

# Aplicaciones del modelo Chaplygin-Jacobi

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# Motivación

Como campo escalar tenemos

$$H(\phi) = H_0 \cosh^{\frac{1}{1+\alpha}} \left[ \sqrt{\frac{6\pi}{m_{Pl}^2}} (1+\alpha)(\phi - \phi_0) \right]$$



Ecuación de estado

$$p_{gceg} = -\frac{B}{\rho_{gceg}^\alpha}$$

## Single-field inflation à la generalized Chaplygin gas

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**Abstract.** In the simplest scenario for inflation, i.e. in the single-field inflation, it is presented an inflaton field with properties equivalent to a generalized Chaplygin gas. Their study is performed using the Hamilton-Jacobi approach to cosmology. The main results are contrasted with the measurements recently released by the Planck data, combined with the WMAP large-angle polarization. If the measurements released by Planck for the scalar spectral index together with its running are taken into account it is found a value for the  $\alpha$ -parameter associated to the generalized Chaplygin gas given by  $\alpha = 0.2578 \pm 0.0009$ .

# Generalización elíptica

$$H(\bar{\phi}, \bar{k}) = H_0 \mathbf{nc}^{\frac{1}{1+\alpha}} \left( [1+\alpha] \Phi \right)$$

$$\mathbf{nc}(x) = 1/\mathbf{cn}(x)$$

$$\mathbf{cn}(x) \equiv \mathbf{cn}(x; \bar{k})$$

Módulo de la función elíptica

Ecuación de estado

$$p(\rho) = -\frac{B \bar{k}}{\rho^\alpha} - \bar{k}' \rho \left( 2 - \frac{1}{B} \rho^{\alpha+1} \right),$$

Con  $\bar{k}=1$  se recupera el esquema original

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## The generalized Chaplygin-Jacobi gas

