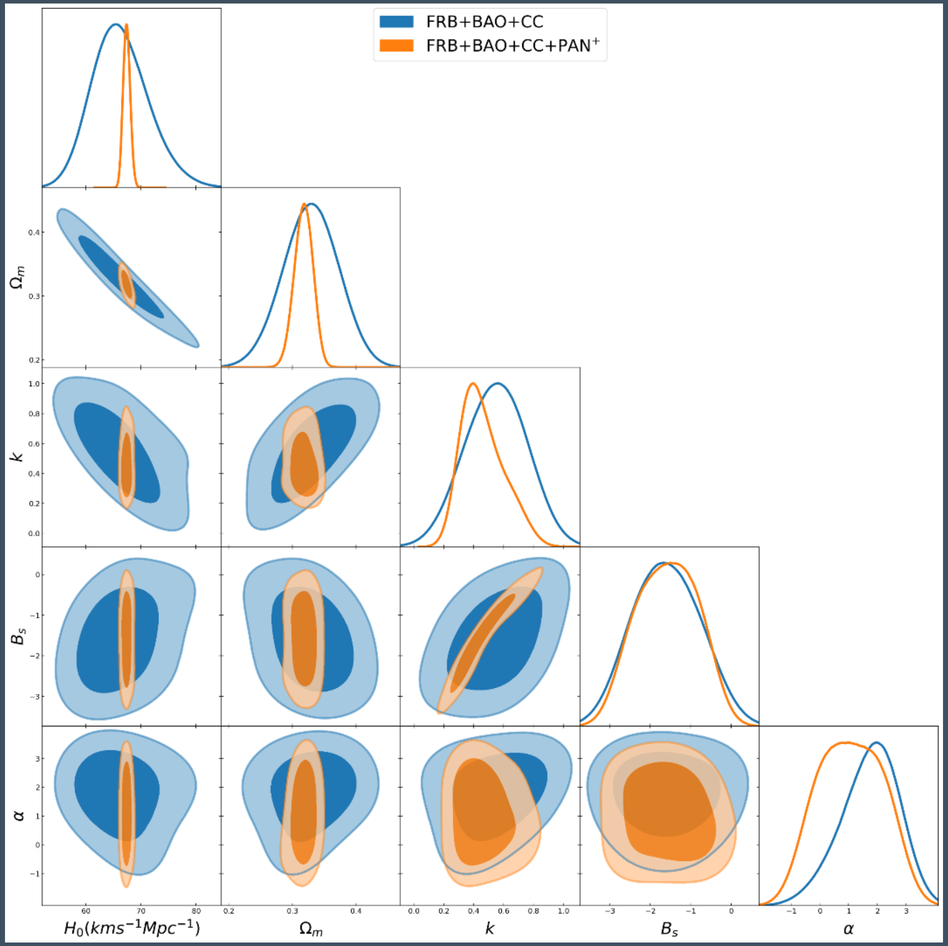


(a) FRB+BAO+CC

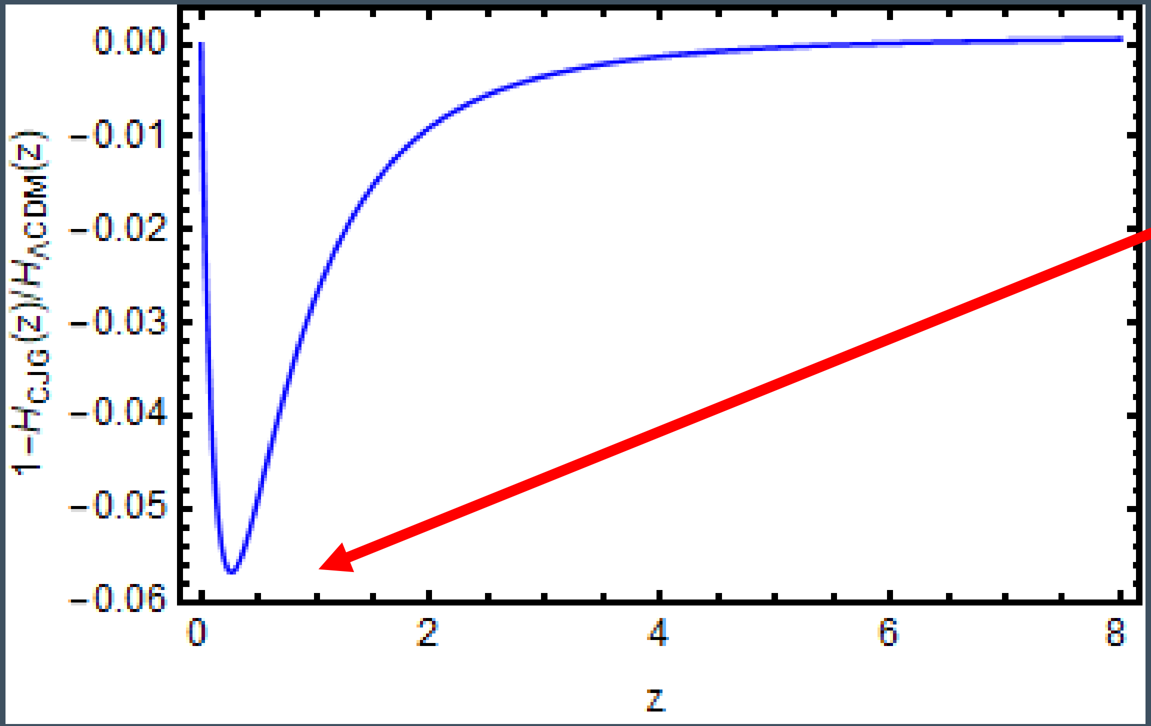
(b) FRB+BAO+CC+PAN$^+$

- Goodness of fit: $\chi_\nu^2 \approx 1$
- Model comparison:

