de Sitter geometric inflation from dynamical singularities

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Based on

- **de Sitter geometric inflation from dynamical singularities** Phys.Rev.D 110 (2024) 8, 084043 • e-Print: 2406.10037 [hep-th]
- **On four-dimensional Einsteinian gravity, quasitopological gravity, cosmology and black holes** Phys.Lett.B 805 (2020) 135435 • e-Print: 1811.06523 [hep-th]

in collaboration with Adolfo Cisterna and Nicolás Grandi

EFTs and field redefinitions

Effective Field Theories (EFTs) are defined perturbatively. Suppose there is a single scale, denoted by α', that controls the integration of high-energy degrees of freedom. A key feature of this framework is the freedom to perform field redefinitions, which alter the explicit form (or frame) of the action without changing any physical observables, such as S-matrix elements. Different frames can be useful to make certain properties of the theory more manifest.

Field redefinitions and $2+1$ gravity

Consider a correction to GR in 3D up to order α

$$I = \int \sqrt{-g}\, d^3x \left(-2\Lambda + R + \alpha(a_1 R^2 + a_2 R_{ab}R^{ab} + a_3 R_{abcd}R^{abcd}) + \mathcal{O}(\alpha^2)\right)$$

Since the Weyl tensor vanishes in three-dimensions

$$I = \int \sqrt{-g}\, d^3x \left(-2\Lambda + R + \alpha(b_1 R^2 + b_2 R_{ab}R^{ab}) + \mathcal{O}(\alpha^2)\right)$$

There is a field redefinition

$$g_{ab} \to g_{ab} + \alpha(c_1 g_{ab} R + c_2 R_{ab}) + \mathcal{O}(\alpha^2)$$

such that the action can be written as

$$I = \int \sqrt{-g}\, d^3x \left(-2\tilde{\Lambda} + R + \mathcal{O}(\alpha^2)\right)$$

This observation is behind the statement that $2+1$-dimensional GR is perturbatively renormalizable.

Field redefinitions and $3 + 1$

Goroff-Sagnotti: all the infinities at two-loop of perturbative quantum gravity in vacuum can be removed by the inclusion of a new counterterm that goes as Weyl^3, namely

$$I = \frac{1}{16\pi G}\int \sqrt{-g}d^4xR + \frac{G^2\lambda}{16\pi}\int \sqrt{-g}d^4xC^{ab}{}_{cd}C^{cd}{}_{ef}C^{ef}{}_{ab} + \mathcal{O}(G^3)$$

with λ a numerical coefficient (see Perturbative quantum gravity by 't Hooft).

This statement relies on advanced computation of QFT, on the topological nature of the four-dimensional Gauss-Bonnet combination in four dimensions, and on the fact that all the counterterms containing Ricci tensors and Ricci scalar can be removed via field redefinitions.

Field redefinitions and String Theory

Metsaev-Tseytlin: From Bosonic and Heterotic string theories

$$I\left[g_{\mu\nu},\phi\right] = \int_M d^d x \sqrt{-g} e^{-2\phi}\left[R + 4\left(\nabla\phi\right)^2 + \alpha\ R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} + \mathcal{O}\left(\alpha^2\right)\right] ,$$

where we denoted $\alpha = \frac{1}{8}\alpha'$. Performing field redefinition $g_{\mu\nu} \to g_{\mu\nu} + \delta g_{\mu\nu}$, $\phi \to \phi + \delta\phi$ with

$$\delta\phi = -\frac{\alpha}{2}\left(R + 4(2d-5)\partial_\mu\phi\partial^\mu\phi\right) ,$$
$$\delta g_{\mu\nu} = -4\alpha\left(R_{\mu\nu} - 4\partial_\mu\phi\partial_\nu\phi + 4g_{\mu\nu}\partial_\alpha\phi\partial^\alpha\phi\right) ,$$

one obtains

$$I\left[g_{\mu\nu},\phi\right] = \int d^d x \sqrt{-g} e^{-2\phi}\Big[R + 4\left(\nabla\phi\right)^2 + \alpha\left(R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho} - 4R_{\mu\nu}R^{\mu\nu} + R^2 - 16\left(\partial_\mu\phi\partial^\mu\phi\right)^2\right) + \mathcal{O}\left(\alpha^2\right)\Big] ,$$

This is how the EGB combination emerges in String Theory.

