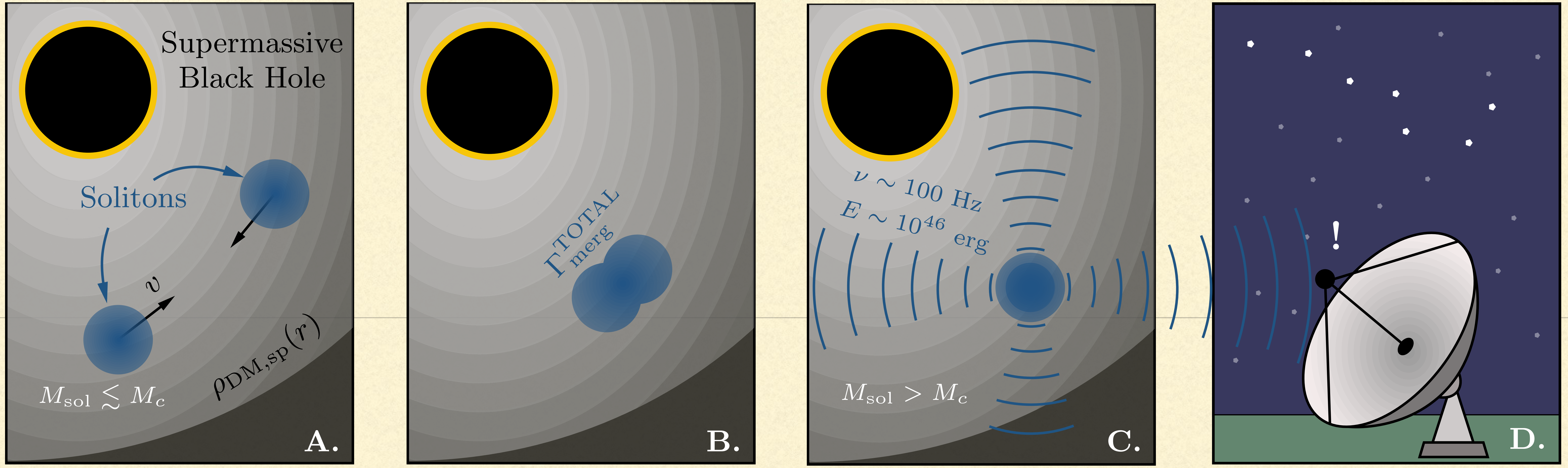
Dark Matter Compact Objects

(Where particle physics, cosmology, and astrophysics meet)



Before Major Merger | Major Merger | Parametric Resonance | Detection

ENRICO D. SCHIAPPACASSE

UNIVERSIDAD SAN SEBASTIAN

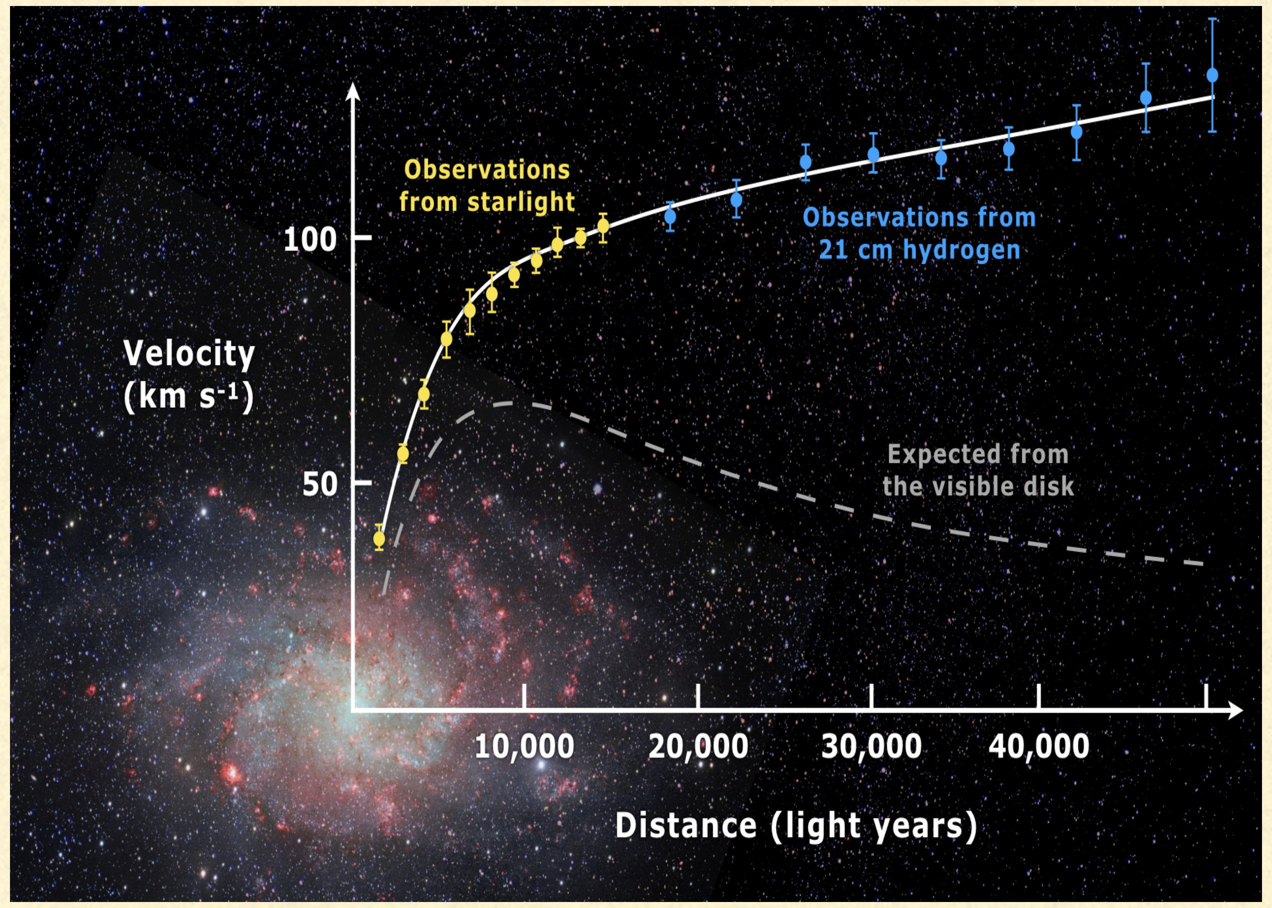
With the collaboration of D.W.P. Amaral (IFAE, Barcelona) and HY. Zhang (SJTU, China)

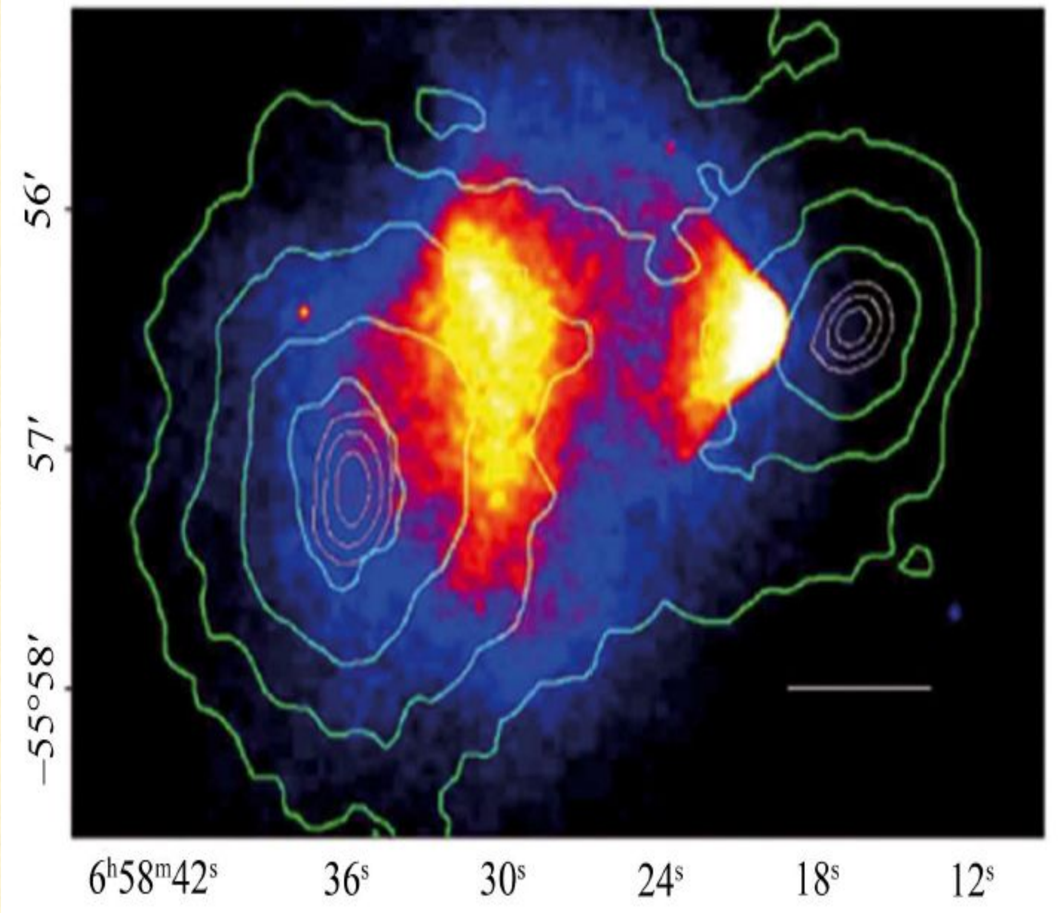
COSMOCONCE y PARTICULAS 5-7 Noviembre 2025, Universidad del Bio-Bio, Campus Concepcion, Chile

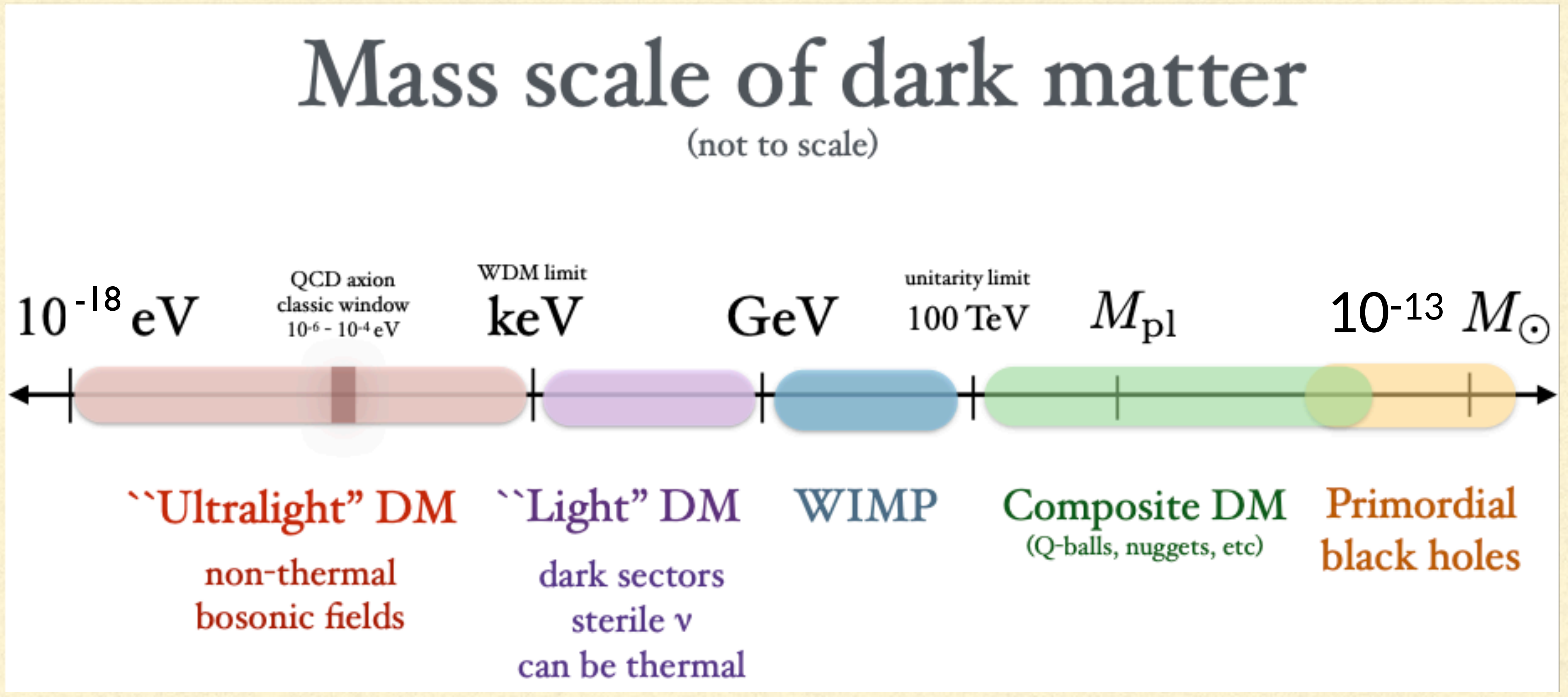
WHAT DO WE DO (NOT) KNOW?

- It is everywhere in the Universe ($\sim 80\,\%$ of total matter).
- Non-relativistic, aggregate itself under the effect of gravity.
- Weak interacting with standard model particles, such as photons.
- Strong evidence (astrophysical/cosmological scales).

- Mass (light, heavy).
- Interactions with ordinary matter.
- Spin (bosonic, fermionic).





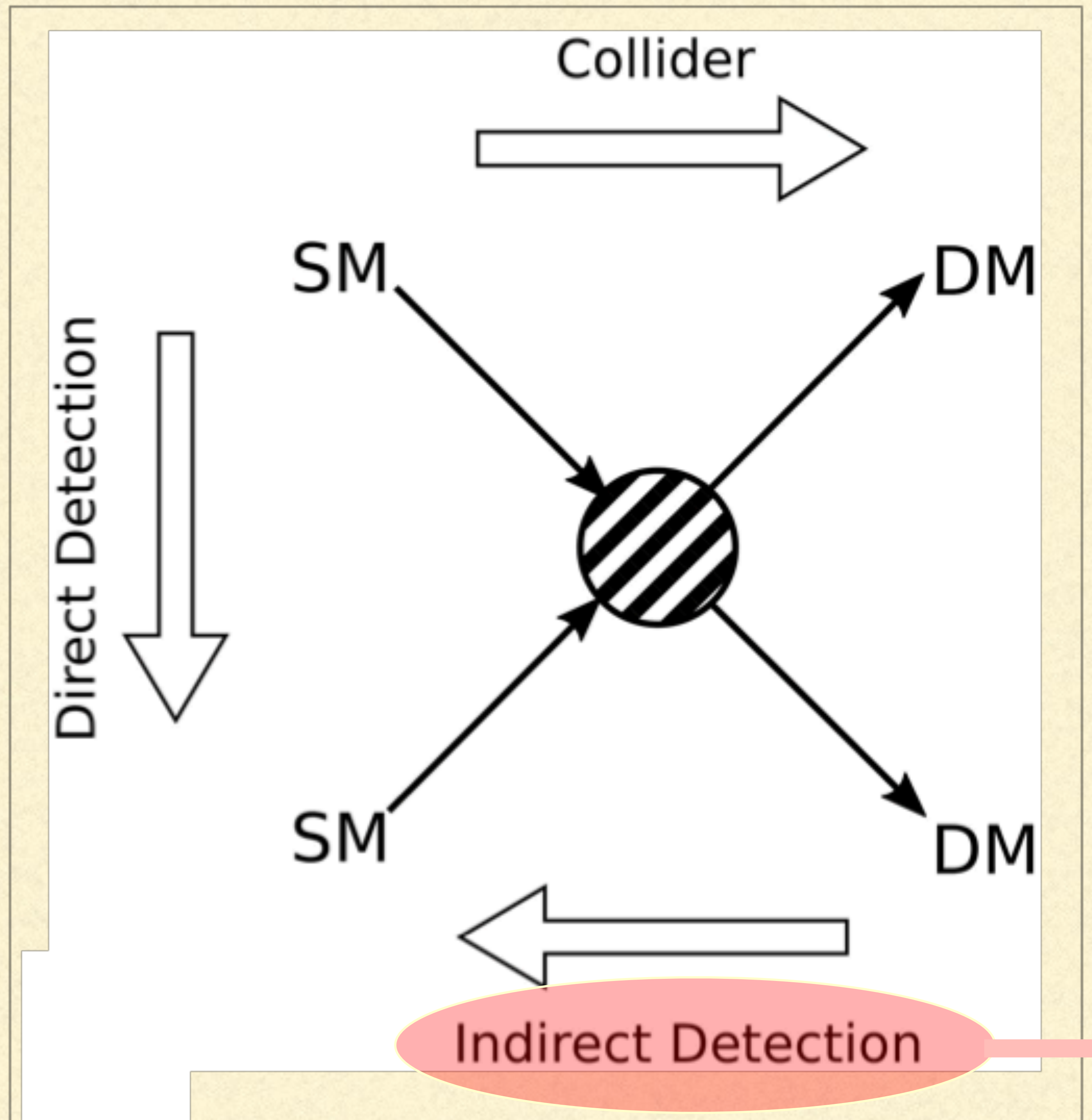


https://astrobites.org/2016/11/04/the-bullet-cluster-a-smoking-gun-for-dark-matter/; MNRAS 311 (2000)

TASI Lectures on dark matter, T. Lin (arXiv:1904.07915)

HOW CAN WE DETECT IT?