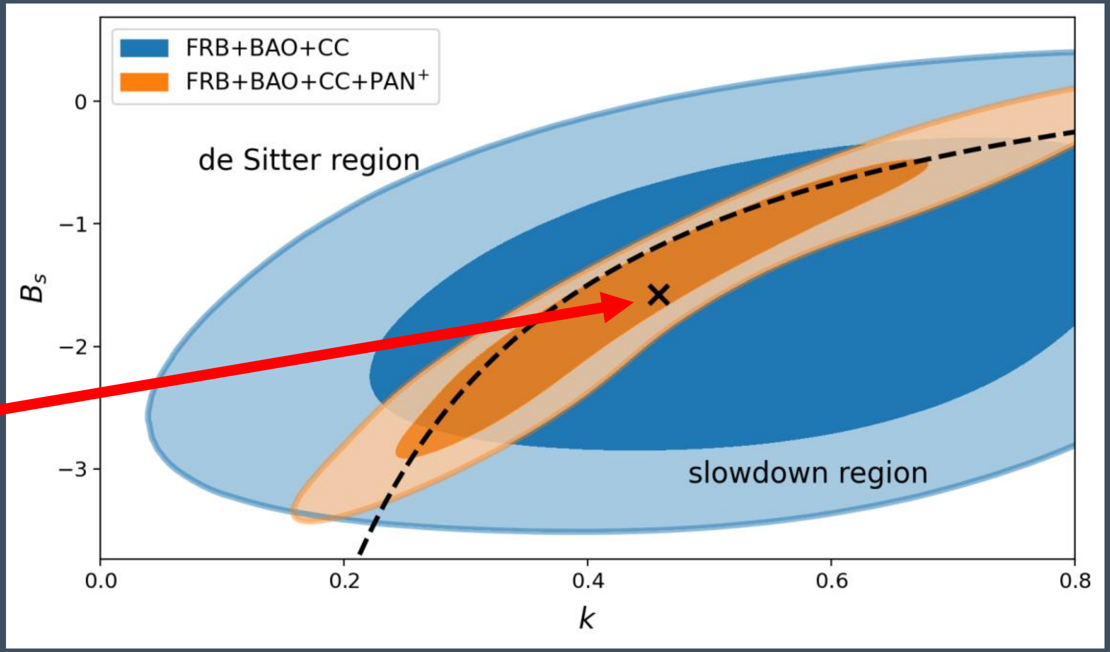
❑ Δ AIC ≈ 30, Δ BIC ≈ 50

❑ ΛCDM **statistically favored**, but CJG **not ruled out**

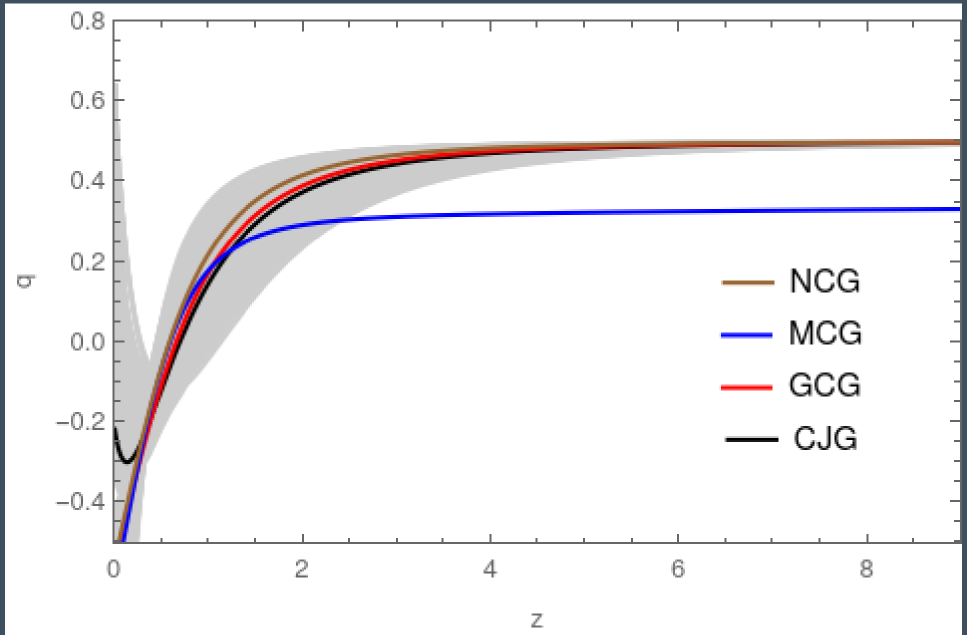
$H(z)$ and transient acceleration

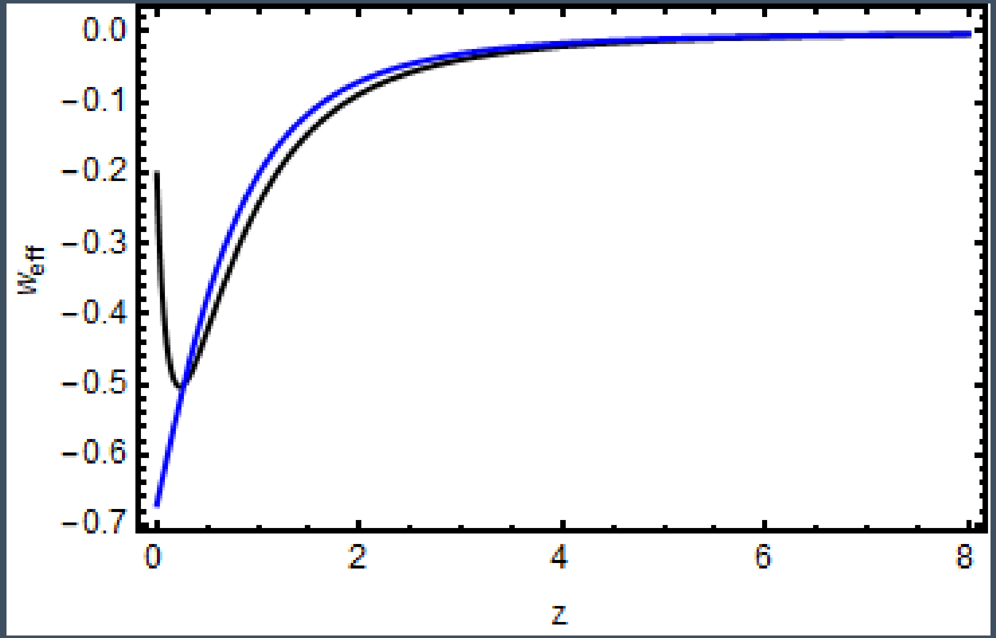