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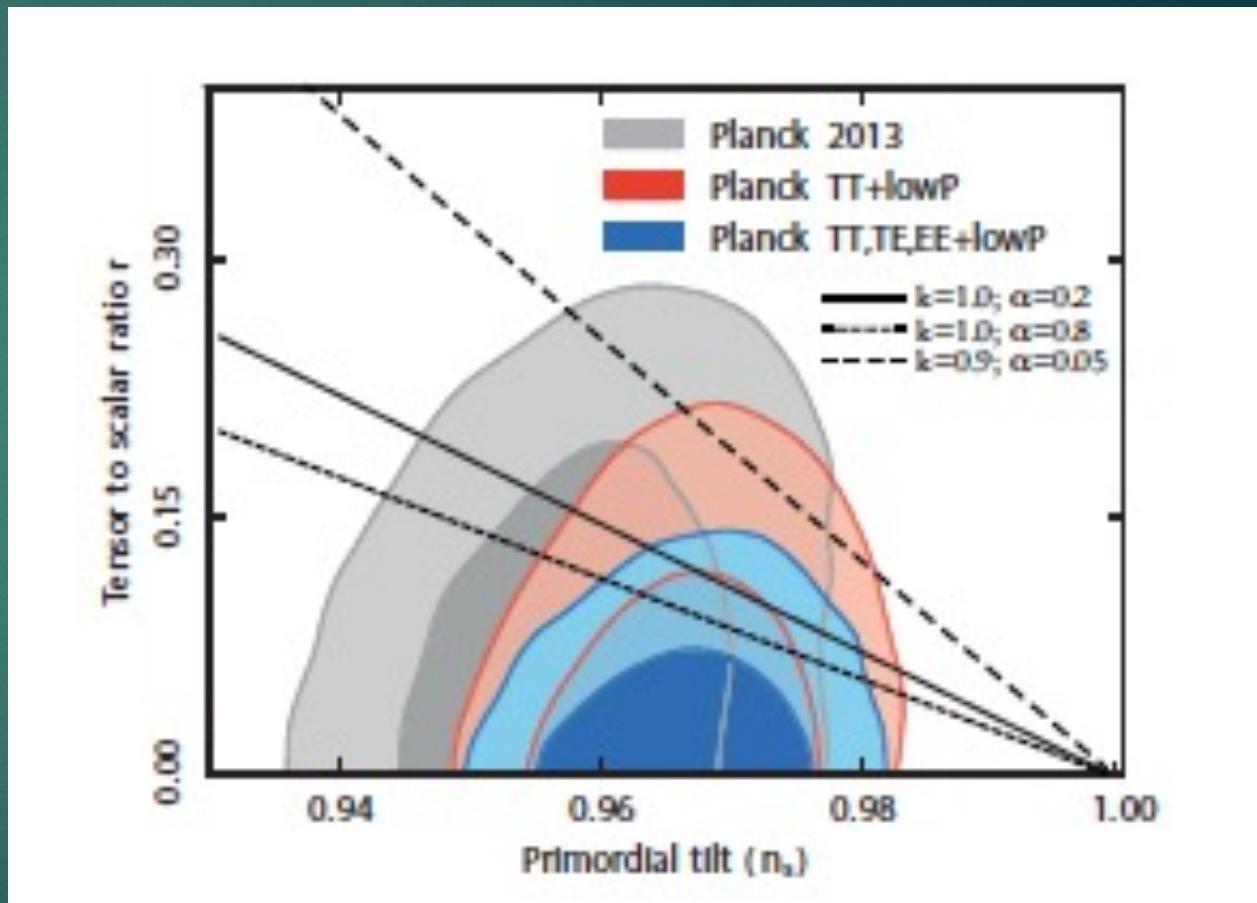
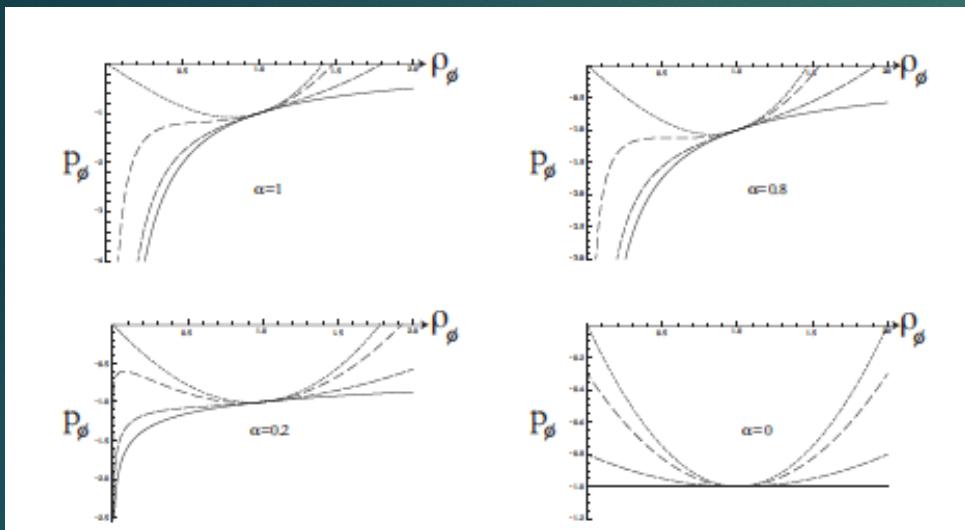
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**Abstract.** The present paper is devoted to find a new generalization of the generalized Chaplygin gas. Therefore, starting from the Hubble parameter associated to the Chaplygin scalar field and using some elliptic identities, the elliptic generalization is straightforward. Thus, all relevant quantities that drive inflation are calculated exactly. Finally, using the measurement on inflation from the *Planck* 2015 results, observational constraints on the parameters are given.

Se asume que  $B > 0$



Sin embargo, el caso  $B < 0$  es interesante

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Full length article

**Black hole in a generalized Chaplygin–Jacobi dark fluid: Shadow and light deflection angle**

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**ABSTRACT**

We investigate a generalized Chaplygin-like gas with an anisotropic equation of state, characterizing a dark fluid within which a static spherically symmetric black hole is assumed. By solving the Einstein equations for this black hole spacetime, we explicitly derive the metric function. The spacetime is parametrized by two critical parameters,  $\mathcal{B}$  and  $a$ , which measure the deviation from the Schwarzschild black hole and the extent of the dark fluid's anisotropy, respectively. We explore the behavior of light rays in the vicinity of the black hole by calculating its shadow and comparing our results with the Event Horizon Telescope observations. This comparison constrains the parameters to  $0 \leq \mathcal{B} \lesssim 0.03$  and  $0 < a \lesssim 0.1$ . Additionally, we calculate the deflection angles to determine the extent to which light is bent by the black hole. These calculations are further utilized to formulate possible Einstein rings, estimating the angular radius of the rings to be approximately  $37.6 \mu\text{as}$ . Throughout this work, we present analytical solutions wherever feasible, and employ reliable approximations where necessary to provide comprehensive insights into the spacetime characteristics and their observable effects.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