- Non-gravitational interactions:
  - Self-interaction
  - Interaction with SM particles
  - Expected to be small for DM



Schematic showing the possible dark matter detection channels.

Looking for DM via solitons

High density

Populate galactic halos

Sizeable fraction of DM

DARK γ-γ INTERACTIONS

- $S[X_\mu(x), A_\mu(x), \mathrm{g}_{\mu\nu}(x)] = \int \mathrm{d}^4x \sqrt{-\mathrm{g}} \left[-\frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{2} m^2 X_\mu X^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_{\mathrm{pl}}^2 \mathrm{R} + \mathscr{L}_{\mathrm{int}} \right]$
- Take the dark photon field $X_\mu = (X_0, \mathsf{X})$ masses $m \ll$ eV being CDM.
- Non-relativistic regime: $k \ll m$ and $\lambda \ll 2\pi/m$.
- Apply perturbative expansion: $|\nabla X_\mu| \sim \lambda^{-1} X_\mu \ll m X_\mu \sim \dot{X}_\mu$.
- Relevant interactions via $\mathscr{L}_{\mathrm{int}} = g^2 \mathcal{O}_i$ (electromagnetic gauge invariance)

Dimension 6 :

$$\mathcal{O}_1 = -\frac{1}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}X_\alpha X^\alpha \approx 2(\mathbf{E}\cdot\mathbf{B})(\mathbf{X}\cdot\mathbf{X})$$

$$\mathcal{O}_2 = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu}X_\alpha X^\alpha \approx (\mathbf{E}\cdot\mathbf{E})(\mathbf{X}\cdot\mathbf{X}) - (\mathbf{B}\cdot\mathbf{B})(\mathbf{X}\cdot\mathbf{X})$$

$$\mathcal{O}_3 = F_{\mu\rho}F^{\nu\rho}X^\mu X_\nu \approx (\mathbf{B}\cdot\mathbf{B})(\mathbf{X}\cdot\mathbf{X}) - (\mathbf{E}\cdot\mathbf{X})^2 - (\mathbf{B}\cdot\mathbf{X})^2$$

$$\mathcal{O}_4 = \tilde{F}_{\mu\rho}\tilde{F}^{\nu\rho}X^\mu X_\nu \approx (\mathbf{E}\cdot\mathbf{E})(\mathbf{X}\cdot\mathbf{X}) - (\mathbf{E}\cdot\mathbf{X})^2 - (\mathbf{B}\cdot\mathbf{X})^2$$

- In the EFT, we may not consider $\mathcal{L}^{(8)}_{\text{int}} = g^2\mathcal{O}_i$ for sufficiently small coupling g^2.
- The system acquires a nonzero vacuum expectation value $\langle X\rangle \sim \bar{X}$, which causes a contribution to lower-order operators.

EFT condition: $g^2\bar{X}^2 \ll 1$

DARK PHOTON SOLITONS

$$S[X_\mu(x), \mathsf{g}_{\mu\nu}(x)] = \int \mathsf{d}^4x \sqrt{-\mathsf{g}} \left[-\frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{2} m^2 X_\mu X^\mu + \frac{1}{2} m_{\mathsf{pl}}^2 \mathsf{R} \right]$$

- For $m \ll$ eV, the field is well described by Classical field theory.
- In the non-relativistic regime we use $\mathsf{g}_{\mu\nu} = \text{diag}(-(1+2\Phi), (1-2\Phi)\delta_{ij})$.
- Express the vector field in terms of a slowly varying function as $\mathbf{X}(t, \mathbf{x}) = \frac{1}{\sqrt{2m}} \left[e^{-imt} \tilde{\mathbf{X}}(t, \mathbf{x}) + e^{imt} \tilde{\mathbf{X}}^*(t, \mathbf{x}) \right]$

$$S[X_\mu(x), \mathsf{g}_{\mu\nu}(x)] = \int \mathsf{d}^4x \left[m_{\mathsf{pl}}^2 \Phi \nabla^2 \Phi + \frac{i}{2} \left(\tilde{\mathbf{X}}^\dagger \cdot \dot{\tilde{\mathbf{X}}} - \tilde{\mathbf{X}} \cdot \dot{\tilde{\mathbf{X}}}^\dagger \right) - \frac{1}{2m} \left(\nabla \tilde{\mathbf{X}}^\dagger \cdot \nabla \tilde{\mathbf{X}} - m\Phi \tilde{\mathbf{X}}^\dagger \cdot \tilde{\mathbf{X}} \right) \right]$$

- From the non-relativistic action of above, we obtain the 3-component SP system:

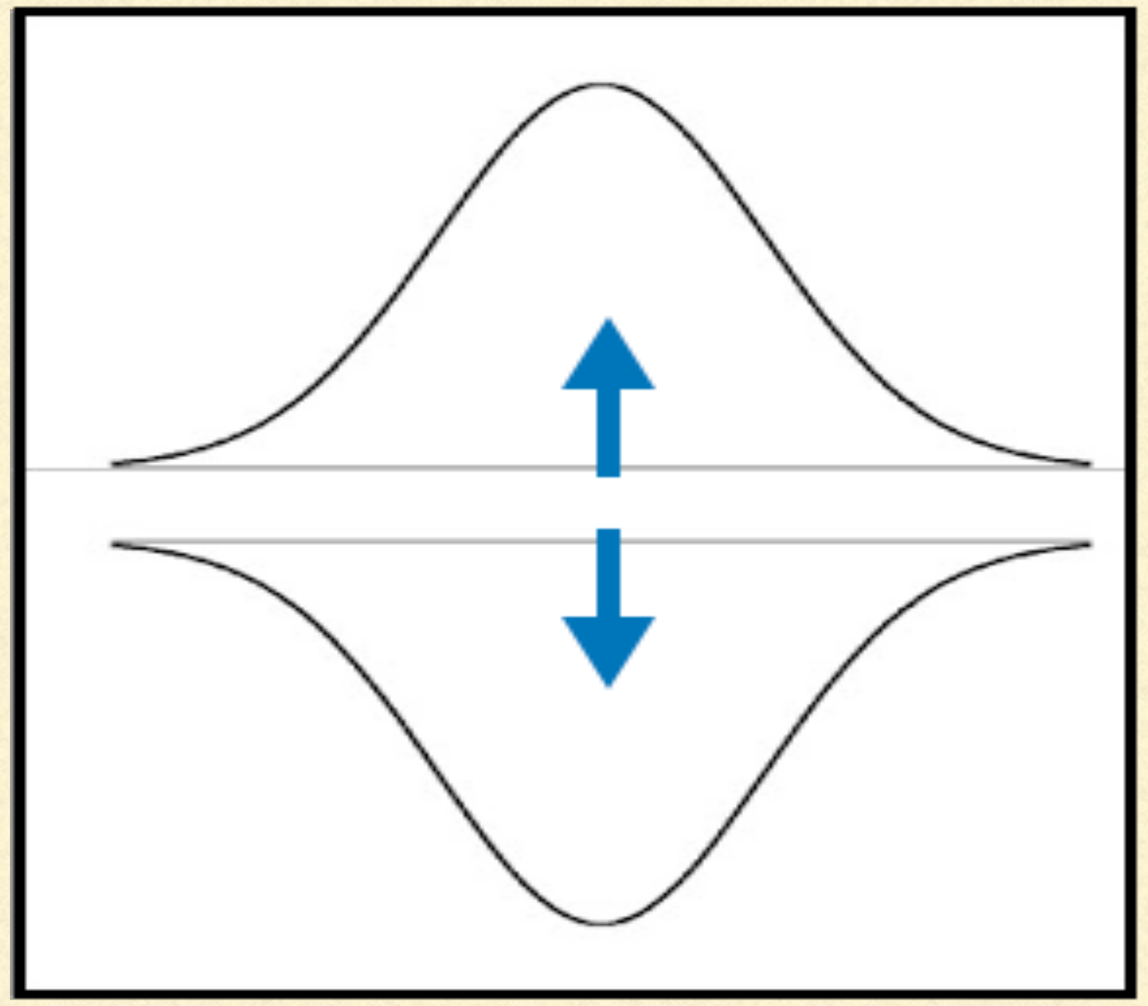
$$i\dot{\tilde{\mathbf{X}}} = -\frac{1}{2m}\nabla^2\tilde{\mathbf{X}} + m\tilde{\mathbf{X}}\Phi \text{ and } \nabla^2\Phi = \frac{m}{2m_{\text{pl}}^2}\tilde{\mathbf{X}}^\dagger \cdot \tilde{\mathbf{X}}$$

$$\tilde{\mathbf{X}}(t, \mathbf{x}) = \frac{1}{2}\sum_\lambda \left[c^\lambda X(r) e^{i(m-\omega)t}\epsilon^\lambda + h.c.\right]$$

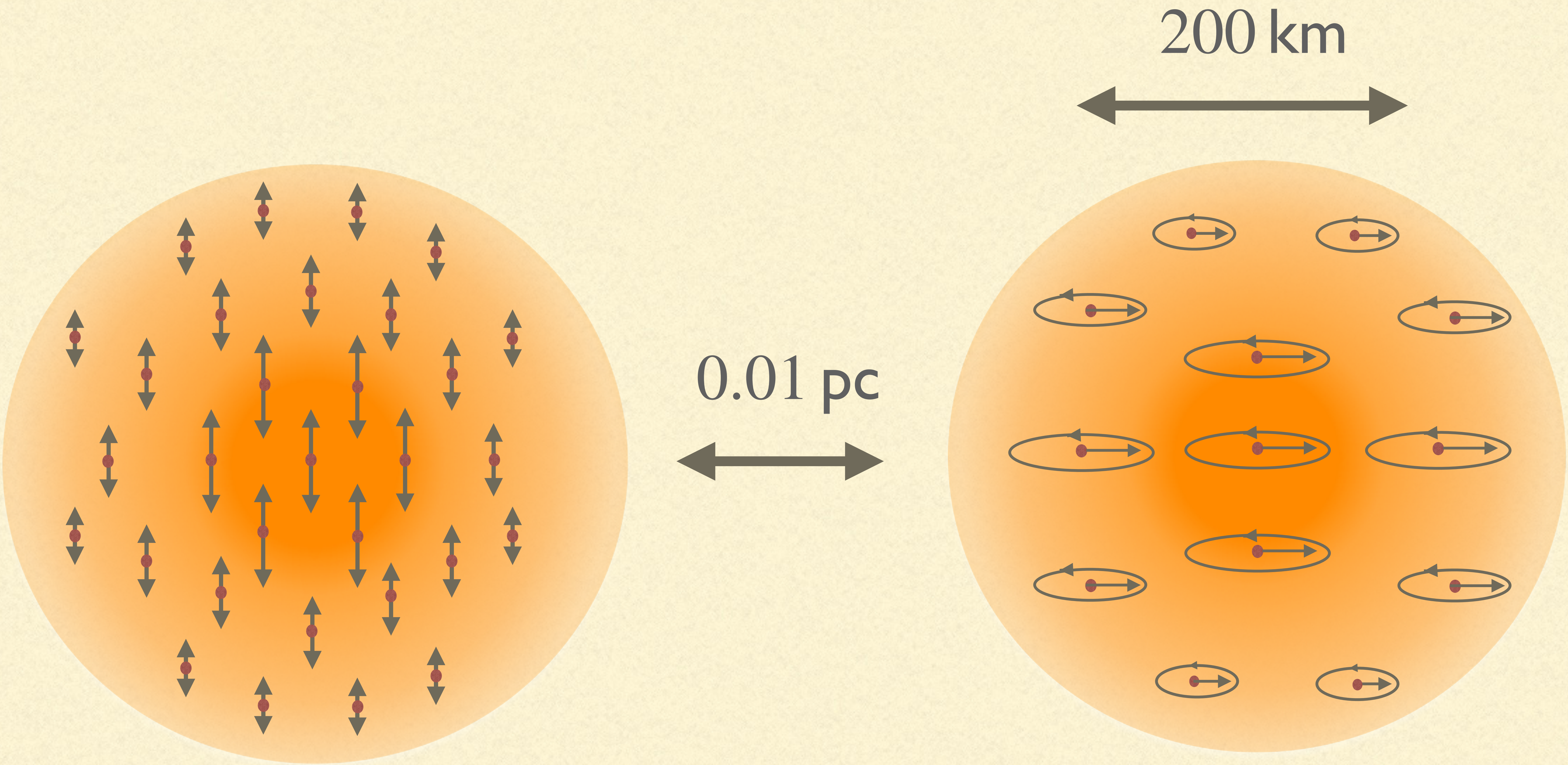
- The field amplitude oscillate in time with an angular frequency $\omega \approx m$.

- The radial profile and Newtonian potential satisfy the time-independent SP system:

$$-(m-\omega)X = -\frac{1}{2mr^2}\frac{\partial}{\partial_r}\left(r^2\frac{\partial X}{\partial r}\right) + m\Phi X, \text{ and } \frac{1}{r^2}\frac{\partial}{\partial_r}\left(r^2\frac{\partial \Phi}{\partial_r}\right) = \frac{m}{2m_{\text{pl}}^2}X^2.$$



Oscillating coherent field
Lowest energy states at fixed N.



$$N \sim 10^{63}\,,\;\; M_{\text{sol}} \sim 10^{-9}\, M_{\odot}$$

$$\bar{\rho} \sim 10^{24}\, M_{\odot}\text{pc}^{-3}$$

$$R_{\text{sol}} \sim 200\,\text{km}\left(\frac{10^{-6}\,\text{eV}}{m}\right)^{2}\left(\frac{10^{-9}\,M_{\odot}}{M_{\text{sol}}}\right)$$

SOLITON PARAMETRIC RESONANCE

- In the background of a coherent oscillating soliton, $\mathbf{X}(t, \mathrm{x})$, operators O_i induce a time-dependent EM field eom.

- EM field's Fourier modes which fall into resonance bands undergo exponential amplification, $A_{\mathbf{k}}(t) = A_{\mathbf{k}}(0)e^{\mu_{\mathbf{k}}t}$.