- Field redefinitions are intrinsic to EFTs.
- Using field redefinitions one can recast the action in a manner useful to make explicit some of the properties of the low energy theory.

Let us start with IIB-SUGRA

The low-energy action, dropping α'-corrections:

$$
\begin{aligned}
S_{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \Big[& R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{2} e^{2\phi} (\partial C_0)^2 \\
& - \frac{1}{12} e^{-\phi} |H_3|^2 - \frac{1}{12} e^{\phi} |F_3|^2 - \frac{1}{240} |F_5|^2 \Big] \\
& - \frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3 \ + \text{fermions} \ .
\end{aligned}
$$

and (NS sector $g_{\mu\nu}, \phi, B_2$ and RR fields C_0, C_2, C_4) such that

$$
\begin{aligned}
& H_3 := dB_2, \ F_3 := dC_2 - C_0 H_3, \\
& F_5 := dC_4 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3, \ F_5 = *F_5.
\end{aligned}
$$

α'-correction implies a term of the form $\alpha'^3 Weyl^4$.

IIB SUGRA On $A_5 \times S^5$

Stringy corrections to IIB SUGRA imply higher curvature corrections. Reducing on S^5 the consistent truncation to the pure gravity sector with the α'^3 correction is

$$I = \frac{1}{16\pi G} \int d^5x \sqrt{-g} \left[R + \frac{12}{L^2} + \frac{\zeta(3)}{8} \alpha'^3 W^4 + \mathcal{O}(\alpha'^4) \right] ,$$

$$W^4 = \left(C_{abcd} C^{ebcf} + \frac{1}{2} C_{adbc} C^{efbc} \right) C^{ag}{}_{he} C^{hd}{}_{fg} ,$$

Many properties of this theory have been studied: holographic thermalization, η/s bound, effects on black hole thermodynamics, entanglement entropy, etc...

The theory has fourth-order field equations, spurious propagations of ghost-modes due to the truncation.

A suitable field redefinition

$$g_{ab} \to g_{ab} - \frac{\zeta(3)}{8} \alpha'^3 \left[\frac{1}{L^4} \left(\hat{C}^{(1)}_{ab} - \frac{g_{ab}}{3} \hat{C}^{(1)} \right) + \frac{1}{L^2} \left(\hat{C}^{(2)}_{ab} - \frac{g_{ab}}{3} \hat{C}^{(2)} \right) + \hat{C}^{(3)}_{ab} - \frac{g_{ab}}{3} \hat{C}^{(3)} \right] ,$$

with

$$\begin{aligned}
\hat{C}^{(3)}_{ab} = & - \frac{6935}{1584} R^{cdef} R_{c}{}^{g}{}_{ea} R_{dgfb} - \frac{18625}{19008} R^{cdef} R_{cd}{}^{g}{}_{a} R_{efgb} + \frac{7}{44} R_{a}{}^{c}{}_{b}{}^{d} R_{efgc} R^{efg}{}_{d} \\
& + \frac{2053}{1584} g_{ab} R_{g}{}^{c}{}_{h}{}^{d} R_{c}{}^{e}{}_{d}{}^{f} R_{e}{}^{g}{}_{f}{}^{h} + \frac{1241}{19008} g_{ab} R_{gh}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{gh} + \frac{6035}{3168} R^{cd} R^{e}{}_{a}{}^{f} \\
& + \frac{24215}{6336} R^{cd} R^{e}{}_{a}{}^{f}{}_{b} R_{ecfd} - \frac{1291}{12672} R^{cd} R^{ef}{}_{ac} R_{efbd} - \frac{767}{1584} R_{ab} R_{cdef} R^{cdef} \\
& + \frac{2155}{12672} g_{ab} R_{ghcd} R^{ghc}{}_{e} R^{de} - \frac{521}{1408} R R_{ghca} R^{ghc}{}_{b} + \frac{1679}{12672} g_{ab} R R_{ghcd} R^{ghcd} \\
& - \frac{25}{44} R^{gh} R_{gahd} R^{d}_{b} - \frac{3569}{3168} R R_{gacb} R^{gc} + \frac{7}{704} g_{ab} R^3 .
\end{aligned}$$