P( $\rho$ )



GCJDF is an anisotropic fluid

$$T_\mu{}^\nu = \rho u_\mu u^\nu + p_r k_\mu k^\nu + p_t \Pi_\mu{}^\nu, \quad T_\mu{}^\nu = -(\rho + p_t) \delta_\mu^t \delta_t^\nu + p_t \delta_\mu^\nu + \Delta \delta_\mu^r \delta_r^\nu,$$

$$\Delta \equiv (p_r - p_t) \longrightarrow \text{Anisotropy factor}$$

$$p(\rho) = p_t + \frac{\Delta}{3} = \frac{2p_t}{3} + \frac{p_r}{3}, \quad p_r = -\rho$$

$$p_t = -\frac{3B\kappa}{2\rho^\alpha} - \frac{(6\kappa' - 1)}{2}\rho + \frac{3\kappa'}{2B}\rho^{\alpha+2}$$

Obtenemos de forma analítica

$$\left(\frac{\rho}{\rho_0}\right)^{1+\alpha} = \mathcal{B} \left[ \frac{\mathcal{k}(1+y) - 1}{\mathcal{k}'(1+y)} \right] \quad y \equiv y(r) = q^2/r^{3(1+\alpha)},$$

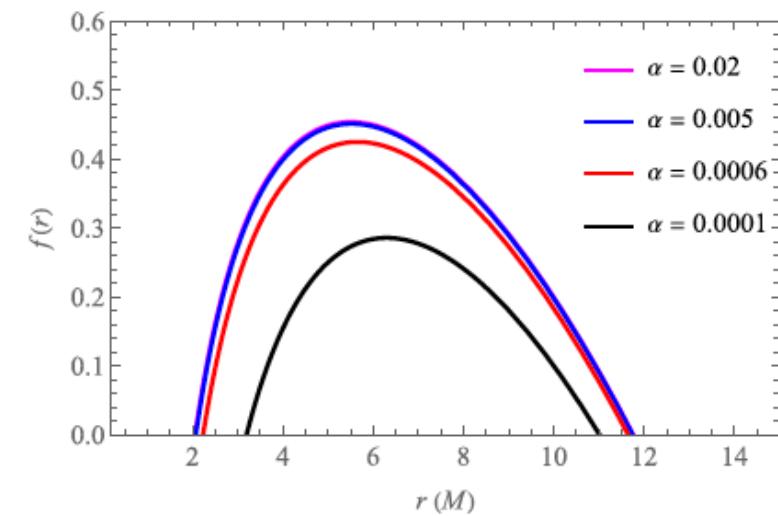
$$\rho(r) \approx \rho_0 \mathcal{B}^{\frac{1}{1+\alpha}} \left[ \frac{1 - \mathcal{k}y}{\mathcal{k}'y} \right]^{\frac{1}{1+\alpha}} \quad y \gg 1$$

$$\rho(r) \approx \rho_0 \mathcal{B}^{\frac{1}{1+\alpha}} = (-B)^{\frac{1}{1+\alpha}} \quad y \ll 1$$