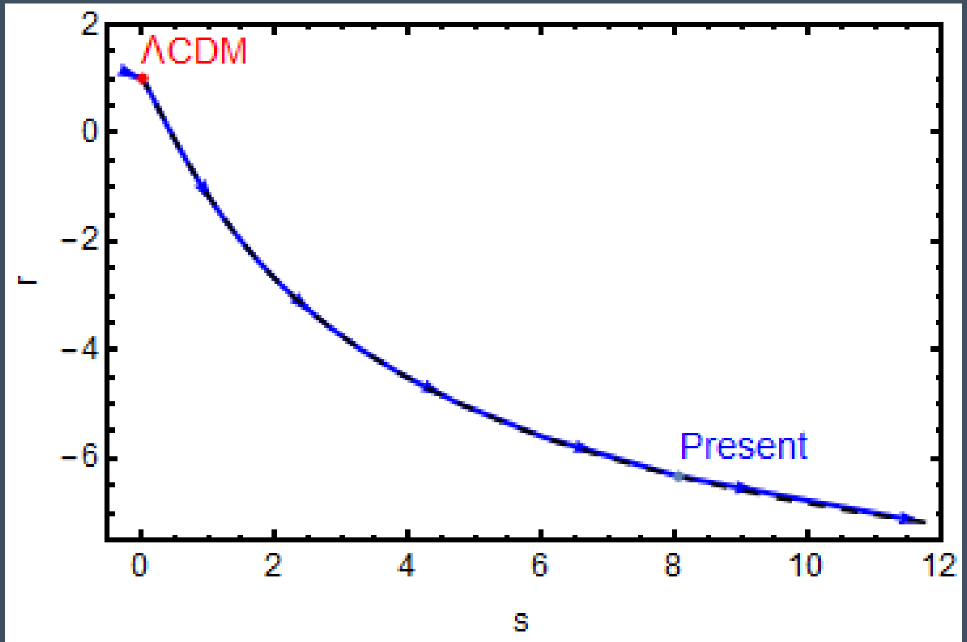
- $H(z)$ nearly identical to ΛCDM for $z > 0.1$
- Deviations appear near the present epoch



- Subregion $\mathcal{N}_4$ compatible with transient acceleration







- Deceleration parameter $q(z)$: transition to deceleration in future