$$\hat{C}^{(2)}_{ab} = \frac{15}{44} R_{acde} R_{b}{}^{cde} - \frac{285}{176} R_{cd} R_{a}{}^{c}{}_{b}{}^{d} + \frac{15}{44} R R_{ab} , \qquad \hat{C}^{(1)}_{ab} = \frac{45}{44} R_{ab} .$$

IIB SUGRA in a quasitopological frame

$$\tilde{I} = \frac{1}{16\pi G}\int d^5x\sqrt{|g|}\left(\frac{12}{L^2} + R + \frac{15\zeta(3)}{88}\alpha'^3\left[\frac{3}{L^6}R + \frac{1}{L^4}\left(R_{abcd}R^{abcd} - 4R_{ab}R^{ab} + R^2\right) + \frac{1}{L^2}Q^{(3)}\left(\mathcal{R}^3\right) + Q^{(4)}\left(\mathcal{R}^4\right)\right]\right) ,$$

with

$$Q^{(3)}\left(\mathcal{R}^3\right) = -\frac{7}{6}R^{ab}{}_{cd}R^{ce}{}_{bf}R^{df}{}_{ae} - R_{ab}R^{cd}{}_{ae}R_{cd}{}^{be} - \frac{1}{2}R_{ab}R^{cd}R^{a}{}_{c}R^{b}{}_{d} + \frac{1}{3}R^{a}{}_{b}R^{b}{}_{c}R^{c}{}_{a} - \frac{1}{2}RR^{a}{}_{b}R^{b}{}_{a} + \frac{1}{12}R^3 ,$$

$$Q^{(4)}\left(\mathcal{R}^4\right) = \frac{11}{15}\left(W^4 + R^{ab}\hat{C}^{(3)}_{ab}\right) .$$

$Q^{(3)}(\mathcal{R}^3)$ discovered in 1003.4773 [gr-qc] J.O. and S. Ray (Birkhoff's Theorem). Also presented in e-Print: 1003.5357 [gr-qc] by R. Myers and B. Robinson (holographic c-function).

More on quasitopological gravities

- $Q^{(4)}(\mathcal{R}^4)$ constructed in e-Print: 1109.4708 [hep-th] Dehghani, Bazrafshan, Mann, Mehdizadeh, Ghanaatian, Vahidinia.
- $Q^{(5)}(\mathcal{R}^5)$ constructed in e-Print: 1702.04676 [hep-th] by Cisterna, Hassaine, Guajardo, Oliva.
- $Q^{(n)}(\mathcal{R}^n)$ in terms of a recurrence relation in 1909.07983 [hep-th] by Bueno, Cano, Hennigar.
- arXiv:2304.08510 [gr-qc] Moreno and Murcia provided an explicit solution for the recurrence relation.

Ok, these theories exist at any order $\mathcal{R}^n$, in any dimension, but what properties do they have?

The quasitopological theories and their spherically symmetric solutions

$$I = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\left(R - 2\Lambda + \sum_{n}^{n_{\max}} \alpha_n \mathcal{Z}_n\right) ,$$

with $\mathcal{Z}_n$ are quasitopological combinations. On

$$ds^2 = -f(t,r)dt^2 + \frac{dr^2}{g(t,r)} + r^2 d\Sigma_\gamma^2 ,$$

lead to $f(t,r) = F(t)f(r)$, $g(t,r) = f(r)$ with

$$\sum_{n=0}^{n_{\max}} \alpha_n r^{D-2n-3}\left(\gamma - f\left(r\right)\right)^n = m\,,$$

with m an integration constant. This is the same equation that one obtains in Lovelock theories (the most general theory giving second order field equations for generic spacetimes), but Lovelock theories exist up to $n_{\max} \leq \left[\frac{D}{2}\right]$, while quasitopological theories exist at any order.