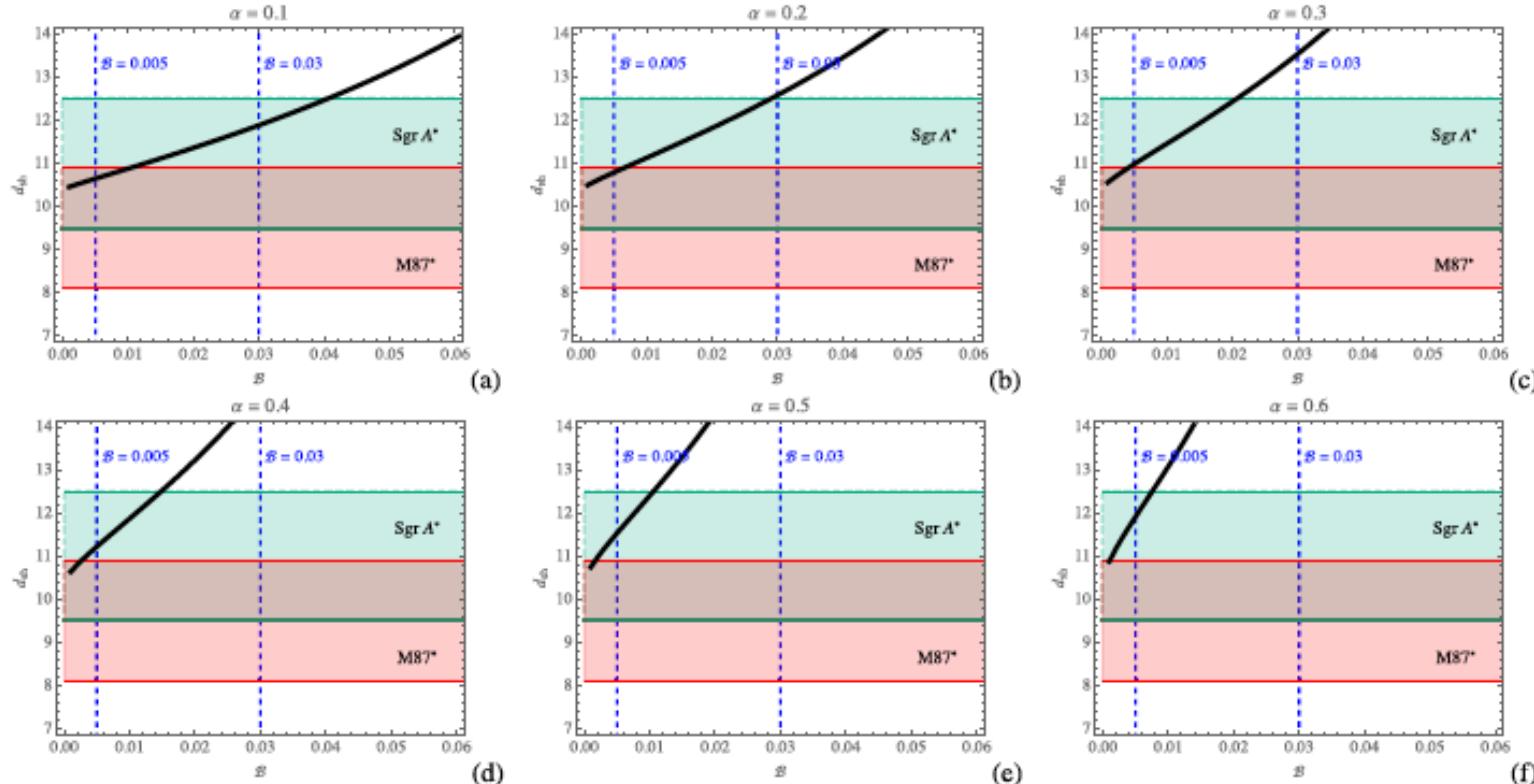
$$f(r) = 1 - \frac{2M}{r} - \frac{1}{3}\lambda(r)r^2,$$

$$\lambda(r) = \rho_0 \left[ \frac{\mathcal{B}(1+y(r))(\hbar y(r) - \hbar')}{1 + \frac{\hbar}{\hbar'} y(r)} \right]^{\frac{1}{1+\alpha}} F_1 \left( -\frac{1}{1+\alpha}; -\frac{1}{1+\alpha}, \frac{1}{1+\alpha}; \frac{\alpha}{1+\alpha}; \frac{\hbar}{\hbar'} y(r), -y(r) \right),$$

Función hipergeométrica de Apell de dos variables



## Comparación con EHT data: diámetro de la sombra



$$0 < \alpha \lesssim 0.1$$

$$\mathcal{B} \lesssim 0.03$$

Fig. 9. The  $\mathcal{B}$ -profiles of the theoretical shadow diameter  $d_{sh}^{\text{theo}}$  (black curves) compared with the observed shadow diameters of M87\* and Sgr A\*, for various values of the  $\alpha$ -parameter. The plots are generated for  $\hbar = 0.4$ ,  $q = 0.1$ , and  $\rho_0 = 1$ .

# Cosmic slowing down of acceleration with the Chaplygin–Jacobi gas as a dark fluid?

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Nuestro problema es resolver las ecuaciones de Einstein usando la métrica FLRW