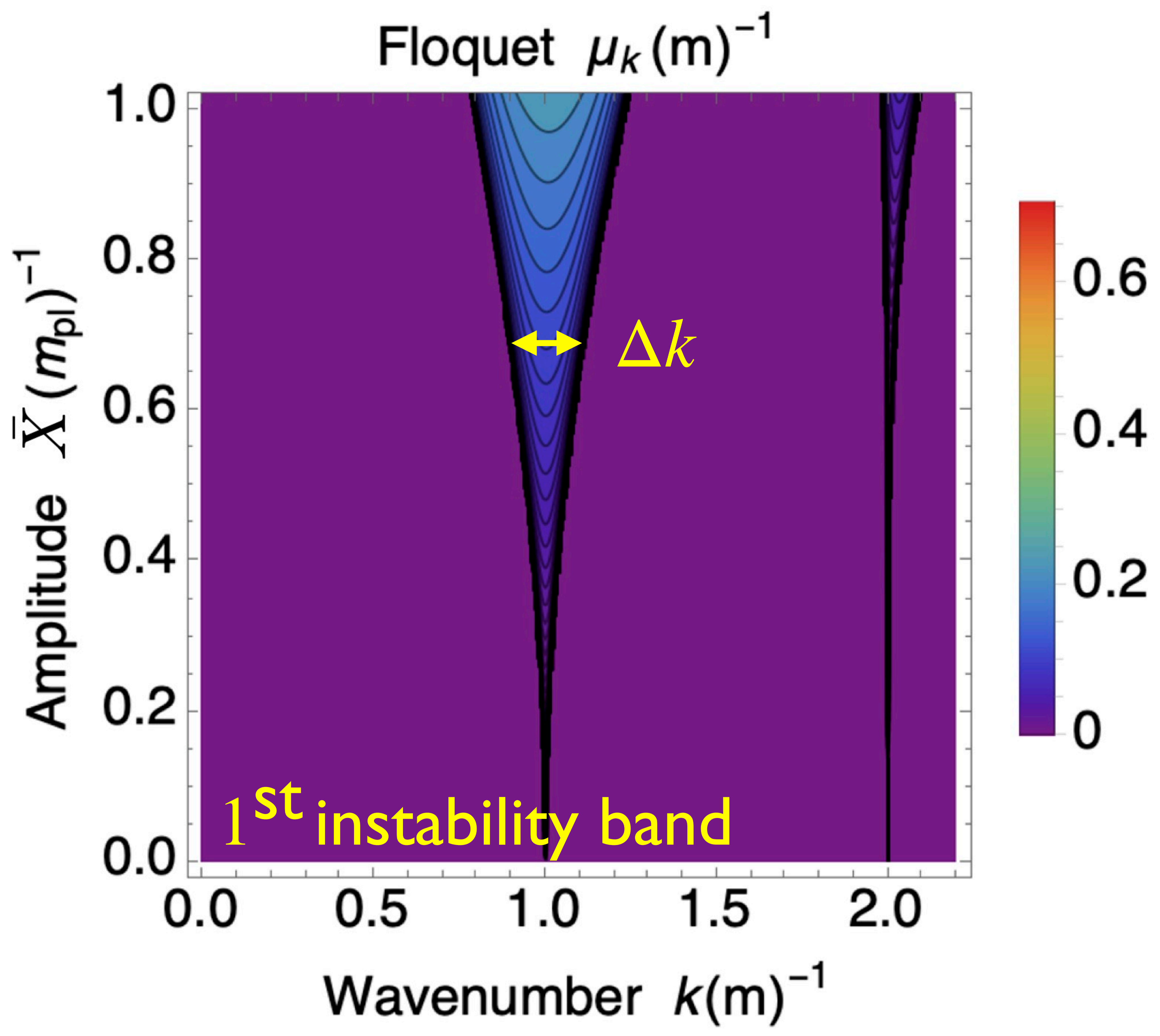
- Let us think we are within the soliton:

$$\mathbf{X} = \bar{X}\cos(mt)\,\hat{z}$$

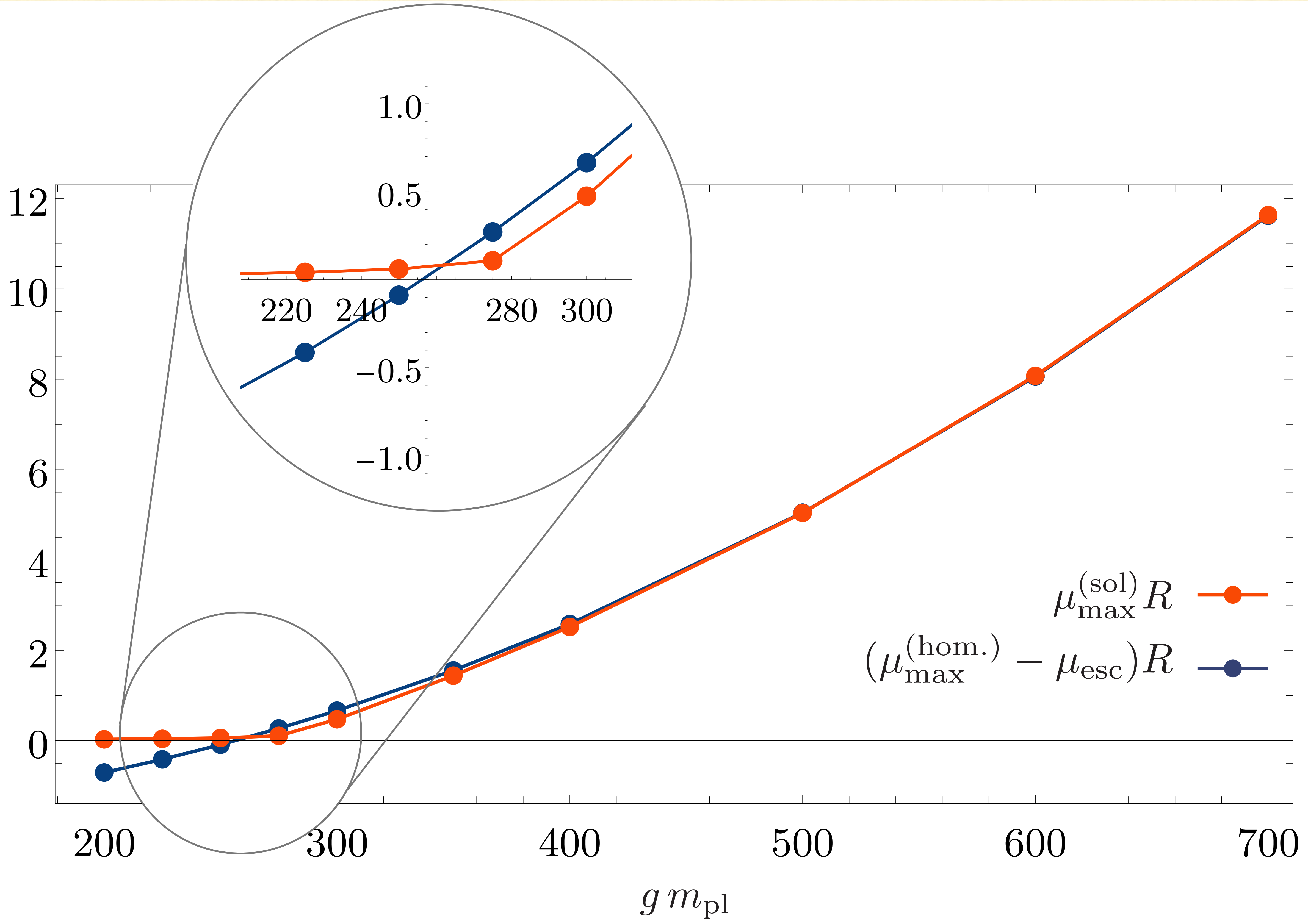
$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{g^2}{2}F_{\mu\nu}F^{\mu\nu}X_\alpha X^\alpha$$

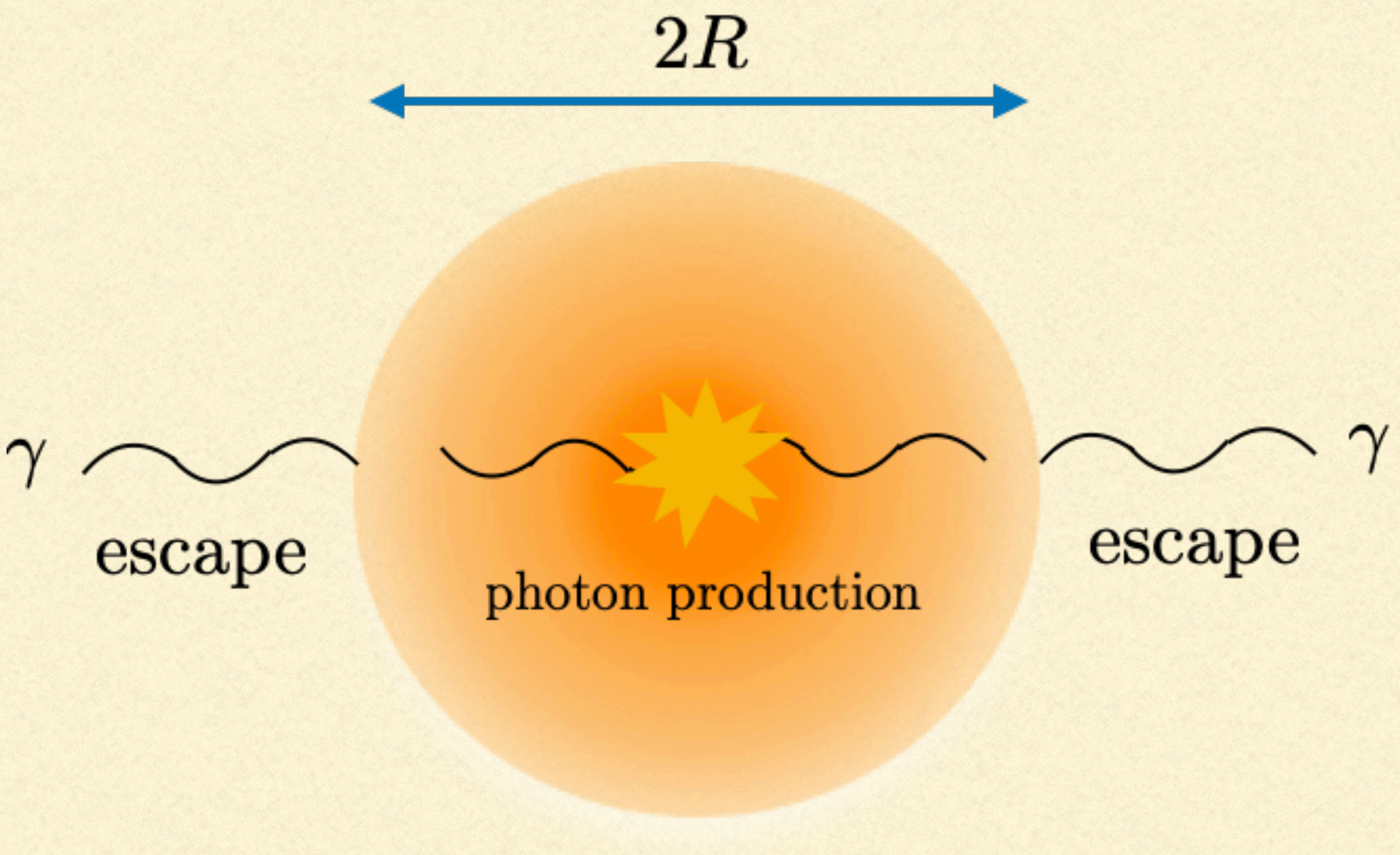
$$\ddot{A}_k + \left[k^2 - 2g^2 m\bar{X}^2 k\sin(2mt)\right]A_k = \ddot{A}_k + \omega^2(t)A_k = 0$$

$$\mu_{\rm max}^{\rm hom} = \frac{1}{2}g^2\bar{X}^2 m \ \text{ and } \ k \approx m\,, \ \Delta k = g^2\bar{X}^2 m$$



- What about the soliton size? Resonance condition: $\mu_{\text{max}}^{\text{hom}} > \mu_{\text{esc}}$



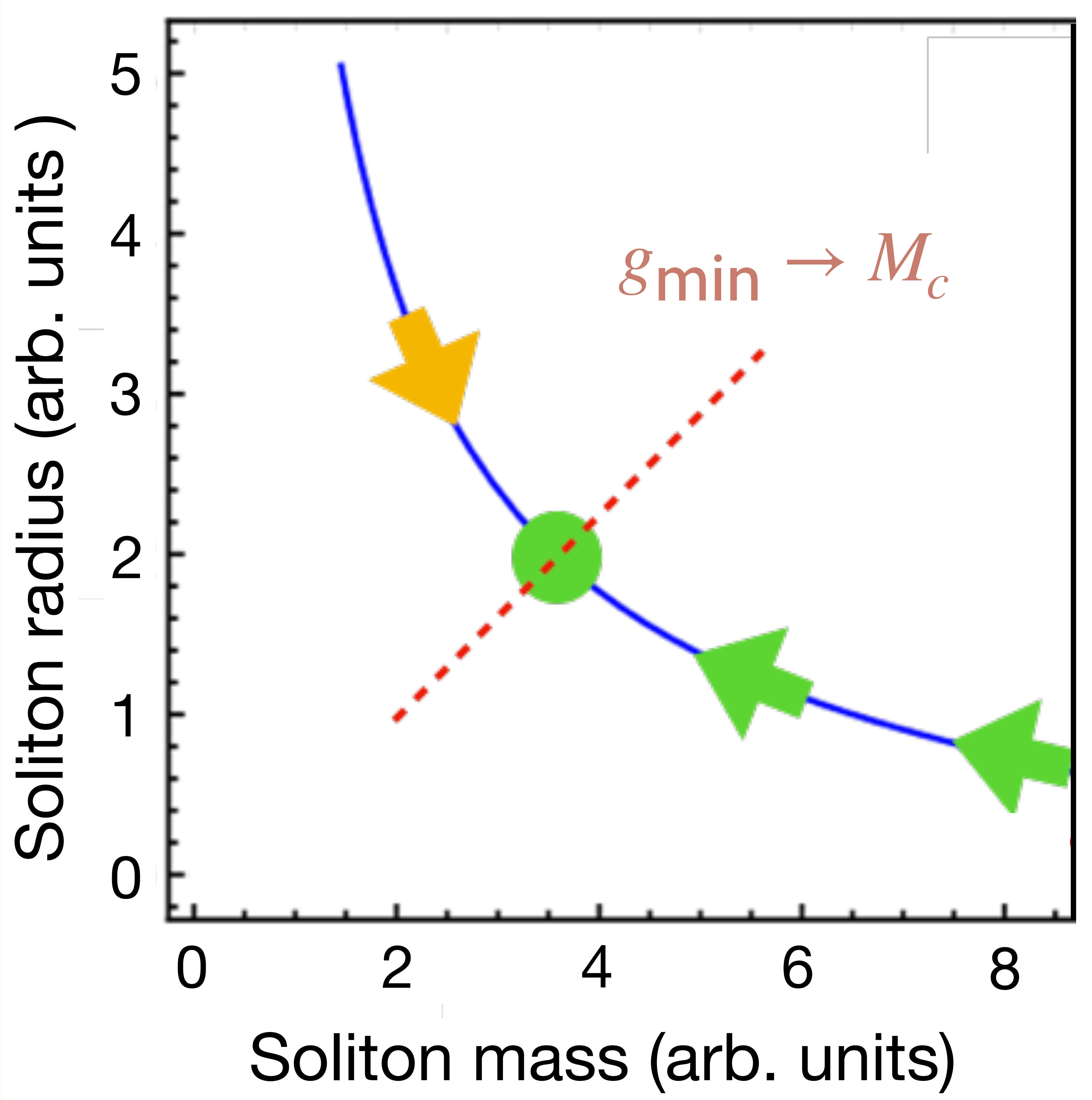


$\nu \sim 0.2\,\text{GHz}\,\bar{m}\,,\, \Delta\nu \sim 0.4\,\text{MHz}\,\bar{m}\,\bar{g}^{-2/3}\,,$

$\tau \sim 20\,\mu\text{s}\,\bar{g}^{2/3}\bar{m}^{-1}\,,\, S_B \sim 10^{18}\,\text{Jy}\,\bar{g}^{-2/3}\bar{m}^{-1}\bar{D}_L^{-2}$

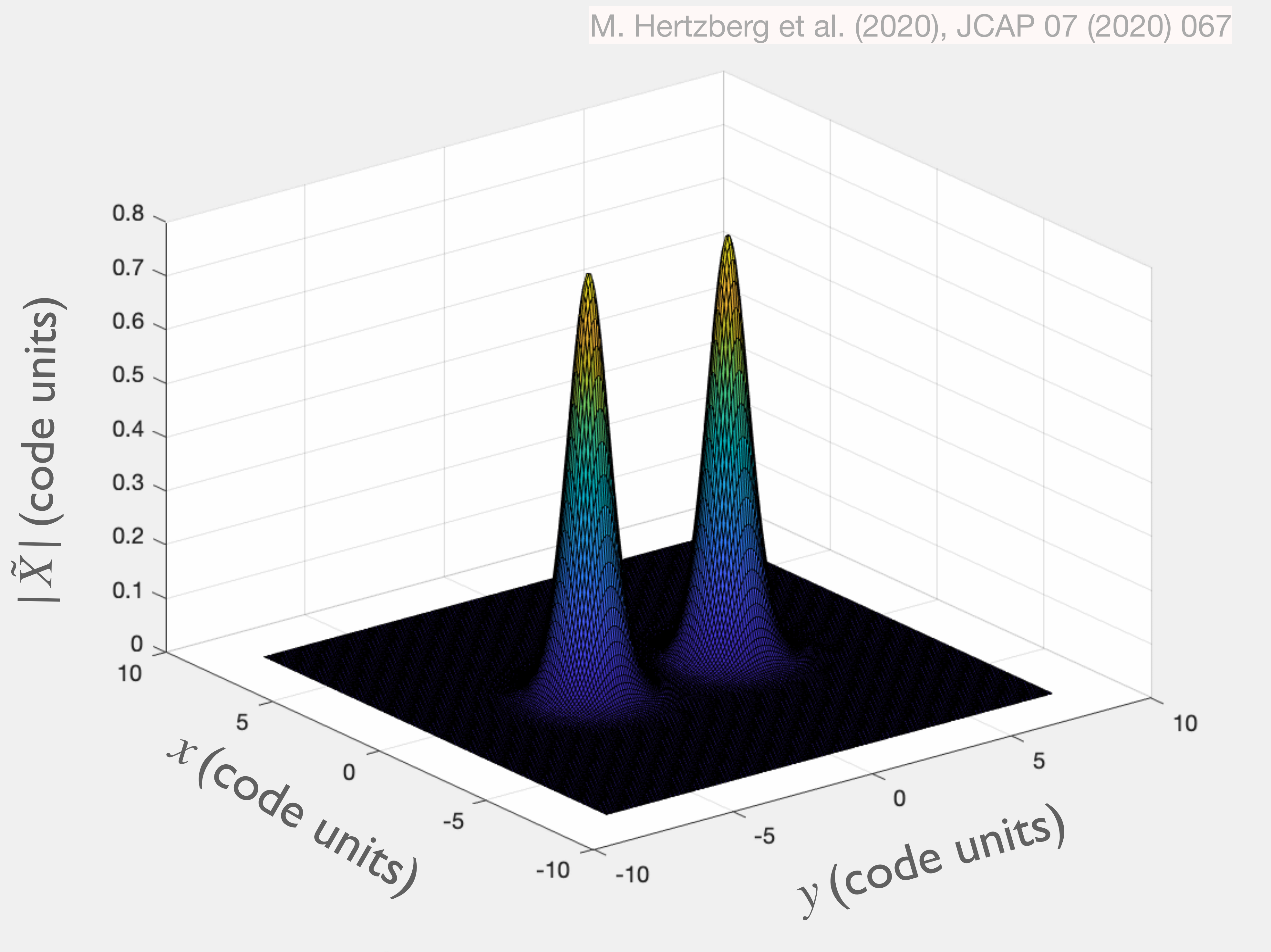
Note: $\bar{g} = 10^{-10}\,\text{GeV}^{-1}, \bar{m} = 10^{-6}$ eV$\,, \bar{D}_L =$ Mpc (signatures) .

