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Nonlinear Scalarization of Schwarzschild Black Hole in Scalar-Torsion Teleparallel Gravity

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Outline

Motivation

Beyond the spontaneous scalarization: New fully nonlinear dynamical mechanism for formation of scalarized black holes

Daniela D. Doneva[1,2,*] and Stoytcho S. Yazadjiev[1,3,4,†]

They consider scalar-Gauss-Bonnet theories with coupling functions that do not allow for a tachyonic instability to occurs, i.e. sGB theories which do not exhibit spontaneous scalarization.

$$I = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - 2\nabla_\mu\varphi\nabla^\mu\varphi + \lambda^2 f(\varphi)\mathcal{R}^2_{GB}\right]$$

Does this new mechanism occur in a scalar-tensor teleparallel theory?

Teleparallel Equivalent of General Relativity

Is a formulation of GR that uses the Weitzenböck connection instead of the Levi-Civita connection [Aldrovandi and Pereira, 2013]. **The Weitzenböck connection has non-zero torsion and zero curvature**.
The equivalence with the Riemannian formulation of GR is written as

$$\mathcal{T} \;=\; -R + 2\, e^{-1}\, \partial_\nu (e\, T_\sigma{}^{\sigma\nu}\,)\,, \tag{1}$$

were the torsion scalar $\mathcal{T}$ is given by

$$\mathcal{T} \;=\; S_\rho{}^{\mu\nu}\;\; T^\rho_{\;\mu\nu} \;=\; \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T_{\rho\mu}{}^\rho T^{\nu\mu}{}_\nu\,. \tag{2}$$

In coordinates, the torsion tensor arises from $T^a = de^a$ and from the Weitzenböck connection $\Gamma^\lambda_{\mu\nu} = e_a{}^\lambda\, \partial_\mu e^a{}_\nu$. The superpotential is defined as

$$S^\rho{}_{\mu\nu} = \frac{1}{4}\,(T^\rho{}_{\mu\nu} - T_{\mu\nu}{}^{\;\rho} + T_{\nu\mu}{}^{\;\rho}) + \frac{1}{2}\,\delta^\rho_\mu\;\, T_{\sigma\nu}{}^{\;\sigma} - \frac{1}{2}\,\delta^\rho_\nu\;\, T_{\sigma\mu}{}^{\sigma}. \tag{3}$$

In TEGR the torsion include all the information concerning to the gravitational field and the action is given by

$$I = \frac{1}{2\kappa} \int d^4x e \left(\mathcal{T} + \mathcal{L}_m \right), \tag{4}$$

where $\kappa = 8\pi G$, $e = \det(e^a{}_\mu) = \sqrt{-g}$ and $\mathcal{L}_m$ is the matter Lagrangian. The variation with respect to the vierbein yields the equations of motion

$$e^{-1}\partial_\mu(e S_a{}^{\mu\nu}) - e_a{}^\lambda T^\rho{}_{\mu\lambda} S_\rho{}^{\nu\mu} - \frac{1}{4} e_a{}^\nu \mathcal{T} = 4\pi G\, e_a{}^\rho \overset{\textbf{em}}{T}{}_\rho{}^\nu \;, \tag{5}$$

so the field equations coincide with those of GR for any choice of geometry. Here, $\overset{\textbf{em}}{T}{}_\rho{}^\nu$ is the usual energy-momentum tensor.

Model of Nonlinear Scalarization

In this work, we consider the scalar-torsion model given by the following action

$$I = \int d^4x\, e\left[\left(\frac{1}{2\kappa} + f(\phi)\right)\mathcal{T} + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\right], \tag{6}$$

where ϕ is the scalar field and $f(\phi)$ is a coupling function of the scalar field. We are interested in **studying nonlinear scalarized black hole solutions** in the model (6), so the coupling function must follow the conditions

$$\frac{df(0)}{d\phi} = 0\,, \quad \frac{d^2f(0)}{d\phi^2} = 0\,. \tag{7}$$

The first condition guarantees the **Schwarzschild solution is also solution** to the field equations, for $\phi = 0$, while the second condition was imposed to the fact that **no tachyonic instability is possible**.