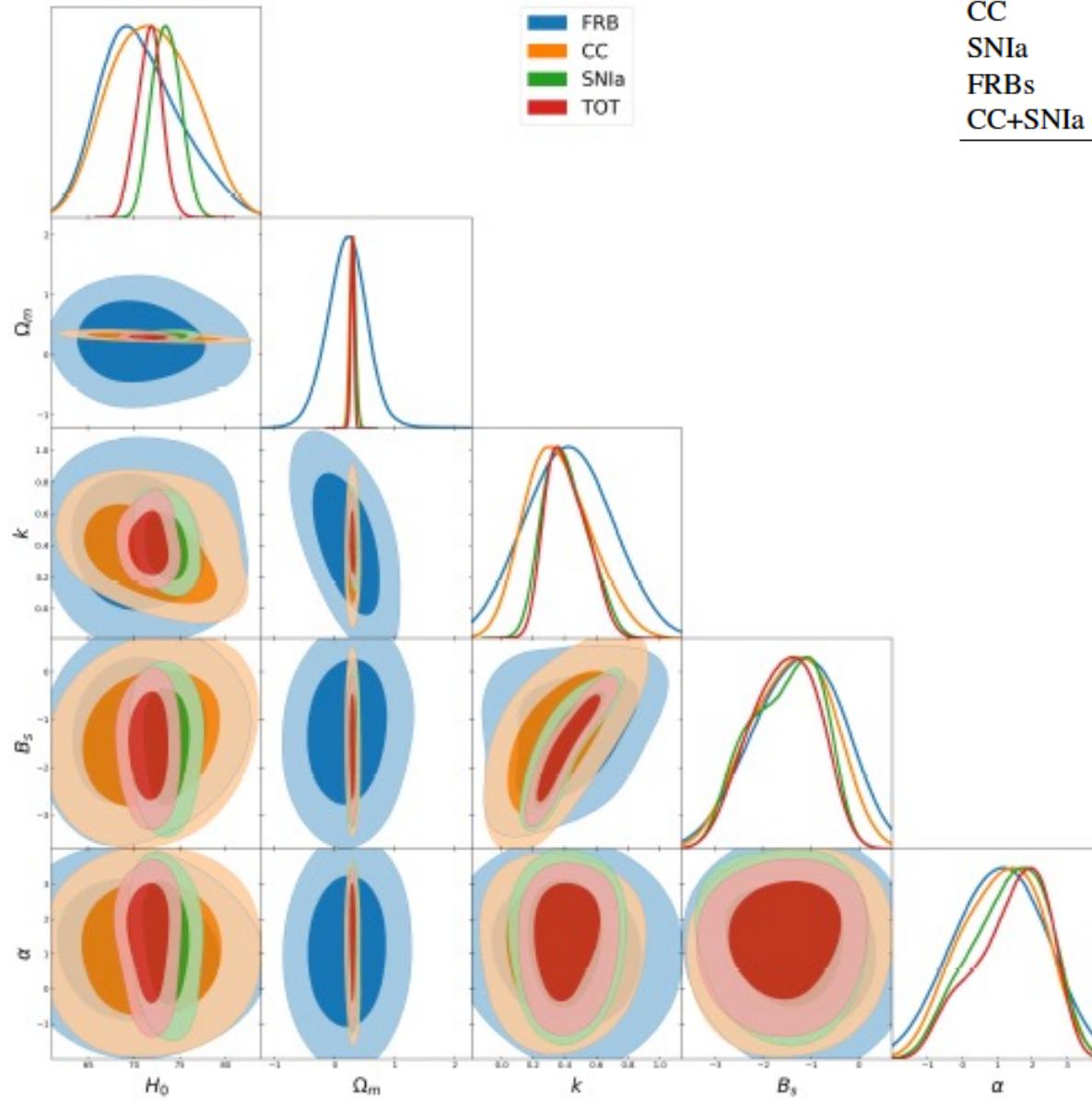
$$\frac{\dot{\rho}}{\rho} = -3\frac{\dot{a}}{a} \left( -\frac{Bk}{\rho^{\alpha+1}} + 2k - 1 + \frac{(1-k)}{B} \rho^{\alpha+1} \right) = -3\frac{\dot{a}}{a} \left( \frac{-Bk - (1-2k)\rho^{\alpha+1} + \frac{1-k}{B}\rho^{2(\alpha+1)}}{\rho^{\alpha+1}} \right)$$



$$\rho(a) = \rho_0 \left[ B_s - \frac{B_s A}{Ak' - a^{3(1+\alpha)}} \right]^{\frac{1}{1+\alpha}}$$

El parámetro de Hubble adimensional

$$E^2(z) = \Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0} - \Omega_{r0}) \left[ B_s + \frac{B_s(1-B_s)(1+z)^{3(1+\alpha)}}{(k' + B_s k) - (1-B_s)k'(1+z)^{3(1+\alpha)}} \right]^{\frac{1}{1+\alpha}}$$