SOLITON MASS PILE-UP



$M_{\text{sol}} + M_{\text{sol}} \approx 0.7(M_{\text{sol}})$ where $M_{\text{sol}} \approx M_c$

$v_{\text{rel}} \lesssim 1.5\,\text{km/s}\,\bar{m}\,\bar{M}_{\text{sol}}$



Note: $\bar{M}_{\text{sol}} = 10^{-9}\,M_\odot\,, \bar{m} = 10^{-6}\,\text{eV}\,.$

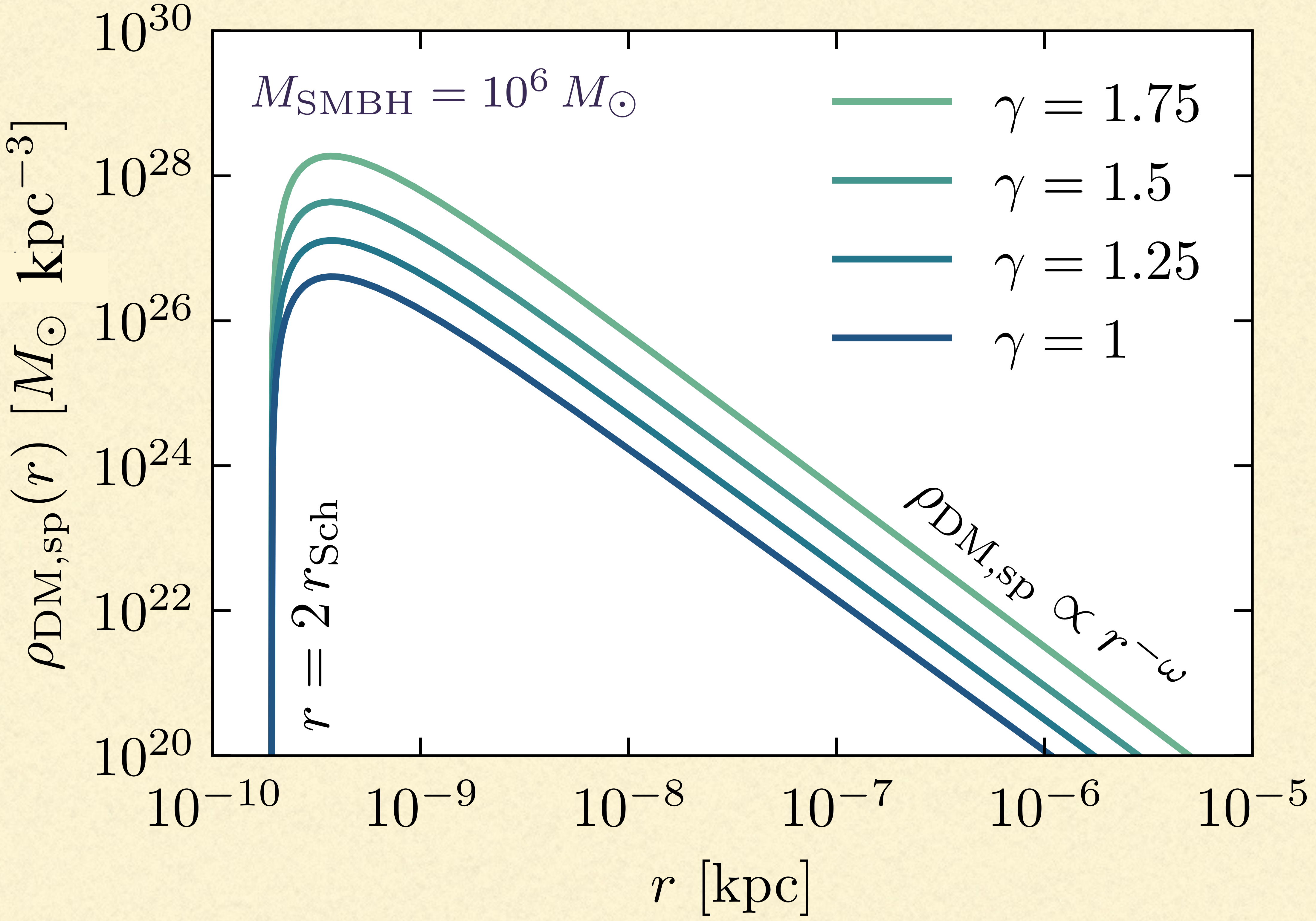
SOLITONS AND SUPERMASSIVE BLACK HOLES

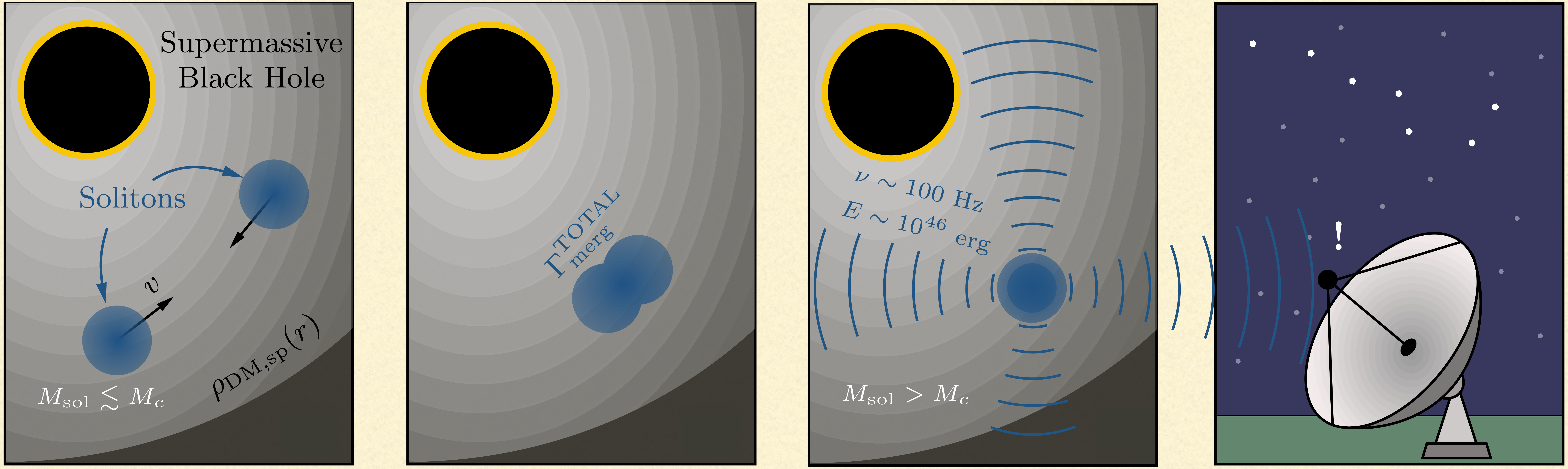
- Since $\Gamma_{\text{merg}} \propto \rho^2_{\text{DM}}$, we expect a significant enhancement of this rate under favorable scenarios ($\lesssim 10^{-10}$ day^{-1}galaxy^{-1} in NFW DM halo galaxies).

- Nearly all large galaxies host supermassive black holes (SMBHs) at their centers .

- The growth of these astrophysical bodies can lead to a highly concentrated dark matter profile in the central regions of galaxies.

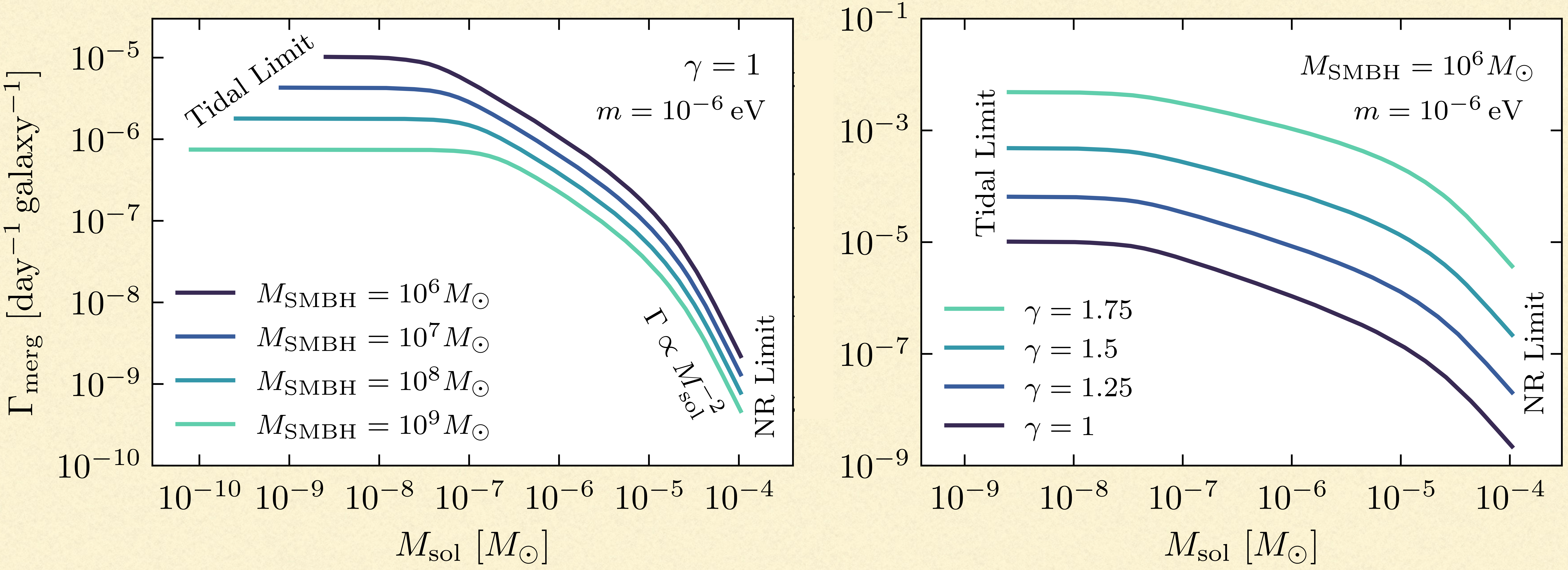
- The initial DM halo density around the SMBH is taken $\rho_{\text{DM}} \sim r^{-\gamma}$.





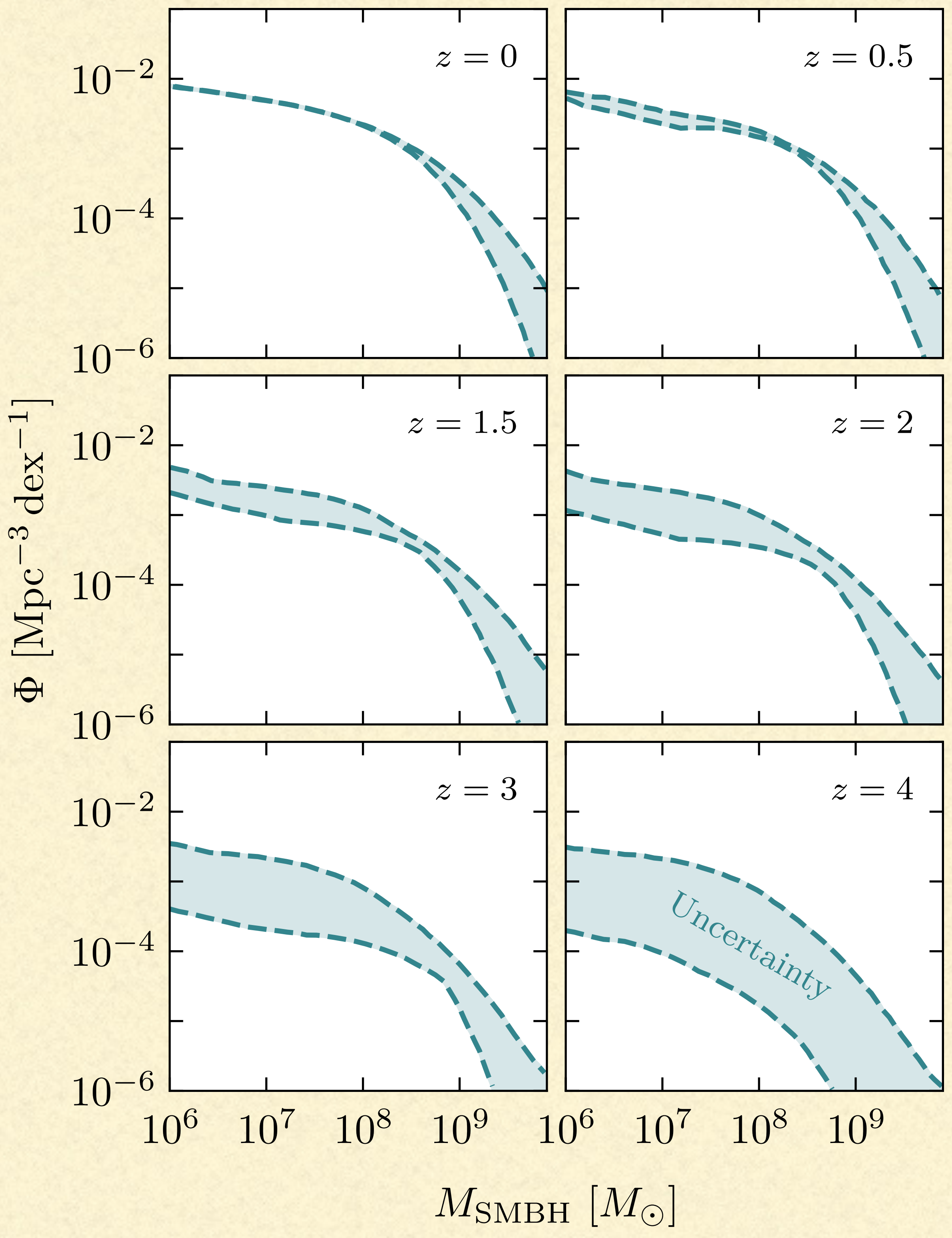
Before Major Merger | Major Merger | Parametric Resonance | Detection