- Effective EoS w_{eff}: evolves from $0 \rightarrow -1 \rightarrow +$ (future deceleration)

- Statefinder (r, s): CGJ deviates from ΛCDM point $\{0,1\}$ at low z

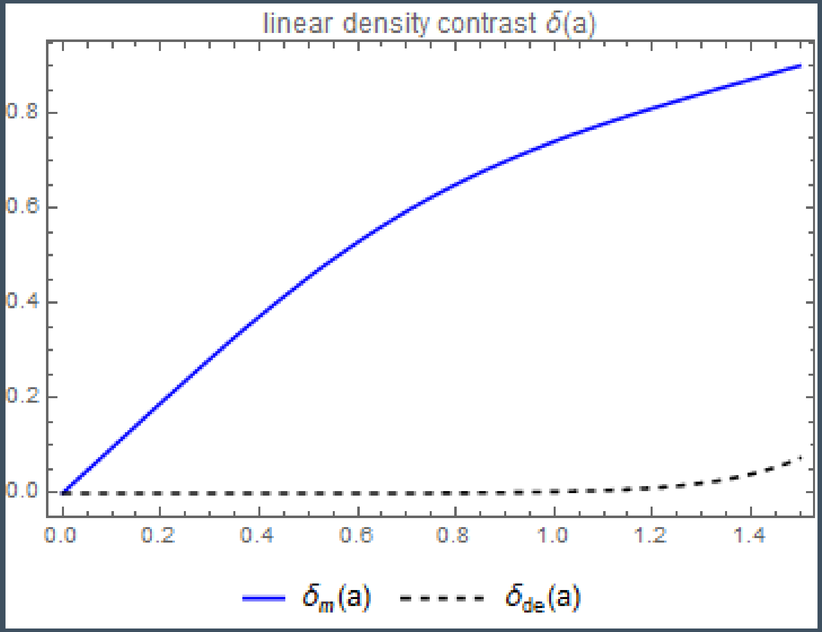
Evolution of linear density fluctuations