The graviton propagator on AdS

Quasitopological gravities have many maximally symmetric solutions. Only one of them is perturbative on the higher curvature couplings, this is the Einstein branch. Let us assume that the maximally symmetric solution is AdS. The equation for the graviton propagating on AdS is

$$\frac{1}{2}\left(1+\frac{2}{3}a_2\lambda-\frac{1}{36}a_3\lambda^2+\frac{1}{324}a_4\lambda^3+\frac{5}{7776}a_5\lambda^4\right)G^L_{\mu\nu}=0\ , \quad (1)$$

where the linearized Einstein tensor is defined by

$$\begin{aligned}G^L_{\mu\nu}=&\bar\nabla_{(\mu|}\bar\nabla_\sigma h^\sigma{}_{|\nu)}-\frac{1}{2}\bar\Box h_{\mu\nu}-\frac{1}{2}\bar\nabla_\mu\bar\nabla_\nu h+5\Lambda h_{\mu\nu}-\Lambda h\bar g_{\mu\nu}\\&-\frac{1}{2}\bar g_{\mu\nu}\left(\bar\nabla^\alpha\bar\nabla^\beta h_{\alpha\beta}-\bar\Box h-4\Lambda h\right)-4\Lambda h_{\mu\nu}\ .\end{aligned}$$

Cosmology? Go to $3+1$ dimensions

- In arXiv:1810.08166 [gr-qc], Arciniega, Edelstein and Jaime discovered a $\mathcal{R}^3$, leading to a first order Friedmann equation! This is unexpected since for generic higher curvature gravities, Friedmann equation contains terms of the form $\ddot{H}$.
- These results led to the topic of Geometric Inflation (see e-Print: 1812.11187 [hep-th] and references thereof), where higher-curvature terms at any order were identified, which lead to a first order Friedmann equation

$$P(H^2) = \kappa\rho/3 + \Lambda_0/3 \tag{2}$$

with $P(x)$ a polynomial.

The 5D origin and more

- In e-Print: 1811.06523 with Cisterna and Grandi, we found that the cubic, quartic and quintic 4D cosmological combination, are the simple dimensional reduction of the $D = 5$ quasitopological Lagrangians. Kinematic argument: 5D static black holes with three-dimensional spacelike sections of constant curvature (spatial geometry of the FLRW ansatz).
- For cosmological perturbations

$$ds^2 = a^2(\tau)\left(-(1+2\epsilon\Phi(\tau,\vec{x}))d\tau^2 + (1-2\epsilon\Psi(\tau,\vec{x}))d\vec{x}^2\right) \ ,$$

follow equations that are of second order in time, and of higher order in the spatial coordinates.

The 5D origin and more

- In JCAP 07 (2020) 041 • e-Print: 2004.03912 [gr-qc], Pookkillath, De Felice and Starobinsky, showed that for the cubic theory, vector perturbations have instatibilites due to the presence of higher derivatives in time.
- In Phys.Rev.D 100 (2019) 12, 126011 • e-Print: 1905.06963 [hep-th] "Duality invariant cosmology to all orders in α'", Hohm and Zwiebach showed that in the cosmological ansatz for a metric, b-field and dilaton, T-duality at every order in α' restricts the form of the corrections leading to a Friedmann equation of the form $P(H) = \cdots$.

Recent news: Regular bhs in QT

A polinomial

$$\sum_{n=0}^{n_{\max}} \alpha_n r^{D-2n-3} \left(\gamma - f\left(r\right)\right)^n = m \,,$$

near the origin leads to

$$f\left(r\right) \propto \frac{1}{r^{\frac{D-2n_{\max}-1}{n_{\max}}}} + \dots \,.$$

Regular center if $n_{\max} \to \infty$ This cannot be achieved in Lovelock theories, but it can be done in QT. Bypass propossed by Bueno, Cano and Hennigar in Phys.Lett.B 861 (2025) 139260 • e-Print: 2403.04827 [gr-qc].

Now we know that a resummed infinite series of higher curvature quasitopological terms provide regular black holes in $D \geq 5$. Since when dimensionally reduce to $D = 4$ these theories lead to simple cosmologies, what are the possible cosmological evolutions when one considers the resummed infinite series of higher curvature terms leading to a first order Friedmann equation?