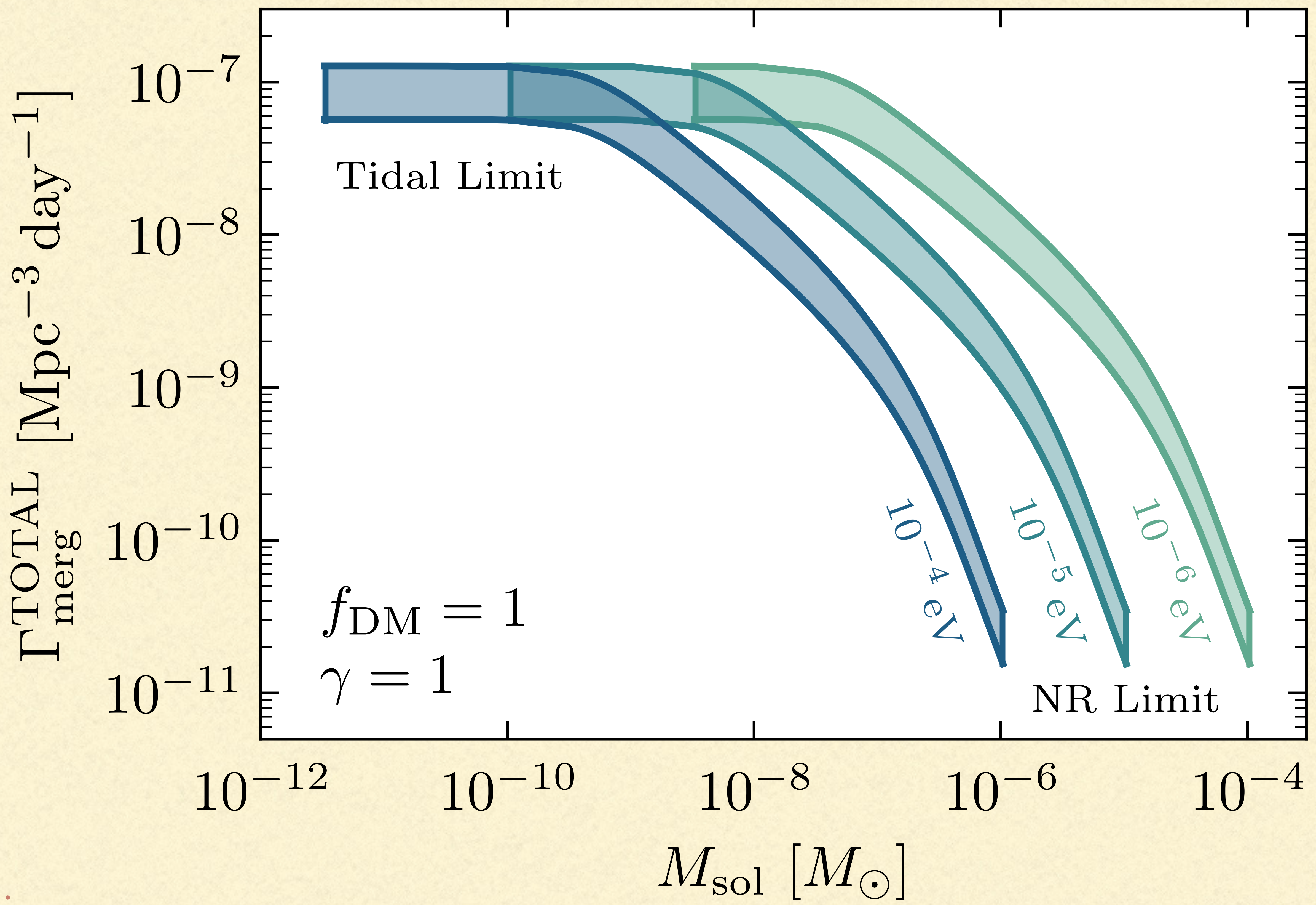
- Since the produced radiation is brief, sharp, and spectrally confined, it is an ideal target for transient radio surveys.
- Existing FRB experiments, such as CHIME, ASKPA and Parkes, have already scanned large regions of the sky with enough sensitivity to detect such bursts (yet, none have been observed). Astrophys. J. 844 (2017); Nature 562 (2018); MNRAS 455 (2016).
- Using this radio silence, we can then constrain $f_{\rm DM}$ or g.



$\Gamma_{\rm merg}$ [day^{-1} galaxy^{-1}]
$M_{\rm sol}$ [$M_\odot$]
Tidal Limit
NR Limit
$\gamma = 1$
$m = 10^{-6}$ eV
$\Gamma \propto M_{\rm sol}^{-2}$
$M_{\rm SMBH} = 10^6 M_\odot$
$M_{\rm SMBH} = 10^7 M_\odot$
$M_{\rm SMBH} = 10^8 M_\odot$
$M_{\rm SMBH} = 10^9 M_\odot$
$M_{\rm SMBH} = 10^6 M_\odot$
$m = 10^{-6}$ eV
$\gamma = 1.75$
$\gamma = 1.5$
$\gamma = 1.25$
$\gamma = 1$

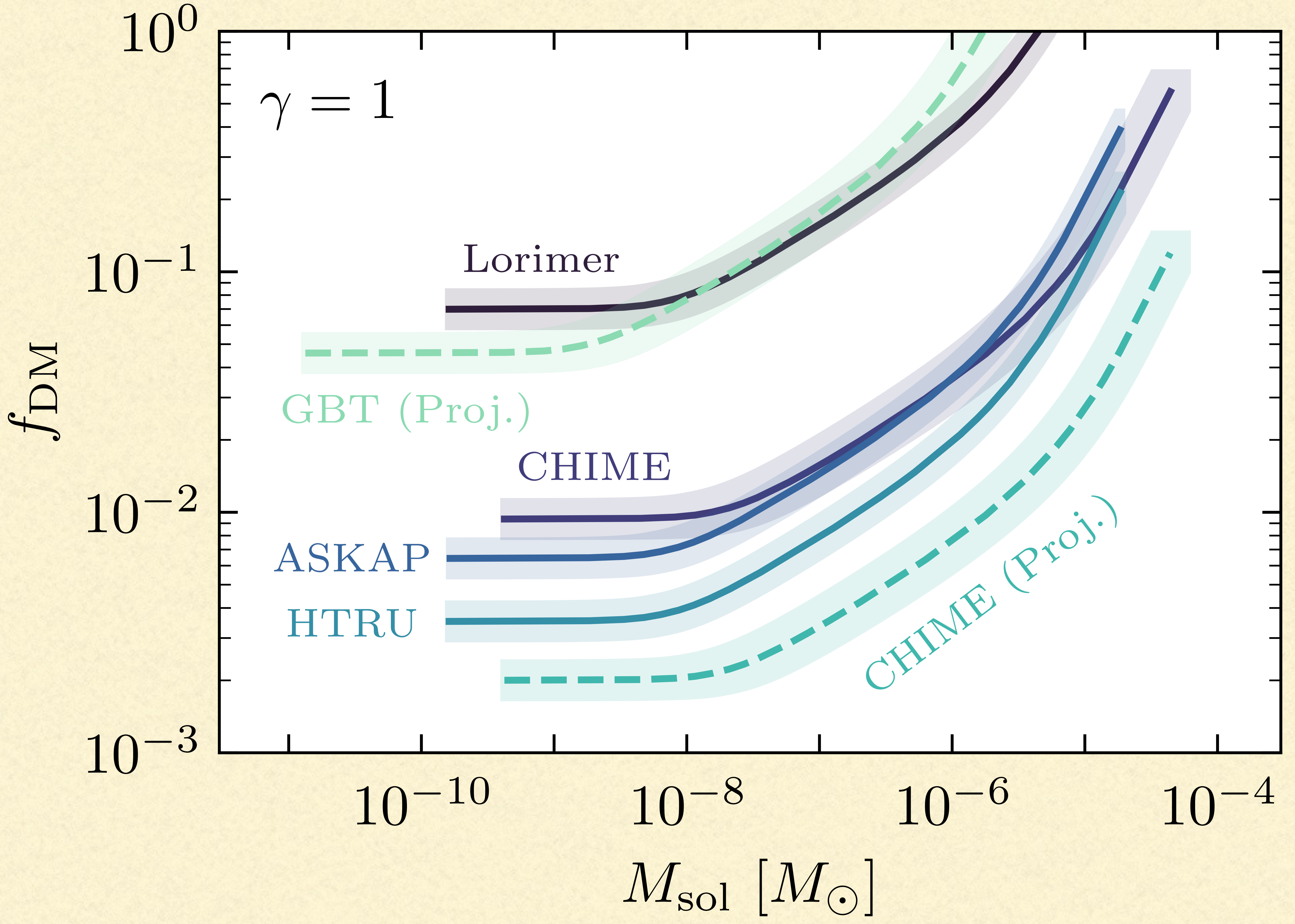
TOTAL MERGER RATE IN DM SPIKES

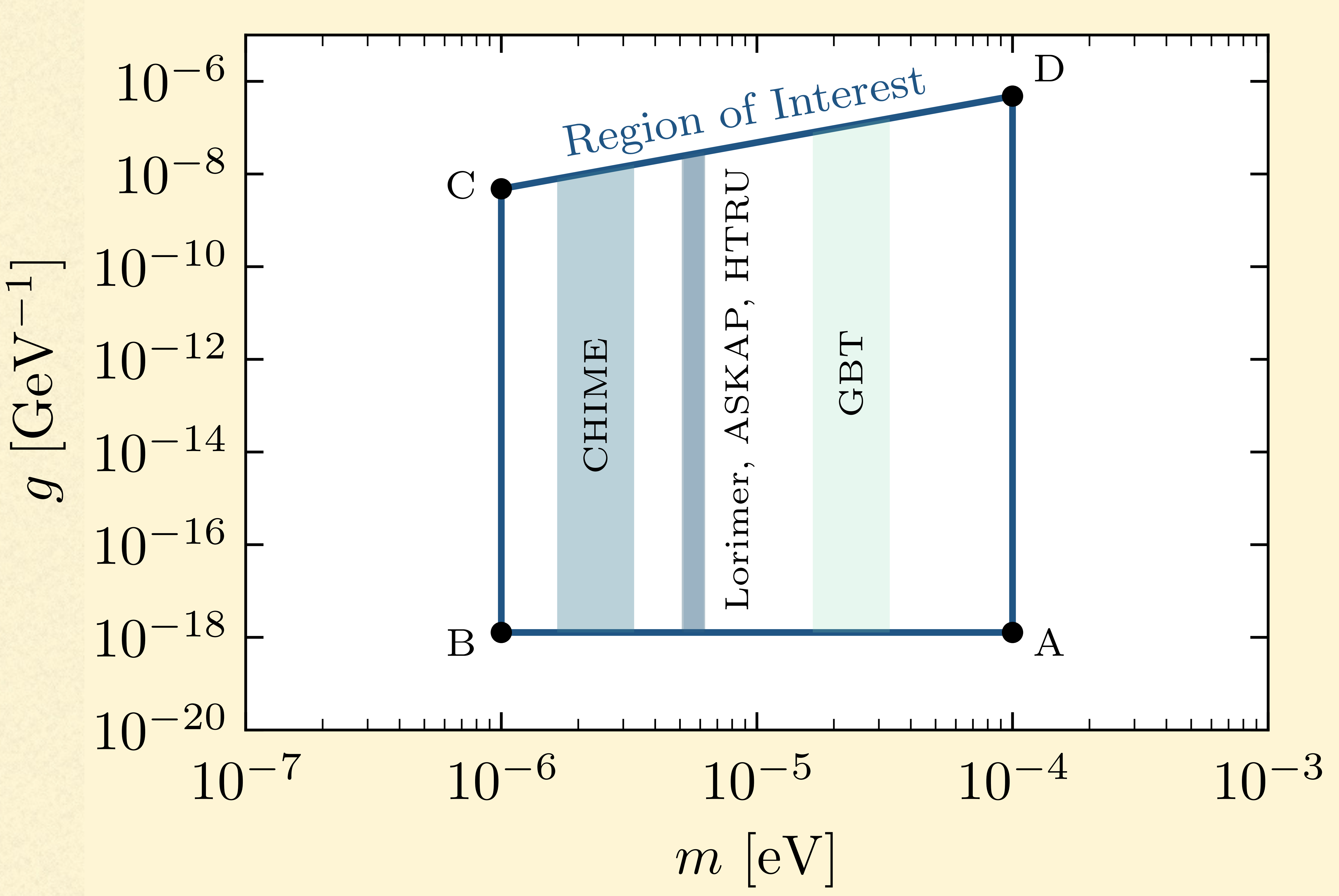




$$\Gamma_{\mathrm{merg}}^{\mathrm{TOTAL}} = \int_0^4 \int_{\log(M_{\mathrm{SMBH}}^{\mathrm{min}}/M_\odot)}^{\log(M_{\mathrm{SMBH}}^{\mathrm{max}}/M_\odot)} \Gamma_{\mathrm{merg}}(M_{\mathrm{SMBH}}, \gamma)\Phi(M_{\mathrm{SMBH}}, z)\, \mathrm{d}\log_{10} M_{\mathrm{SMBH}}\, \mathrm{d}z\,.$$

CONSTRAINTS ON f_{DM} AND g





CONCLUSIONS

- First constraints on the dark matter fraction in spin-1 dark photon solitons by considering their mergers in the dense astrophysical environments surrounding supermassive black holes.

- Using the non-observation of such radio transients—radio silence—in fast radio burst (FRB) surveys, we statistically constrained the soliton merger rate and $f_{\rm DM}$.

BACK UP SLIDES

DARK PHOTON-PHOTON ACTION

- $S[X_\mu(x), A_\mu(x), \mathrm{g}_{\mu\nu}(x)] = \int \mathrm{d}^4 x \sqrt{-\mathrm{g}} \left[-\frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{2} m^2 X_\mu X^\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_{\mathrm{pl}}^2 \mathrm{R} + \mathscr{L}_{\mathrm{int}} \right]$
- Take the dark photon field $X_\mu = (X_0, \mathrm{X})$ masses $m \ll \mathrm{eV}$ being CDM.
- Allow interactions between $X_\mu(x)$ and the electromagnetic field $A_\mu(x)$.
- Enumerate the relevant interactions via $\mathscr{L}_{\mathrm{int}}$.

NON-RELATIVISTIC MODES FOR $X_\mu(x)$

- Only non-relativistic modes will propagate: $k \ll m$ and $\lambda \ll 2\pi/m$.
- Apply perturbative expansion: $|\nabla X_\mu| \sim \lambda^{-1} X_\mu \ll m X_\mu \sim \dot{X}_\mu$.
- At leading order, we set $\nabla X_\mu = 0$.
- In terms of theories that we study: $X_0 = (\nabla^2 - m^2)^{-1}(\nabla \cdot \dot{\mathbf{X}})$.
- We set $X_0(x) = 0$ at leading order in the gradient expansion.

INTERACTIONS WITH ELECTROMAGNETISM

- Working in an EFT:
  - Operators respecting electromagnetic gauge invariance.
  - Organizing them via their mass dimension $\mathscr{L}_{\text{int}} = g^2 \mathcal{O}_i$.

$$\text{Dimension } 4: \quad \mathscr{L}^{(4)}_{\text{int}} \supset F_{\mu\nu} X_{\alpha\beta}$$

$$\text{Dimension } 5: \quad \text{odd numbers of Lorentz indices}$$

Dropping terms containing $X_0, \nabla X_\mu$

Dimension 6 :

$$\mathcal{O}_1 = -\frac{1}{2} F_{\mu\nu}\tilde{F}^{\mu\nu} X_\alpha X^\alpha \approx 2(\mathbf{E}\cdot\mathbf{B})(\mathbf{X}\cdot\mathbf{X})$$

$$\mathcal{O}_2 = -\frac{1}{2} F_{\mu\nu}F^{\mu\nu} X_\alpha X^\alpha \approx (\mathbf{E}\cdot\mathbf{E})(\mathbf{X}\cdot\mathbf{X}) - (\mathbf{B}\cdot\mathbf{B})(\mathbf{X}\cdot\mathbf{X})$$