Now, varying the action (6) with respect to the vierbein and the scalar field, the equations of motion respectively are

$$\left(\frac{2}{\kappa}+4f(\phi)\right)\left[e^{-1}\partial_\mu(eS_a{}^{\mu\nu})-e_a{}^{\lambda}T^{\rho}{}_{\mu\lambda}S_\rho{}^{\nu\mu}-\frac{1}{4}e_a{}^{\nu}\mathcal{T}\right] \\ -\frac{1}{2}e_a{}^{\nu}\partial_\mu\phi\partial^\mu\phi+4f'(\phi)(\partial_\mu\phi)S_a{}^{\mu\nu}+e_a{}^{\mu}\partial^\nu\phi\partial_\mu\phi=0\,, \tag{8}$$

$$\frac{1}{e}\partial_\nu(eg^{\mu\nu}\partial_\mu\phi)-f'(\phi)\mathcal{T}=0\,. \tag{9}$$

In the following we consider the case $\boldsymbol{f(\phi)=\eta\phi^4}$, where $\boldsymbol{\eta}$ is a constant which shows the strength of the interaction.

For finding **asymptotically flat black hole solutions**, a static and spherically symmetric diagonal metric ansatz is used

$$ds^2 = A(r)dt^2 - \frac{1}{B(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,. \tag{10}$$

A tetrad that satisfy this condition within the Weitzenböck gauge is

$$e^a{}_\mu = \begin{pmatrix} 0 & \frac{i}{\sqrt{B(r)}} & 0 & 0 \\ i\sqrt{A(r)}\cos\varphi\sin\theta & 0 & -r\sin\varphi & -r\cos\theta\cos\varphi\sin\theta \\ i\sqrt{A(r)}\sin\theta\sin\varphi & 0 & r\cos\varphi & -r\cos\theta\sin\theta\sin\varphi \\ i\sqrt{A(r)}\cos\theta & 0 & 0 & r\sin^2\theta \end{pmatrix}\,. \tag{11}$$

Model of Nonlinear Scalarization

Using the metric (10) and tetrad (11), the field equations (8) and (9) yield

$$
\begin{aligned}
E^t_t &\equiv \frac{(rB'(r)+B(r)-1)\left(2\eta\kappa\phi(r)^4+1\right)}{\kappa r^2}+\frac{16\eta B(r)\phi(r)^3\phi'(r)}{r}+\frac{1}{2}B(r)\phi'(r)^2=0\,,\\
E^r_r &\equiv \frac{(rB(r)A'(r)+A(r)(B(r)-1))\left(2\eta\kappa\phi(r)^4+1\right)}{\kappa r^2 A(r)}-\frac{1}{2}B(r)\phi'(r)^2=0\,,\\
E^\theta_\theta &\equiv -rB(r)A'(r)^2\left(2\eta\kappa\phi(r)^4+1\right)+A(r)\Big(2rB(r)A''(r)\left(2\eta\kappa\phi(r)^4+1\right)\\
&+A'(r)\left(\left(rB'(r)+2B(r)\right)\left(2\eta\kappa\phi(r)^4+1\right)+16\eta\kappa rB(r)\phi(r)^3\phi'(r)\right)\Big)\\
&+2A(r)^2\left(B'(r)\left(2\eta\kappa\phi(r)^4+1\right)+\kappa B(r)\phi'(r)\left(16\eta\phi(r)^3+r\phi'(r)\right)\right)=0\,,\\
E_\phi &\equiv B(r)\phi''(r)+\frac{\phi'(r)\left(rB(r)A'(r)+A(r)\left(rB'(r)+4B(r)\right)\right)}{2rA(r)}\\
&+\frac{8\eta\phi(r)^3\left(rB(r)A'(r)+A(r)(B(r)+1)\right)}{r^2A(r)}=0\,.
\end{aligned}
\tag{12}
$$