The ansatz

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2\right) \tag{3}$$

the equation

$$P\left(H^2 + \frac{k}{a^2}\right) = \frac{\kappa}{3}\left(\frac{\rho_r^0}{a^4} + \frac{\rho_m^0}{a^3}\right) + \frac{\Lambda}{3} \tag{4}$$

The dynamics is controlled by the function P, which is function obtained from a resummed infinite polynomial. Our analisis relies on the pole structure of the function P. We assume $P(x) \sim x$ for small x (recover GR for small curvature).

$$P\left(H^2+\frac{k}{a^2}\right)=\frac{\kappa}{3}\left(\frac{\rho_r^0}{a^4}+\frac{\rho_m^0}{a^3}\right)+\frac{\Lambda}{3}$$

Near a regular point

$$P(\bar{x})+P'(\bar{x})(H^2-\bar{x})=\frac{c}{a^{2q}} \tag{5}$$

therefore

$$H^2=\frac{\tilde{c}}{a^{2q}}+\bar{x} \tag{6}$$

- Nonlinear regime triggered at small Universe leads to $H^2 \sim (t-\bar{t})^{-2}$.
- Nonlinear regime triggered at large Universe leads to de Sitter phase $H^2 \sim \bar{x}$.

$$P\left(H^2 + \frac{k}{a^2}\right) = \frac{\kappa}{3}\left(\frac{\rho_r^0}{a^4} + \frac{\rho_m^0}{a^3}\right) + \frac{\Lambda}{3}$$

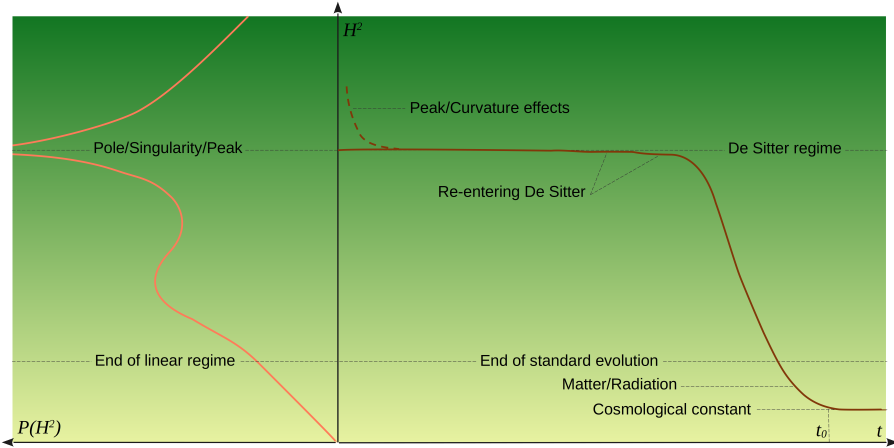
Near a regular singular point

$$\frac{r_p}{(H^2 - \bar{x})^p} = \frac{c}{a^{2q}} \tag{7}$$

therefore

$$H^2 = \tilde{c}a^{2q/p} + \bar{x} \tag{8}$$

- Nonlinear regime triggered at small Universe leads to de Sitter phase.
- Nonlinear regime triggered at large Universe leads to $H^2 \sim (t - \bar{t})^{-2}$.



H^2
Peak/Curvature effects
Pole/Singularity/Peak
De Sitter regime
Re-entering De Sitter
End of linear regime
End of standard evolution
Matter/Radiation
Cosmological constant
$P(H^2)$
t_0
t

Conclusions

- The local dynamics around a generic regular point can be power-law or de Sitter.
- Near a pole, or a peak of the function $P(x)$, one has a de Sitter phase, and as we evolve into the past, a power-law phase could be reached again.
- We confirmed these results also around some types of essential singularities, namely

$$P(x) \sim e^{\frac{w_p}{(x-x_i)^p}} \tag{9}$$

- What about more general, resurgent-like transeries behavior?
- Detail evolution requires a detail knowledge of the function $P(x)$, which in turns could in principle be computed from the whole series of α' corrections in String Theory. Is this approach really feasible?