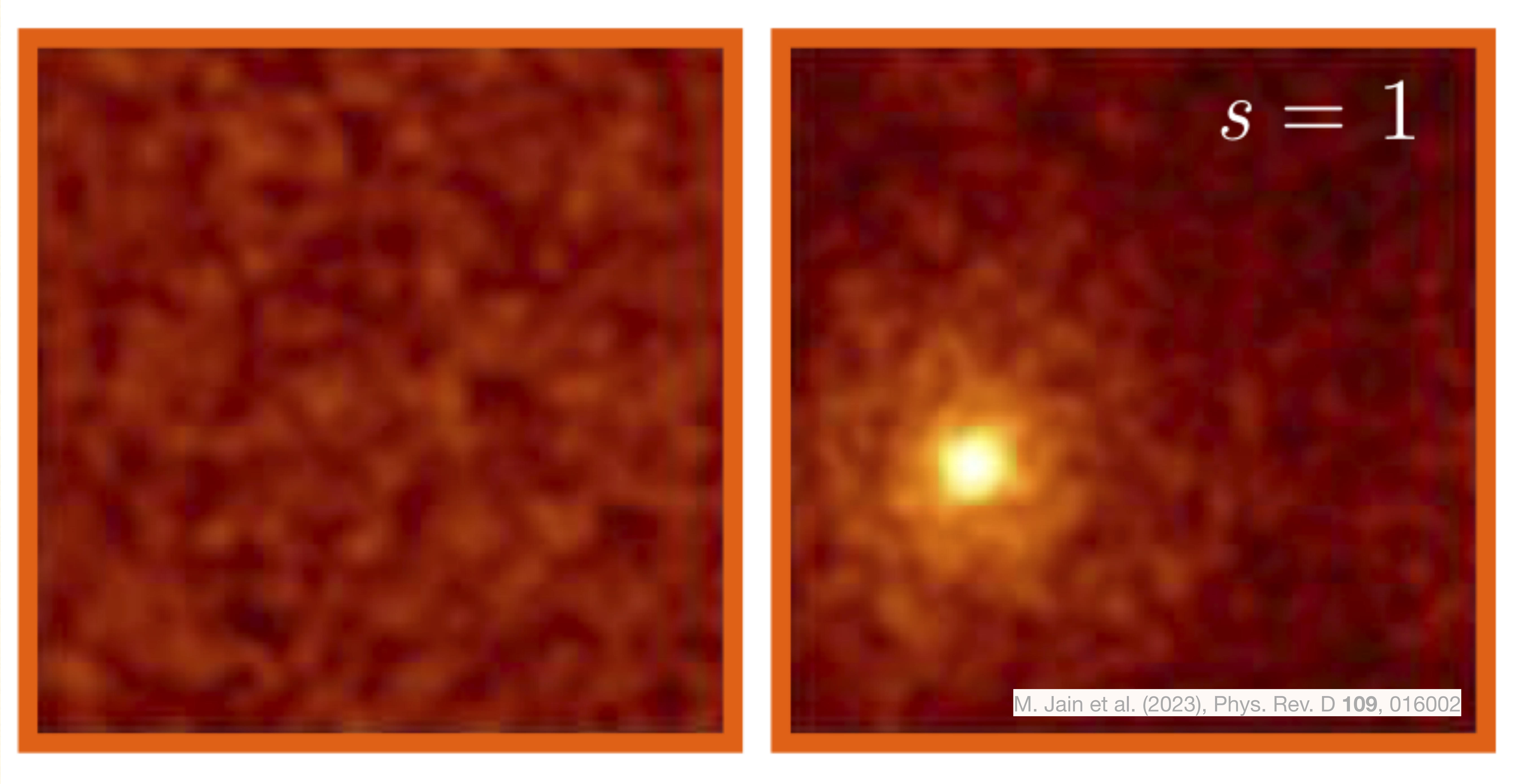
$$\mathcal{O}_3 = F_{\mu\rho}F^{\nu\rho} X^\mu X_\nu \approx (\mathbf{B}\cdot\mathbf{B})(\mathbf{X}\cdot\mathbf{X}) - (\mathbf{E}\cdot\mathbf{X})^2 - (\mathbf{B}\cdot\mathbf{X})^2$$

$$\mathcal{O}_4 = \tilde{F}_{\mu\rho}\tilde{F}^{\nu\rho} X^\mu X_\nu \approx (\mathbf{E}\cdot\mathbf{E})(\mathbf{X}\cdot\mathbf{X}) - (\mathbf{E}\cdot\mathbf{X})^2 - (\mathbf{B}\cdot\mathbf{X})^2$$

- We write $\mathscr{L}_{\text{int}} = g^2 \mathcal{O}_i$ and study the effect of each operator one at a time.
- In the EFT, we may not consIder $\mathscr{L}^{(8)}_{\text{int}} = g^2 \mathcal{O}_i$ for sufficiently small coupling g^2.
- We take g to be the DM-photon coupling with $g^2 \ll G_f \sim 10^{-5}\text{GeV}^{-2}$.
- The system acquires a nonzero vacuum expectation value $\langle X \rangle \sim \bar{X}$, which causes a contribution to lower-order operators.

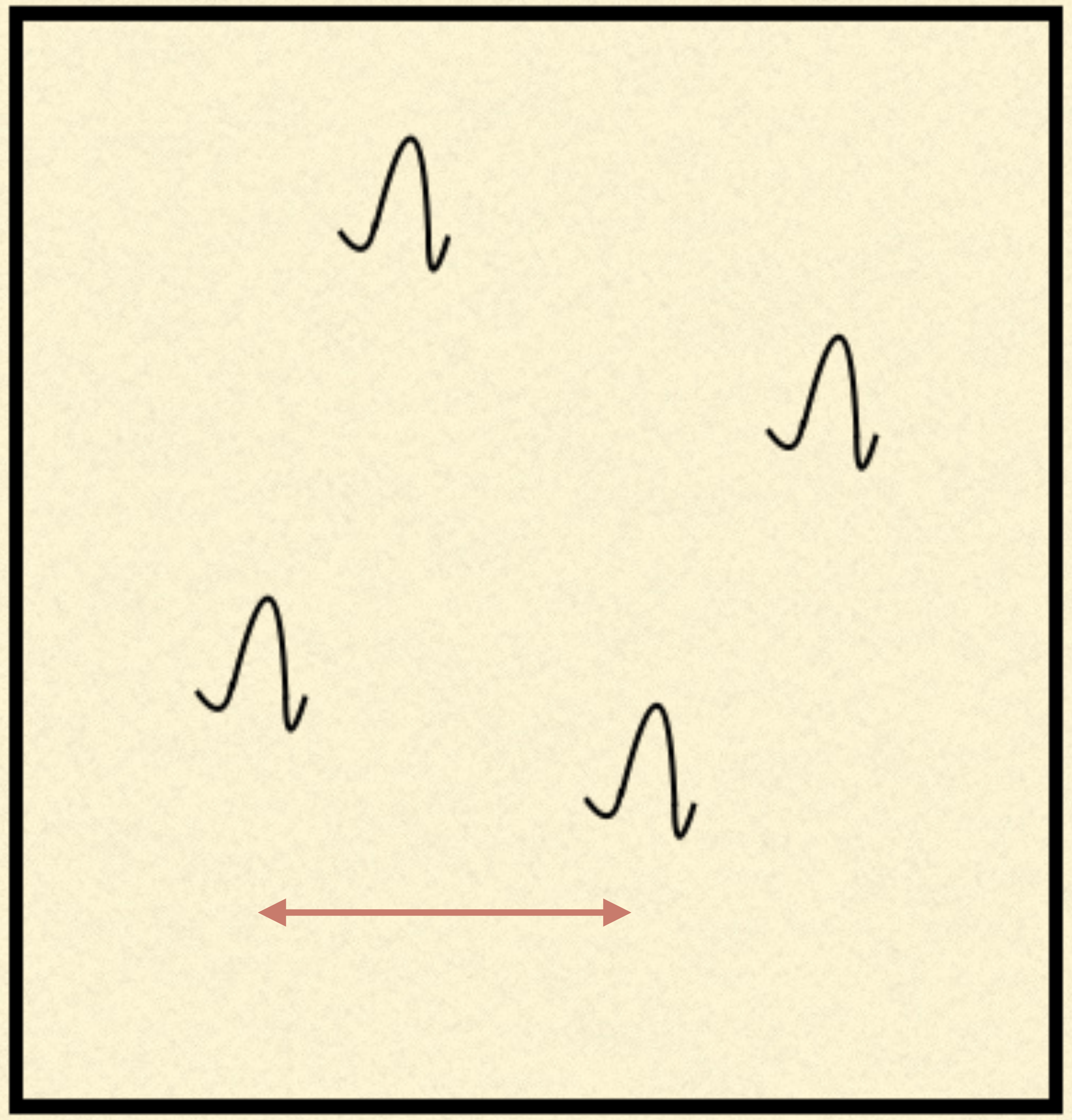
EFT condition: $g^2 \bar{X}^2 \ll 1$

DARK PHOTON SOLITONS



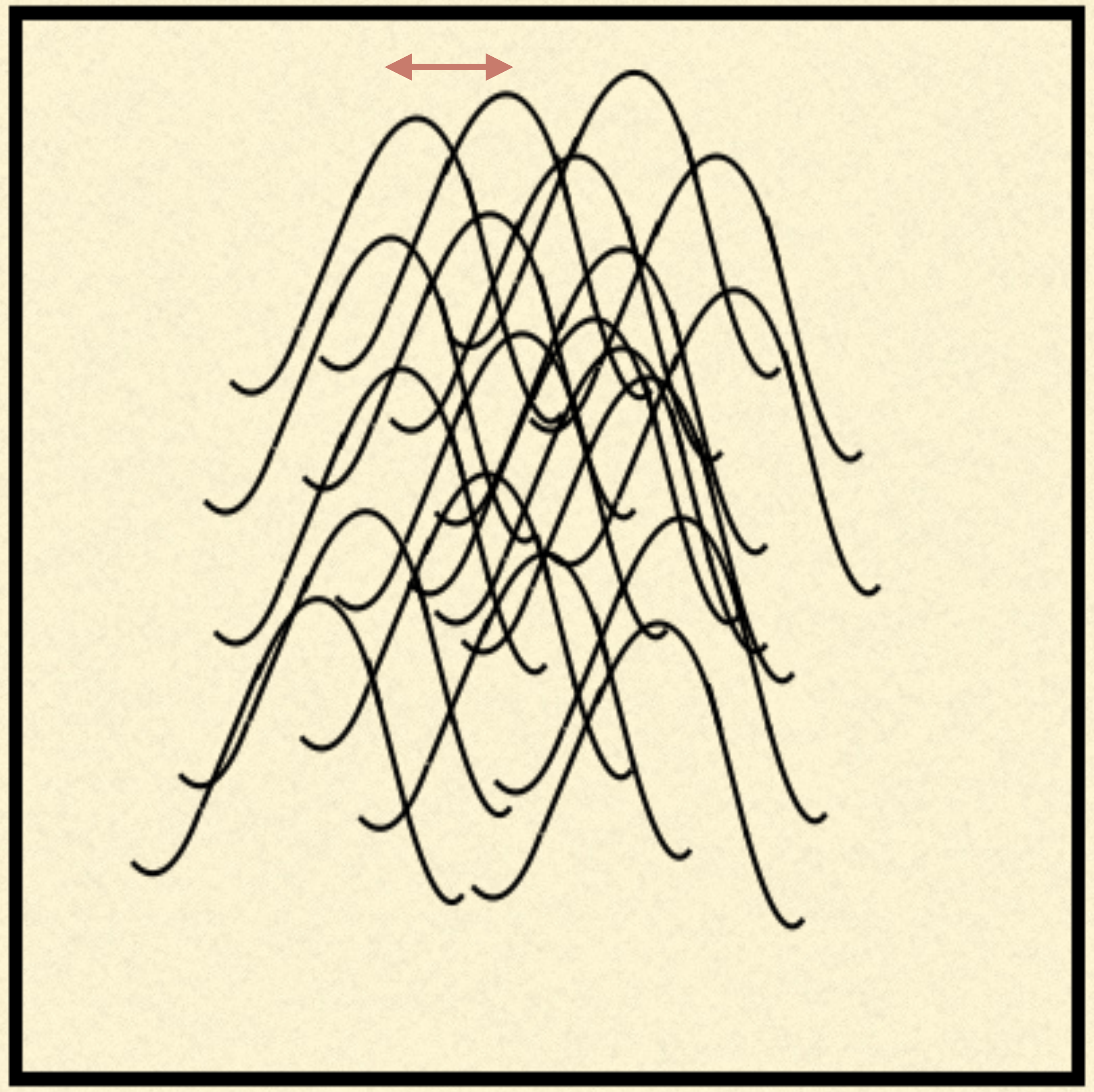
WAVE DARK MATTER

$\lambda_{\mathrm{DB}} \ll d$



Particle Dark Matter

$\lambda_{\mathrm{DB}} \gg d$



Light Dark Photon Dark Matter

$$N = \frac{\rho_{\mathrm{DM}}\lambda_{\mathrm{dB}}^3}{m} \sim 10^{30}\left(\frac{10^{-6}\,\mathrm{eV}}{m}\right)^4$$

- For $m \ll$ eV, the field is well described by Classical field theory.

NONRELATIVISTIC ACTION

$$S[X_\mu(x), \mathrm{g}_{\mu\nu}(x)] = \int \mathrm{d}^4x \sqrt{-\mathrm{g}} \left[-\frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{1}{2} m^2 X_\mu X^\mu + \frac{1}{2} m_{\mathrm{pl}}^2 \mathrm{R} \right]$$

- In the non-relativistic regime we use a perturbed Minkoswki metric as $\mathrm{g}_{\mu\nu} = \mathrm{diag}(-(1+2\Phi), (1-2\Phi)\delta_{ij})$.

- The real vector field is expressed in terms of a slowly varying function as

$$\mathbf{X}(t, \mathbf{x}) = \frac{1}{\sqrt{2m}} \left[e^{-imt} \tilde{\mathbf{X}}(t, \mathbf{x}) + e^{imt} \tilde{\mathbf{X}}*(t, \mathbf{x}) \right]$$

$$S[X_\mu(x), g_{\mu\nu}(x)] = \int \mathrm{d}^4x \left[m_{\mathrm{pl}}^2 \Phi \nabla^2 \Phi + \frac{i}{2}\left(\tilde{\mathbf{X}}^\dagger \cdot \dot{\tilde{\mathbf{X}}} - \tilde{\mathbf{X}} \cdot \dot{\tilde{\mathbf{X}}}^\dagger\right) - \frac{1}{2m}\left(\nabla \tilde{\mathbf{X}}^\dagger \cdot \nabla \dot{\tilde{\mathbf{X}}} - m\Phi \tilde{\mathbf{X}}^\dagger \cdot \tilde{\mathbf{X}}\right)\right]$$

- From the non-relativistic action of above, we obtain the 3-component SP system:

$$i\dot{\tilde{\mathbf{X}}} = -\frac{1}{2m}\nabla^2 \tilde{\mathbf{X}} + m\tilde{\mathbf{X}}\Phi \quad \text{and} \quad \nabla^2 \Phi = \frac{m}{2m_{\mathrm{pl}}^2}\tilde{\mathbf{X}}^\dagger \cdot \tilde{\mathbf{X}}$$

- The symmetries in the non-relativistic effective theory are linked to conservation laws.
- Particle number/Energy/Spin angular momentum/Orbital angular momentum.

$$N = \int d^3x \, \tilde{\mathbf{X}}^\dagger \cdot \tilde{\mathbf{X}}$$

POLARIZED VECTOR SOLITONS

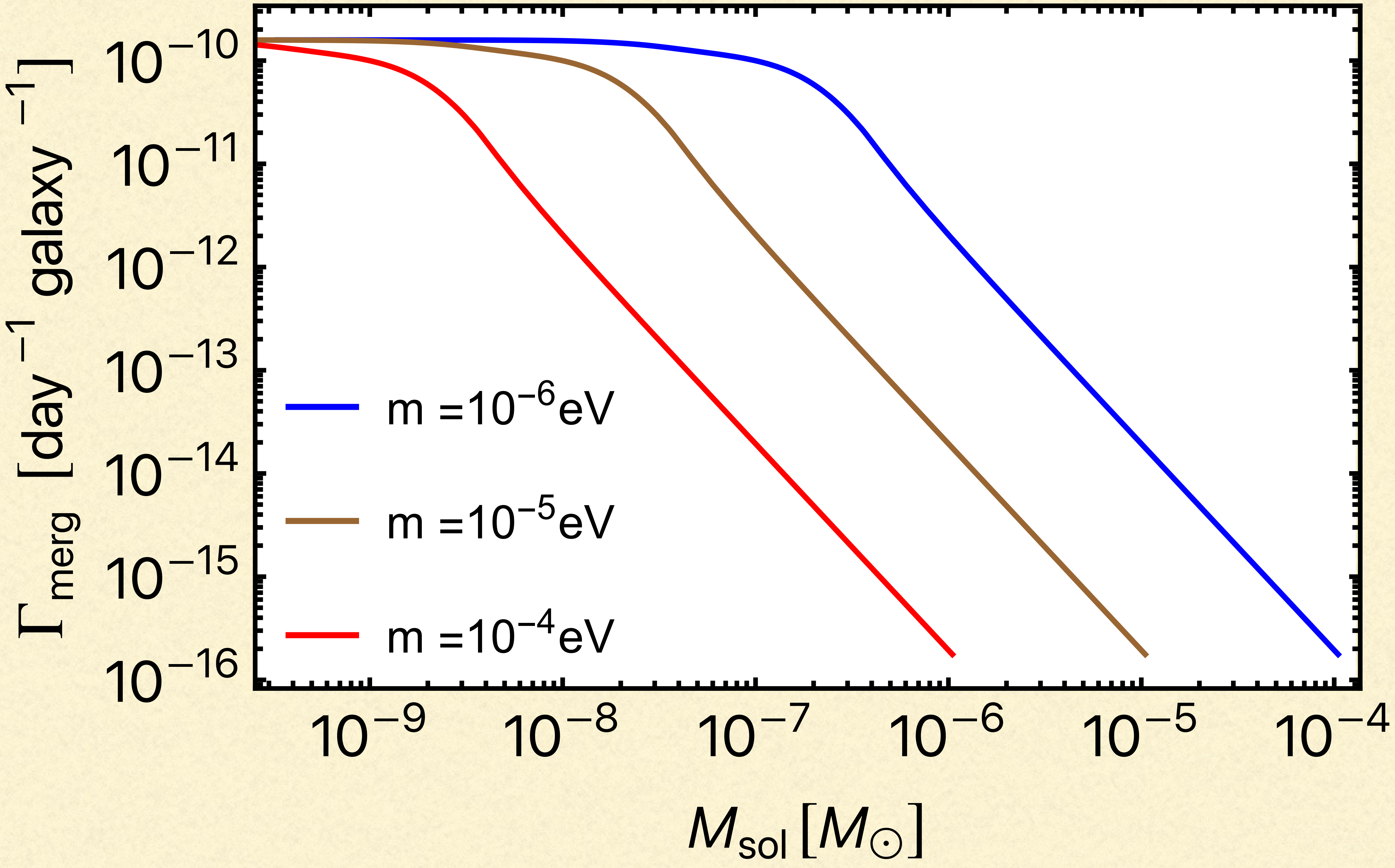
$$\tilde{\mathbf{X}}(t,\mathsf{x}) = \frac{1}{2}\sum_{\lambda}\left[c^{\lambda}X(r)e^{i(m-\omega)t}\epsilon^{\lambda} + h.c.\right]$$

$$\mathbf{X}(t,\mathsf{x}) = \frac{X(r)}{\sqrt{m}}\left[\frac{c^{-1}}{\sqrt{2}}\big(\cos(\omega t)\hat{x} - \sin(\omega t)\hat{y}\big) + c^{0}\cos(\omega t)\hat{z} + \frac{c^{+1}}{\sqrt{2}}\big(\cos(\omega t)\hat{x} + \sin(\omega t)\hat{y}\big)\right]$$