Scalarized Black Hole Solutions

To find scalarized black hole solutions, we solve the system of equations (12) numerically. To facilitate the numerical integration, we introduce a bounded radial coordinate $\boldsymbol{z = 1 - r_H/r}$ and redefine the metric functions and scalar field as

$$A(z) = za(z), \quad B(z) = zb(z), \quad \phi(z) = (1-z)\psi(z).$$

The differential equations are solved using the shooting method, which requires the Taylor expansions of the metric functions and the scalar field near the event horizon ($z = 0$) and at spatial infinity ($z \to 1$).

Event Horizon

$$a(z) = a_H + a_{H1}(a_H, \psi_H, \eta)z + \dots,$$
$$b(z) = 1 + b_{H1}(b_H, \psi_H, \eta)z + \dots,$$
$$\psi(z) = \psi_H + \psi_{H1}(\psi_H, \eta)z + \dots$$

Spatial Infinity

$$a(z) = 1 + a_1(b_1)(1-z) + \dots,$$
$$b(z) = 1 + b_1(1-z) + \dots,$$
$$\psi(z) = \psi_\infty + \psi_1(b_1, \psi_\infty)(1-z) + \dots$$

The asymptotically flat scalarized black holes solutions are characterized by five parameters: $\{\eta, a_H, \psi_H, \psi_\infty, b_1\}$.

Scalarized Black Hole Solutions

For fixed η, specific values of $\{a_H, \psi_H, \psi_\infty, b_1\}$ yield bounded scalar solutions whose behavior depends on the number of nodes. **While only node-less solutions exist for small η, increasing η leads to multiple solutions with varying numbers of nodes**.

Type of BH	η	**Nodes**	a_H	ψ_H	ψ_∞	b_1	**Label**
Schwarzschild			1	0	0	0	
Scalarized	2	0	0.979	0.250	0.125	-0.005	IIa
		1	0.004	1.410	-0.540	-0.070	IIb
		1	0.205	0.764	-0.719	-0.068	IIc

Table: Parameters of the Schwarzschild and scalarized black hole solutions for $\eta = 2$.

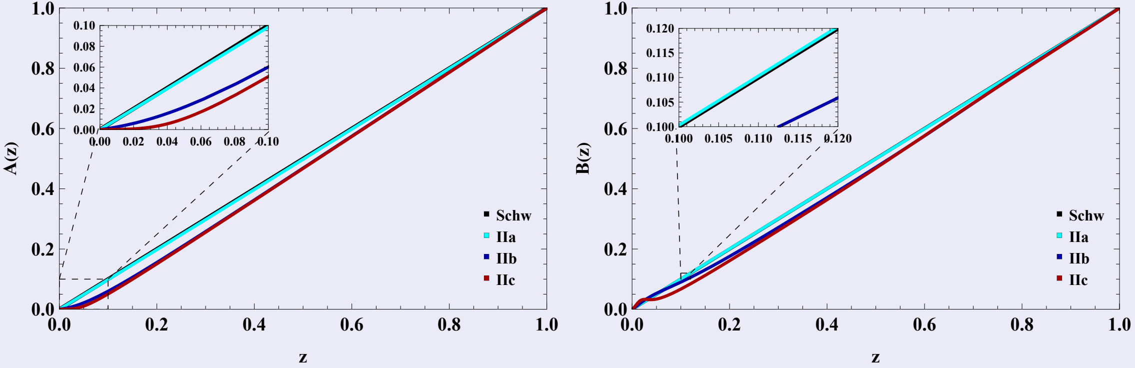

Figure: Left panel for the behavior of $A(z)$ as a function of z. Right panel for the behavior of $B(z)$ as a function of z. Black line for Schwarzschild, cyan line for IIa, blue line for IIb, and red line for IIc.