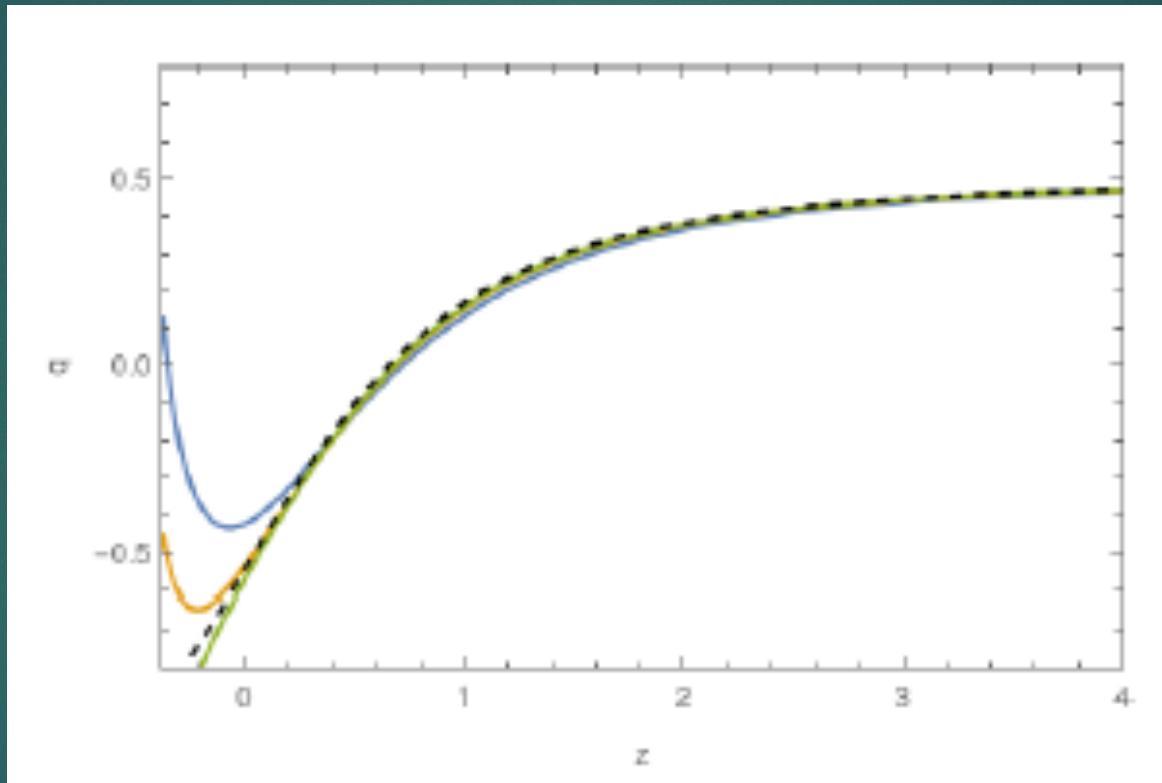


Data	$H_0$	$\Omega_m$	$B_s$	$k$	$\alpha$
CC	$72.0 \pm 4.3$	$0.30 \pm 0.05$	$-1.40^{+0.94}_{-0.82}$	$0.36^{+0.18}_{-0.23}$	$1.1^{+1.3}_{-1.1}$
SNIa	$73.5 \pm 1.5$	$0.31^{+0.04}_{-0.04}$	$-1.52^{+0.91}_{-0.67}$	$0.41^{+0.12}_{-0.16}$	$1.35^{+1.3}_{-0.67}$
FRBs	$70.8^{+3.6}_{-5.1}$	$0.25 \pm 0.23$	$-1.27^{+1.0}_{-0.8}$	$0.43^{+0.27}_{-0.30}$	$1.1 \pm 1.2$
CC+SNIa+FRBs	$71.7^{+1.4}_{-1.3}$	$0.30 \pm 0.03$	$-1.55^{+0.79}_{-0.67}$	$0.41^{+0.11}_{-0.15}$	$1.42^{+1.3}_{-0.77}$

MCMC analysis

FIG. 2.  $1\sigma$  and  $2\sigma$  C.L. curves from the MCMC analysis using the SNIa (Pantheon+), CC, and FRBs datasets, alongside the joint analysis

El gas de Chaplygin–Jacobi admite regiones de parámetros que llevan a una desaceleración del universo posterior a la aceleración actual.



En ambos contextos tenemos valores de parámetros que se superponen, pero debemos interpretar cuál es el significado físico de esto.

Tenemos trabajo futuro: estudio termodinámico, completar la estructura geodésica, estudio de interacción, etc.

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