- The field amplitude oscillate in time with an angular frequency $\omega \approx m$.
- The radial profile and Newtonian potential satisfy the time-independent SP system:

$$-(m-\omega)X = -\frac{1}{2mr^2}\frac{\partial}{\partial_r}\left(r^2\frac{\partial X}{\partial r}\right) + m\Phi X\,, \text{ and } \frac{1}{r^2}\frac{\partial}{\partial_r}\left(r^2\frac{\partial \Phi}{\partial_r}\right) = \frac{m}{2m_{\text{pl}}^2}X^2\,.$$

TYPICAL MERGER RATE



Astrophysical Signal Properties

$$\nu \sim 200\,\mathrm{MHz}\left(\frac{m}{10^{-6}\,\mathrm{eV}}\right),$$

$$\Delta\nu \sim 40\,\mathrm{kHz}\left(\frac{g}{10^{-10}\,\mathrm{GeV}^{-1}}\right)^{-2/3}\left(\frac{m}{10^{-6}\,\mathrm{eV}}\right),$$

$$\tau \sim 20\,\mu\mathrm{s}\left(\frac{g}{10^{-10}\,\mathrm{GeV}^{-1}}\right)^{2/3}\left(\frac{m}{10^{-6}\,\mathrm{eV}}\right)^{-1},$$

$$S_B \sim 3\times 10^{18}\,\mathrm{Jy}\left(\frac{g}{10^{-10}\,\mathrm{GeV}^{-1}}\right)^{-2/3}\left(\frac{m}{10^{-6}\,\mathrm{eV}}\right)^{-1}\left(\frac{D_L}{1\,\mathrm{Mpc}}\right)^{-2}.$$

$$\Gamma_{\rm merg} = \int^{r_{\rm vir}} \frac{4\pi r^2}{2}\langle\sigma_{\rm eff} v_{\rm rel}\rangle_{\rm merg}\left(\frac{\rho_{\rm halo}(r) f_{\rm DM}}{M_{\rm sol}}\right)^2 dr$$

SPIKY DARK MATTER AROUND SMBH

- Since $\Gamma_{\text{merg}} \propto \rho^2_{\text{DM}}$, we anticipate a significant enhancement of this rate under favorable astrophysical scenarios.

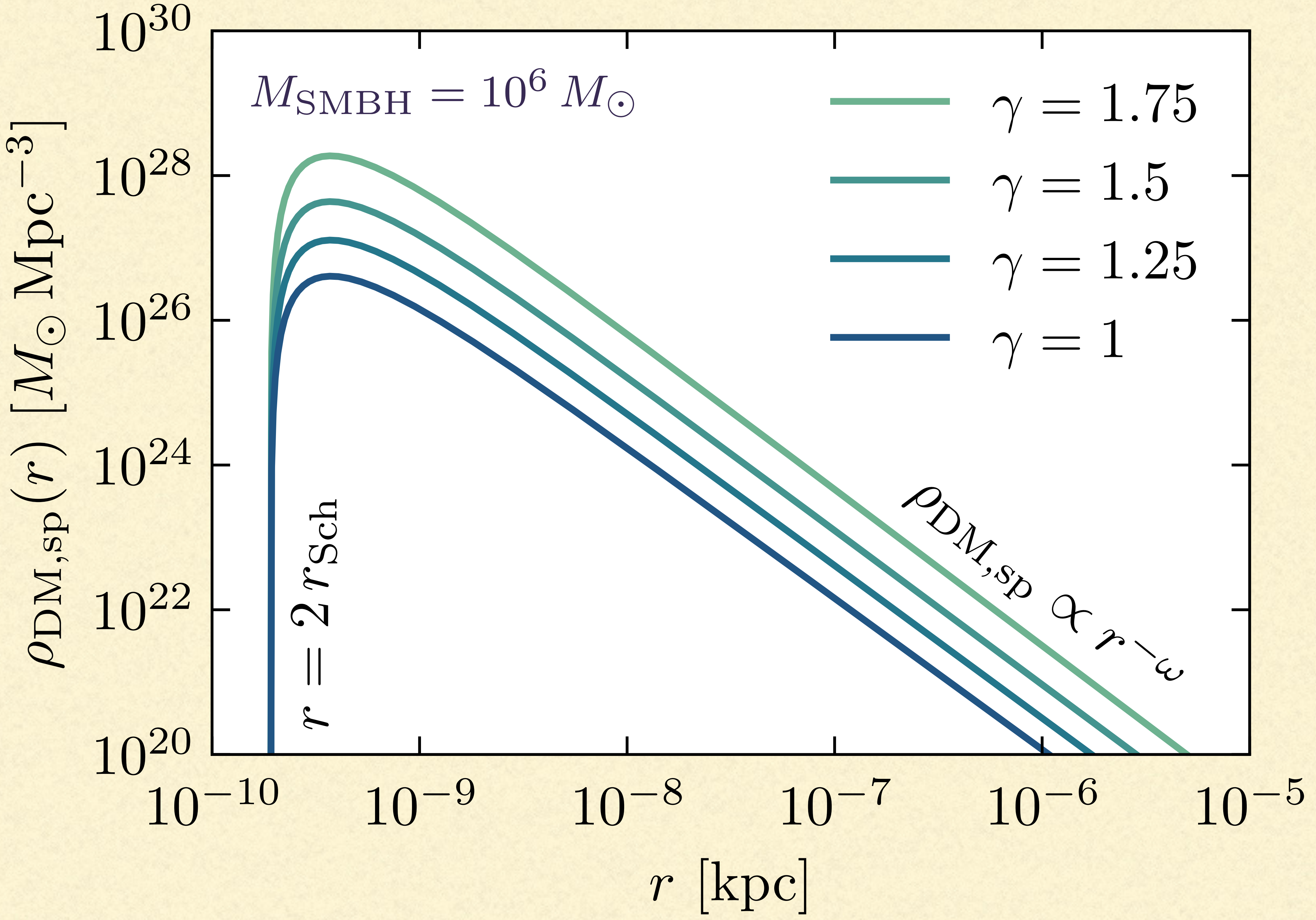
- Nearly all large galaxies host supermassive black holes (SMBHs) at their centers .
  Annu. Rev. Astron. Astrophys. 33 (1995); Annu. Rev. Astron. Astrphys. 51 (2013).

- The growth of these astrophysical bodies can lead to a highly concentrated dark matter profile in the central regions of galaxies. Phys. Rev. Lett. 83 (1999)..; Mon. Not. Roy. Astron. Soc. 337 (2002); Z..Zhang et al. (2025) arXiv:2503.02573.

- Evidence for DM spike around the SMBH in OJ 287. Astrophys. J. Lett. 962 (2024).

- A SMBH adiabatically grows placed at the galactic center.
- The initial DM halo density around the SMBH is taken $\rho_{\text{DM}} \sim r^{-\gamma}$.
- $0.9 < \gamma < 1.5$ (N-body baryonic simulations), $1.7 < \gamma < 2.1$ (N-body baryonic-DM simulations).

Nature 454 (2008); Mon.Not.Roy.Astron.Soc. 402 (2010); Phys. Rev. D 74 (2006).



$$\rho_{\text{DM,sp}}(r) = 0.0263\, M_\odot/\text{pc}^3 \left[10^b \left(\frac{M_{\text{SMBH}}}{M_\odot}\right)^a \left(\frac{G_N M_{\text{SMBH}}}{r}\right)^\omega \left(1 - \frac{4 G_N M_{\text{SMBH}}}{r}\right)^\eta\right].$$

Z..Zhang et al. (2025) arXiv:2503.02573.

MERGER RATE IN SPIKES

$$\Gamma_{\mathrm{merg}} = \int_{2r_{Sch}}^{r_h} \frac{4\pi r^2}{2} \langle \sigma_{\mathrm{eff}} v_{\mathrm{rel}} \rangle_{\mathrm{merg}} \left(\frac{\rho_{\mathrm{DM,halo}}(r) f_{\mathrm{DM}}}{M_{\mathrm{sol}}} \right)^2 dr$$

Averaged effective cross section

$$\langle \sigma_{\mathrm{eff}}\, v_{\mathrm{rel}} \rangle_{\mathrm{merg}} = \int_0^{\min\left(2v_{\mathrm{esc}},\, v^*_{\mathrm{rel}}\right)} \left(\sigma_{\mathrm{eff}}\, v_{\mathrm{rel}}\right) p(v_{\mathrm{rel}})\, \mathrm{d}v_{\mathrm{rel}} \,.$$

$$p(v_{\mathrm{rel}}) dv_{\mathrm{rel}} = 4\pi \left(\frac{3}{2\pi\sigma^2_{\mathrm{rel}}} \right)^{3/2} \exp\left(-\frac{3v^2_{\mathrm{rel}}}{2\sigma^2_{\mathrm{rel}}} \right) v^2_{\mathrm{rel}} dv_{\mathrm{rel}} \,, \quad v_{\mathrm{rel}} \lesssim 1.5\,\mathrm{km/s} \left(\frac{m}{10^{-6}\,\mathrm{eV}} \right) \left(\frac{M_{\mathrm{sol}}}{10^{-9}\, M_\odot} \right),$$

$$\sigma_{\mathrm{eff}}(v_{\mathrm{rel}}) = 4\pi R^2_{\mathrm{sol}} \left(1 + \frac{M_{\mathrm{sol}}}{4\pi m^2_{\mathrm{pl}} R_{\mathrm{sol}} v^2_{\mathrm{rel}}} \right),$$

$$\sim 5 \times 10^5 \mathrm{km}^2 \left(\frac{10^{-9}\, M_\odot}{M_{\mathrm{sol}}} \right)^2 \left(\frac{10^{-6}\,\mathrm{eV}}{m} \right)^4 \left[1 + 10^{-5} \left(\frac{m}{10^{-6}\,\mathrm{eV}} \right)^2 \left(\frac{M_{\mathrm{sol}}}{10^{-9}\, M_\odot} \right)^2 \left(\frac{\sqrt{2} \times 220\,\mathrm{km/s}}{v_{\mathrm{rel}}} \right)^2 \right],$$

Jeans Equation

$$\frac{\partial(\rho_{\mathrm{DM}}(r)\sigma^2_{\mathrm{DM}})}{\partial r} + \rho_{\mathrm{DM}}(r) \frac{\partial \Phi(r)}{\partial r} = 0 \quad \text{and} \quad \rho \sim r^{-\omega} \,,$$

$$\Phi(r) = -\frac{G_N M_{\mathrm{SMBH}}}{r}\,, \quad \sigma_{\mathrm{rel}}(r) = \sqrt{2}\sigma_{\mathrm{DM}}(r) = (1+\omega)^{-1/2} \left(\frac{G_N M_{\mathrm{SMBH}}}{r} \right)^{1/2} .$$

Gravitational Influence radius

$$r_h = G_N M_{\mathrm{SMBH}} / \sigma^{*2} \,,$$

$$\log(M_{\mathrm{SMBH}}/M_\odot) = 8.12 - 4.24 \log(200\,\mathrm{km/s}/\sigma^*) \,.$$

Astrophys. J. 539 (2000)

LIMITS FOR M_{sol} AND g

- M_{sol} upper limit to be within the non-relivistic limit.

$$M_{\mathrm{sol}} \lesssim 10^{-4}\, M_\odot \left(\frac{10^{-6}\,\mathrm{eV}}{m}\right)$$

- g upper limit from soliton tidal resistance.

$$g \lesssim 5 \times 10^{-9}\,\mathrm{GeV}^{-1} \left(\frac{M_{\mathrm{SMBH}}}{10^6\, M_\odot}\right)\left(\frac{m}{10^{-6}\,\mathrm{eV}}\right)$$

$$g \lesssim 7 \times 10^{-9}\,\mathrm{GeV}^{-1} \left(\frac{M_{\mathrm{SMBH}}}{10^6\, M_\odot}\right)^{2/3}\left(\frac{10^{-9}\, M_\odot}{M_{\mathrm{sol}}}\right)^{2/3}$$

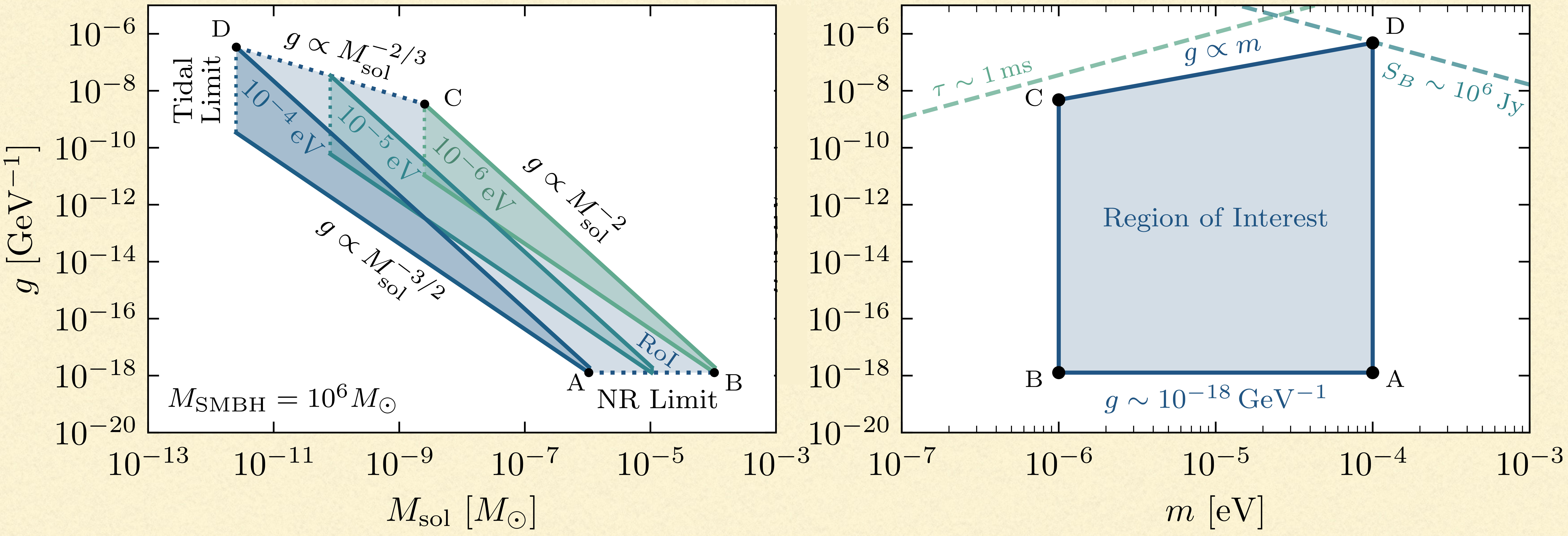
- g upper limit for the EFT validity.

$$g \ll 2 \times 10^{-8}\,\mathrm{GeV}^{-1} \left(\frac{10^{-9}\, M_\odot}{M_{\mathrm{sol}}}\right)^{2}\left(\frac{10^{-6}\,\mathrm{eV}}{m}\right)^{2}$$

- g lower limit to resonance occurs.

$$g \gtrsim 10^{-11}\,\mathrm{GeV}^{-1} \left(\frac{10^{-9}\, M_\odot}{M_{\mathrm{sol}}}\right)^{3/2}\left(\frac{10^{-6}\,\mathrm{eV}}{m}\right)^{3/2}$$

PARAMETER OF SPACE



FAST RADIO BUSRTS SURVEYS

- We can compare the characteristics of the electromagnetic radiation emitted during soliton mergers to those of FRBs.

- FRBs are also brief and highly energetic bursts of radio-frequency energy whose astrophysical origins remain mysterious. Prog. Part. Nucl. Phys. 103 (2018); Astron. Astrophys. Rev. 27 (2019).