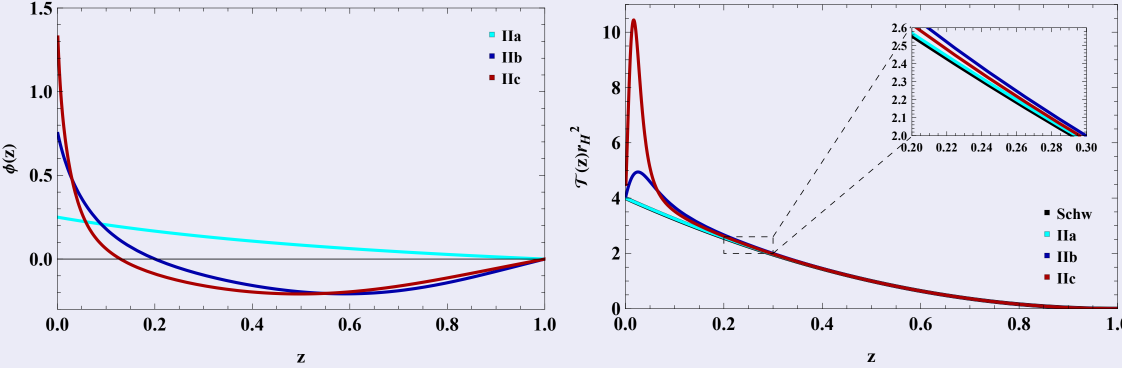

Figure: Left panel for the behavior of $\phi(z)$ as a function of z. Right panel for the behavior of $\mathcal{T}(z)r_H^2$ as a function of z. Black line for Schwarzschild, cyan line for IIa, blue line for IIb, and red line for IIc.

We thermodynamically analyze the solutions by deriving the entropy and mass of the scalarized BH solutions using two different methods:

Padmanabhan Method [Padmanabhan, 2012]

Mass

$$\mathcal{M} = \frac{a_H\left(\frac{1}{\kappa} + 2\eta\psi_H^4\right)}{T}$$

Entropy

$$\mathcal{S} = \frac{a_H(\frac{1}{\kappa} + 2\eta\psi_H^4)}{2T^2}$$

Wald's Formalism [Iyer and Wald, 1994]

ADM Mass

$$\mathcal{E} = \frac{(1 - b_1)\sqrt{a_H}}{\kappa T}$$

Entropy

$$\mathcal{S} = \frac{a_H(\frac{1}{\kappa} + 2\eta\psi_H^4)}{2T^2}$$

The temperature is given by $T = \frac{\sqrt{a_H}}{4\pi r_H}$. Both approaches lead to the same expression for the entropy, $\mathcal{S} = \frac{A}{4G} + 4\pi f(\psi_H)A$, revealing corrections to the standard area law due to the scalar coupling $f(\phi) = \eta\phi^4$, with $A = 4\pi r_H^2$.