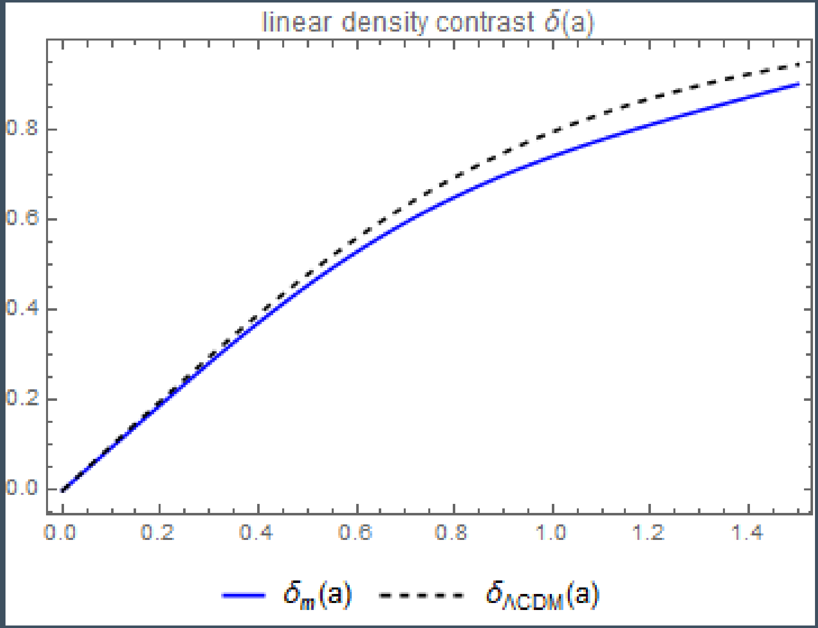
Newtonian gauge and sub-horizon limit $(k \gg aH)$:

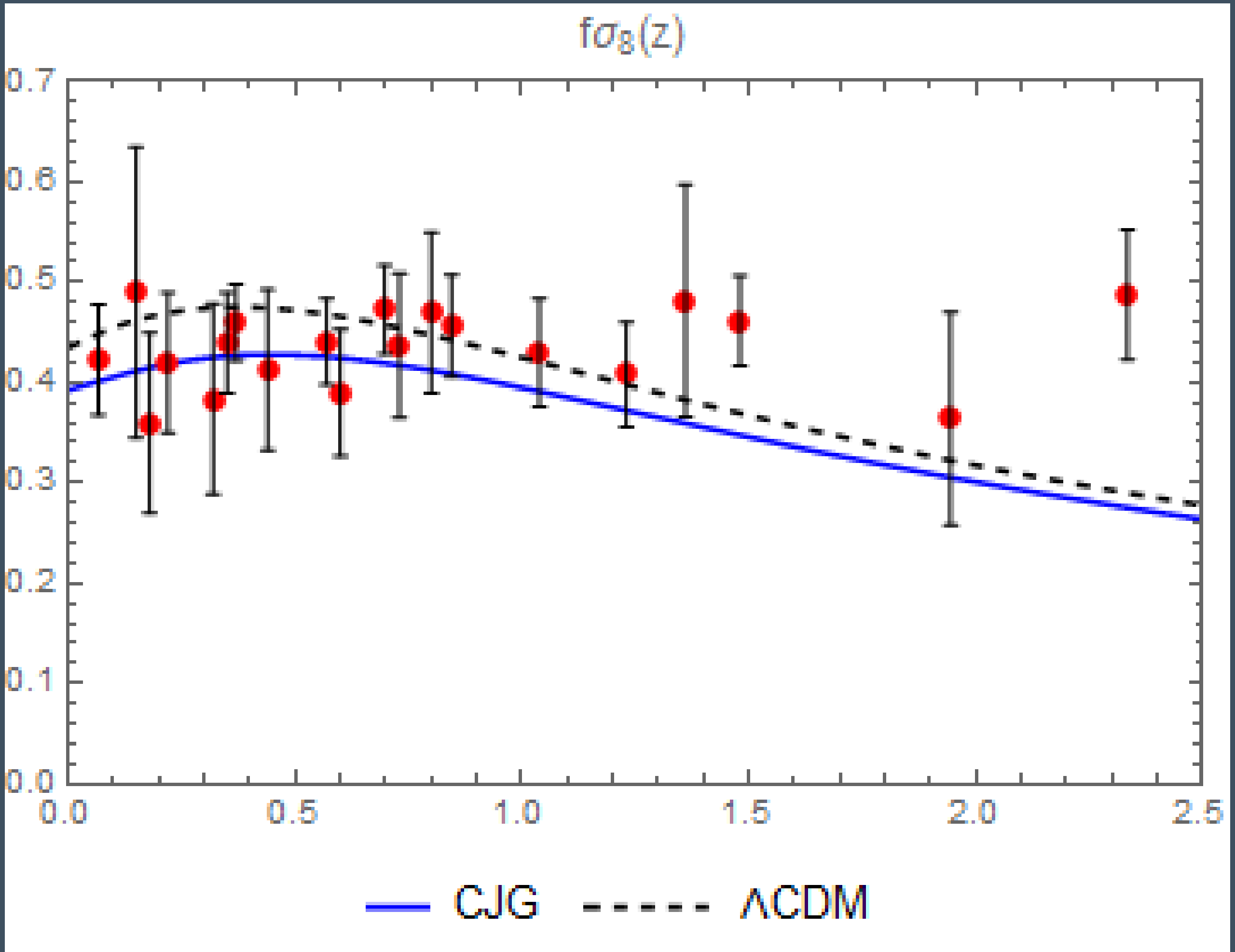
$$\delta_m'' + \left(\frac{3}{a} + \frac{E'(a)}{E(a)}\right)\delta_m' - \frac{3}{2}\frac{\Omega_{m0}}{a^5E^2(a)}\delta_m = \frac{3}{2}\frac{\Omega_{de0}(1+3w_{de}(a))}{a^5E^2(a)}\delta_{de},$$

$$\delta_{de}'' + \left(\frac{3}{a} + \frac{E'(a)}{E(a)} - \frac{w_{de}'(a)}{1+w_{de}(a)}\right)\delta_{de}' + \left[w_{de}(a)\left(\frac{k}{aH_0E(a)}\right)^2 - (1+w_{de}(a))\frac{\Omega_{de0}(1+3w_{de}(a))}{a^5E^2(a)}\right]\delta_{de} = (1+w_{de}(a))\frac{3}{2}\frac{\Omega_{m0}}{a^5E^2(a)}\delta_m.$$

Density contrast and $f\sigma_8(z)$







Gold + eBOSS Compilation [**Beutler et al. (2012), Alam et al. (2021)**]

Concluding Remarks

- CJG extends Chaplygin gas models using the **Hamilton–Jacobi formalism** with elliptic functions.
- Predicts **cosmic slowing down** (transient acceleration).
- Best-fit values for the joint analysis support the cosmic slowing down phenomenon with a maximum of acceleration at $z{\sim}0.1$.
- Statistically disfavored vs. ΛCDM (AIC/BIC), but not ruled out.
- Slower growth at low z → could ease S_8 tension


GRACIAS