- Frequency range $\sim (100\,\mathrm{MHz} - 10\,\mathrm{GHz})$, with durations of ~ 1 ms and flux densities and fluences in the range of $(5 \times 10^{-4} - 10^{2})$ Jy and $(5 \times 10^{-4} - 10^{2})$ Jy ms, respectively.

- Extremal observed values are $\sim 10^{6}$ Jy and $\sim 10^{6}$ Jy ms for FRB200428 (3×10^{4} ly).

Nature 587 (2020); Publ. Astron. Soc. pac. 132 (2020).

- We cannot claim to explain FRBs with soliton mergers.

- To produce ~ 1 ms bursts, we already saturate our allowed parameter space, requiring $m \sim 10^{-6}$ eV and $g \sim 10^{-9}$ GeV^{-1}.

- For the flux density and fluence, we fiducialize to our largest considered cosmic distances at $z = 4$ ($D_L \sim 10$ Gpc). Our best recovered quantities are $S_B \sim 10^7$ Jy and $\mathscr{F} \sim 10^8$ Jy ms (larger than the extremal values from FRB 200428).

- The non-observation of our predicted signals provides a powerful means of constraining the merger rate, and in turn, the dark matter fraction in vector solitons.

- The number of events detected within a patch of the sky N should follow a Poisson distribution, $N \sim \mathrm{Pois}(\lambda)$.

$$\lambda \equiv \frac{\varepsilon \mathscr{V}(z)}{4\pi} \Gamma_{\mathrm{merg}}^{\mathrm{TOTAL}} \lesssim 3$$

- A $95\ \%$ confidence level limit of non-observation, we requiere an expectation value distribution $\lambda < 3$.

$$\Gamma_{\mathrm{merg}}^{\mathrm{TOTAL}} \lesssim \Gamma_{\mathrm{lim}}^{\mathrm{TOTAL}} \equiv 3 \frac{4\pi}{\varepsilon \mathscr{V}(z=4)}$$

- We write the upper limit on the merger rate taking a comoving volume for $z = 4$.

$$f_{\mathrm{DM}} \lesssim \sqrt{\frac{\Gamma_{\mathrm{lim}}^{\mathrm{merg}}}{\Gamma_{\mathrm{lim}}^{\mathrm{TOTAL}}(M_{\mathrm{sol}}; f_{\mathrm{DM}} = 1)}}$$

- For each soliton mass, we identify an upper limit on the fraction of dark matter fraction that solitons can compose, $f_{\mathrm{DM}}^{\mathrm{lim}}$, e.g. a lower bound for M_{sol} given the DM fraction, $M_{\mathrm{sol}}^{\mathrm{lower}}$.

Telescope	ε [deg^2 hr]	f [GHz]
Lorimer et al.	4.3×10^3	1.23–1.52
CHIME PF	2.4×10^5	0.400–0.800
HTRU	1.7×10^6	1.23–1.52
CHIME (Proj.)	5.3×10^6	0.400–0.800
GBT (Proj.)	1.0×10^4	4.00–8.00

Science 318 (2007); Astrophys. J. 844 (2017); Nature 562 (2018); MNRAS 455 (2016).

- First constraints on the dark matter fraction in spin-1 dark photon solitons by considering their mergers in the dense astrophysical environments surrounding supermassive black holes.

- Using the non-observation of such radio transients—radio silence—in fast radio burst (FRB) surveys, we statistically constrained the soliton merger rate.

- Combining these observational limits with our theoretical predictions, we derived upper bounds on the fraction of dark matter f_{DM} that can reside in vector solitons.

Telescope	$\Gamma^{\mathrm{lim}}_{\mathrm{merg}}$ [Mpc^{-3} day^{-1}]	$f^{\mathrm{min}}_{\mathrm{DM}}$	M_{sol} [$M_\odot$]	m [eV]	$g_{\max}$ [GeV^{-1}]
Lorimer et al. [75]	4.2×10^{-10}	7.0×10^{-2}	1×10^{-10}–2×10^{-5}	$(5.1$–$6.3) \times 10^{-6}$	3.0×10^{-8}
CHIME PF [77]	7.5×10^{-12}	9.4×10^{-3}	4×10^{-10}–6×10^{-5}	$(1.7$–$3.3) \times 10^{-6}$	1.6×10^{-8}
ASKAP [78]	3.5×10^{-12}	6.4×10^{-3}	2×10^{-10}–2×10^{-5}	$(5.2$–$6.2) \times 10^{-6}$	3.0×10^{-8}
HTRU [76]	1.1×10^{-12}	3.5×10^{-3}	2×10^{-10}–2×10^{-5}	$(5.1$–$6.3) \times 10^{-6}$	3.0×10^{-8}
CHIME (Proj.) [109]	3.4×10^{-13}	2.0×10^{-3}	4×10^{-10}–6×10^{-5}	$(1.7$–$3.3) \times 10^{-6}$	1.6×10^{-8}
GBT (Proj.) [79]	1.8×10^{-10}	4.6×10^{-2}	1×10^{-11}–6×10^{-6}	$(1.7$–$3.3) \times 10^{-5}$	1.6×10^{-7}

OPERATORS

$$S[X_\mu(x), A_\mu(x), \mathrm{g}_{\mu\nu}(x)] = \int \mathrm{d}^4x \sqrt{-\mathrm{g}}\Big[-\frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{2}m^2 X_\mu X^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_{\mathrm{pl}}^2 \mathrm{R} + \mathscr{L}_{\mathrm{int}}\Big]$$

$$\mathscr{L}_{\mathrm{int}}^{(6)} \supset F_{\mu\nu}F_{\rho\sigma}X_\alpha X_\beta\,, \quad F_{\mu\nu}F_{\rho\sigma}\partial_\alpha X_\beta\,, \quad F_{\mu\nu}X_\rho X_\sigma \partial_\alpha X_\beta\,, \quad F_{\mu\nu}\partial_\rho X_\sigma \partial_\alpha X_\beta\,, \quad F_{\mu\nu}\partial_\rho\partial_\sigma\partial_\alpha X_\beta\,.$$

$$\begin{aligned}
\mathcal{O}_1 &= -\tfrac{1}{2}F_{\mu\nu}\tilde{F}^{\mu\nu}(X\cdot X) && \approx 2(\boldsymbol{E}\cdot\boldsymbol{B})(\boldsymbol{X}\cdot\boldsymbol{X})\\
\mathcal{O}_2 &= -\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu}(X\cdot X) && \approx (\boldsymbol{E}\cdot\boldsymbol{E})(\boldsymbol{X}\cdot\boldsymbol{X}) - (\boldsymbol{B}\cdot\boldsymbol{B})(\boldsymbol{X}\cdot\boldsymbol{X})\\
\mathcal{O}_3 &= F_{\mu\rho}F^{\nu\rho}X^\mu X_\nu && \approx (\boldsymbol{B}\cdot\boldsymbol{B})(\boldsymbol{X}\cdot\boldsymbol{X}) - (\boldsymbol{E}\cdot\boldsymbol{X})^2 - (\boldsymbol{B}\cdot\boldsymbol{X})^2\\
\mathcal{O}_4 &= \tilde{F}_{\mu\rho}\tilde{F}^{\nu\rho}X^\mu X_\nu && \approx (\boldsymbol{E}\cdot\boldsymbol{E})(\boldsymbol{X}\cdot\boldsymbol{X}) - (\boldsymbol{E}\cdot\boldsymbol{X})^2 - (\boldsymbol{B}\cdot\boldsymbol{X})^2\\
\mathcal{O}_5 &= F_{\mu\rho}F^{\nu\rho}\partial^\mu X_\nu && \approx (\boldsymbol{E}\times\boldsymbol{B})\cdot\dot{\boldsymbol{X}}\,.
\end{aligned}$$

ULTRAVIOLET EMBEDDING

- Suppose that X_μ is the vector potential associated with a dark $U(1)_d$ gauge symmetry and suppose that the UV theory includes a dark Higgs field $\phi(x)$ with $D_\mu \phi = \partial_\mu \phi - i g_d X_\mu \phi$.

- If the dark Higgs acquires a nonzero vacuum expectation value $\langle \phi \rangle = v/\sqrt{2}$, then $\mathcal{O}_2$ may arise from the dimension-8 operator

$$\mathscr{L}_8 = -\frac{1}{8} M^{-4} |D_\alpha \phi|^2 F_{\mu\nu} F^{\mu\nu}.$$

- In our EFT, we have $g^2 \sim g_d^2 v^2 / M^4 \sim m^2 / M^4$ where $m \sim g_d v$ is the mass scale of the dark photon and M is the UV scale of new physics.

ELECTROMAGNETIC EQUATION OF MOTION

- Working in the Coulomb gauge $\nabla \cdot \mathbf{A} = \mathbf{0}$, the electromagnetic equation of motion admits a Fourier representation of the form:.

$$\mathbb{O}_{ij}\ddot{A}_j + \mathbb{P}_{ij}\dot{A}_j + \mathbb{Q}_{ij}A_j = 0\,,$$

$$\mathbb{O}_{ij} = \begin{cases} \delta_{ij} & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_1 \\ \delta_{ij} + \left(2g^2|\boldsymbol{X}|^2\right)\delta_{ij} & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_2 \\ \delta_{ij} + \left(-2g^2\right)X_iX_j + \left(2g^2\frac{\boldsymbol{k}\cdot\boldsymbol{X}}{|\boldsymbol{k}|^2}\right)k_iX_j & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_3 \\ \delta_{ij} + \left(2g^2|\boldsymbol{X}|^2\right)\delta_{ij} + \left(2g^2\frac{\boldsymbol{k}\cdot\boldsymbol{X}}{|\boldsymbol{k}|^2}\right)k_iX_j + \left(-2g^2\right)X_iX_j & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_4 \\ \delta_{ij} & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_5 \end{cases}$$

$$\mathbb{P}_{ij} = \begin{cases} 0 & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_1 \\ \left(4g^2\boldsymbol{X}\cdot\dot{\boldsymbol{X}}\right)\delta_{ij} & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_2 \\ \left(-2g^2\right)\dot{X}_iX_j + \left(-2g^2\right)X_i\dot{X}_j + \left(2g^2\frac{\boldsymbol{k}\cdot\boldsymbol{X}}{|\boldsymbol{k}|^2}\right)k_i\dot{X}_j + \left(2g^2\frac{\boldsymbol{k}\cdot\dot{\boldsymbol{X}}}{|\boldsymbol{k}|^2}\right)k_iX_j & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_3 \\ \left(4g^2\boldsymbol{X}\cdot\dot{\boldsymbol{X}}\right)\delta_{ij} + \left(-2g^2\right)\dot{X}_iX_j + \left(-2g^2\right)X_i\dot{X}_j + \left(2g^2\frac{\boldsymbol{k}\cdot\dot{\boldsymbol{X}}}{|\boldsymbol{k}|^2}\right)k_iX_j + \left(2g^2\frac{\boldsymbol{k}\cdot\boldsymbol{X}}{|\boldsymbol{k}|^2}\right)k_i\dot{X}_j & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_4 \\ \left(-2ig^2\boldsymbol{k}\cdot\dot{\boldsymbol{X}}\right)\delta_{ij} & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_5 \end{cases}$$

$$\mathbb{Q}_{ij} = \begin{cases} |\boldsymbol{k}|^2\delta_{ij} + \left(4ig^2\boldsymbol{X}\cdot\dot{\boldsymbol{X}}\right)\epsilon_{ijk}k_k & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_1 \\ |\boldsymbol{k}|^2\delta_{ij} + \left(2g^2|\boldsymbol{k}|^2\,|\boldsymbol{X}|^2\right)\delta_{ij} & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_2 \\ |\boldsymbol{k}|^2\delta_{ij} + \left(-2g^2(\boldsymbol{k}\cdot\boldsymbol{X})^2\right)\delta_{ij} + \left(-2g^2|\boldsymbol{k}|^2\right)X_iX_j + \left(2g^2\boldsymbol{k}\cdot\boldsymbol{X}\right)k_iX_j & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_3\,. \\ |\boldsymbol{k}|^2\delta_{ij} + \left(-2g^2(\boldsymbol{k}\cdot\boldsymbol{X})^2 + 2g^2|\boldsymbol{k}|^2\,|\boldsymbol{X}|^2\right)\delta_{ij} + \left(-2g^2|\boldsymbol{k}|^2\right)X_iX_j + \left(2g^2\boldsymbol{k}\cdot\boldsymbol{X}\right)k_iX_j & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_4 \\ |\boldsymbol{k}|^2\delta_{ij} + \left(-ig^2\boldsymbol{k}\cdot\ddot{\boldsymbol{X}}\right)\delta_{ij} & , \quad \mathscr{L}_{\text{int}} = g^2\mathcal{O}_5 \end{cases}$$

FLOQUET THEORY (REDUCED SYSTEM)

- Before applying Floquet theory to analyse the solutions of the Fourier representation of the electromagnetic modes, we impose $\mathbf{k} \cdot \mathbf{A} = 0$ and eliminate A_3. Explicity, we have $A_3 = -k_3^{-1}(k_2 A_2 + k_1 A_1)$ for $k_3 \neq 0$.

$$\tilde{\mathbb{O}}_{ij}\ddot{A}_j + \tilde{\mathbb{P}}_{ij}\dot{A}_j + \tilde{\mathbb{Q}}_{ij}A_j = 0, \quad \text{where} \quad \tilde{\mathbb{O}}_{ij} \equiv \mathbb{O}_{ij} - \mathbb{O}_{i3}k_j/k_3 \quad (\text{similarly for } \tilde{\mathbb{P}}, \tilde{\mathbb{Q}} \text{ and } i,j = 1,2).$$

$$\dot{\boldsymbol{q}}(t) = \tilde{\mathbb{U}}(t)\,\boldsymbol{q}(t) \qquad \text{with} \qquad \boldsymbol{q}(t) = \begin{pmatrix} \boldsymbol{A}(t) \\ \dot{\boldsymbol{A}}(t) \end{pmatrix} \quad \text{and} \quad \tilde{\mathbb{U}}(t) = \left(\begin{array}{c|c} 0 & \mathbb{1} \\ \hline -\tilde{\mathbb{O}}^{-1}\tilde{\mathbb{Q}} & -\tilde{\mathbb{O}}^{-1}\tilde{\mathbb{P}} \end{array}\right).$$

- If $\tilde{U}(t+T) = \tilde{U}(t)$ is periodic with period T, then Floquet's theorem guarantees a general solution of the form $\mathbf{q}(t) = \sum_{s=1}^{4} c_s \mathbf{P}_s(t) e^{\mu_s t}$ where $\mathbf{P}_s(t+T) = \mathbf{P}_s(t)$ and μ_s are called Floquet exponents.

- If $\mathscr{R}(\mu_s) > 0$ for any s, then the equation of motion admits exponentially growing solutions.

ELECTROMAGNETIC RADIATION

$$\boldsymbol{X}(t,\boldsymbol{x}) = \bar{X}\,\cos(mt)\,\hat{\boldsymbol{z}}\,,$$

$$\mu_{\max} = \begin{cases} \frac{1}{2} g^2 \bar{X}^2 m & , \quad \text{for } \mathscr{L}_{\rm int} = g^2 \mathcal{O}_1 \\ \frac{1}{2} g^2 \bar{X}^2 m & , \quad \text{for } \mathscr{L}_{\rm int} = g^2 \mathcal{O}_2 \\ \frac{1}{2} g^2 \bar{X}^2 m & , \quad \text{for } \mathscr{L}_{\rm int} = g^2 \mathcal{O}_3 \\ \frac{1}{2} g^2 \bar{X}^2 m & , \quad \text{for } \mathscr{L}_{\rm int} = g^2 \mathcal{O}_4 \\ O(g^4) & , \quad \text{for } \mathscr{L}_{\rm int} = g^2 \mathcal{O}_5 \end{cases} ,$$

$$\boldsymbol{X}(t,\boldsymbol{x}) = \frac{\bar{X}}{\sqrt{2}}\left(\cos(mt)\,\hat{\boldsymbol{x}} + \sin(mt)\,\hat{\boldsymbol{y}}\right),$$

$$\mu_{\max} = \begin{cases} 0 & , \quad \text{for } \mathscr{L}_{\rm int} = g^2 \mathcal{O}_1 \\ 0 & , \quad \text{for } \mathscr{L}_{\rm int} = g^2 \mathcal{O}_2 \\ \frac{1}{2} g^2 \bar{X}^2 m & , \quad \text{for } \mathscr{L}_{\rm int} = g^2 \mathcal{O}_3 \\ \frac{1}{2} g^2 \bar{X}^2 m & , \quad \text{for } \mathscr{L}_{\rm int} = g^2 \mathcal{O}_4 \\ O(g^4) & , \quad \text{for } \mathscr{L}_{\rm int} = g^2 \mathcal{O}_5 \end{cases} .$$