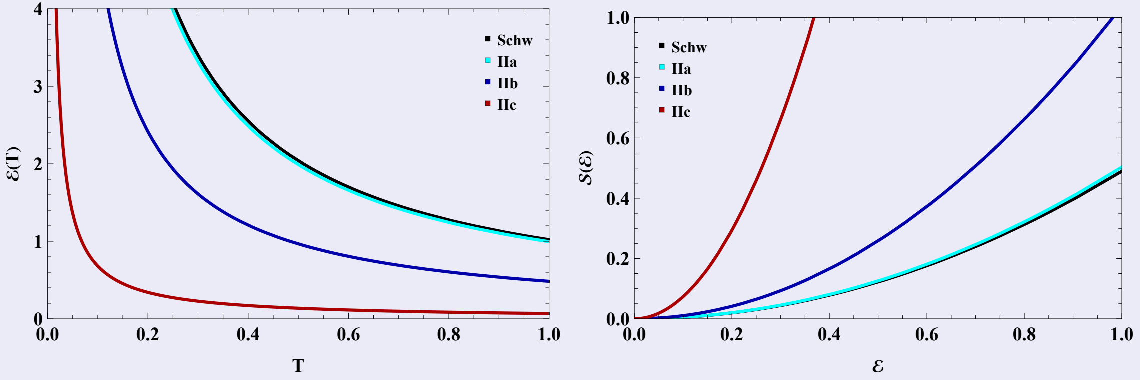

Figure: Left panel for the behavior of the ADM mass $\mathcal{E}$ as a function of temperature. Right panel for the behavior of the entropy as a function of the ADM mass $S(\mathcal{E})$. Black line for Schwarzschild, cyan line for IIa, blue line for IIb, red line for IIc.

Scalarized BHs are always entropically preferred over the Schwarzschild BH.

We study first-order and second-order phase transitions between scalarized and Schwarzschild black holes by analyzing the behavior of the free energy $\mathcal{F}$ and the heat capacity $\mathcal{C}$, which allow us to determine the thermodynamic stability and the nature of the transition.

$$\mathcal{F} = \mathcal{E} - T\mathcal{S} = \frac{\sqrt{a_H}}{T}\left(\frac{1-b_1}{\kappa} - \frac{\sqrt{a_H}}{2}\left(\frac{1}{\kappa} + 2\eta\psi_H^4\right)\right) ,$$

$$\mathcal{C} = \frac{\partial \mathcal{E}}{\partial T} = -\frac{(1-b_1)\sqrt{a_H}}{\kappa T^2} = -\frac{\mathcal{E}}{T} .$$

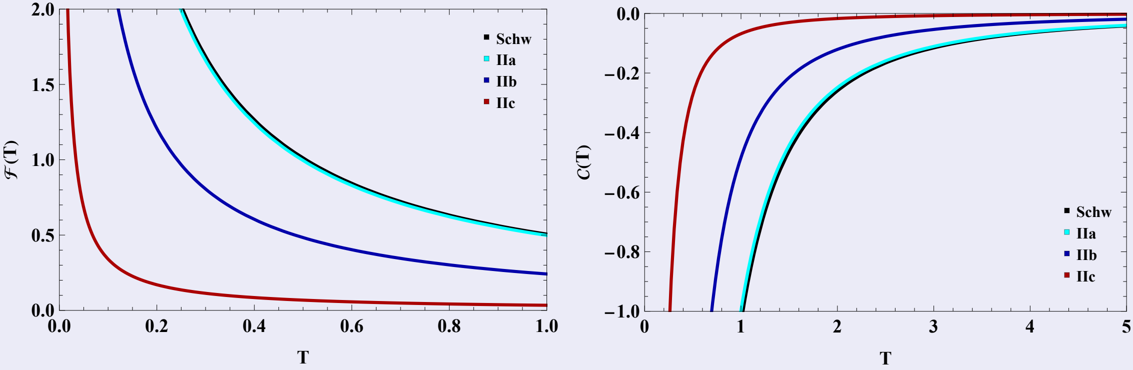

Figure: Left panel for the behavior of $\mathcal{F}$ as a function of temperature. Right panel for the behavior of $\mathcal{C}$ as a function of temperature.

Scalarized black hole solutions are meta-stable and don't exhibit first or second-order phase transitions.

Final Remarks

Summary

A scalar–tensor theory within the framework of Teleparallel Gravity was studied, where the scalar field couples to the torsion scalar through a function that avoids tachyonic instabilities.

Numerical results show the **formation of new black holes with scalar hair**, arising from the scalarization of the Schwarzschild solution.

Asymptotically flat solutions with zero, one, and two scalar-field nodes were analyzed using Padmanabhan's and Wald's formalisms. The **scalarized black holes are meta-stable and show no first- or second-order phase transitions.**

Finally, **scalarized black holes are thermodynamically favored** over the Schwarzschild solution due to their lower free energy and higher entropy.

Final Remarks

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