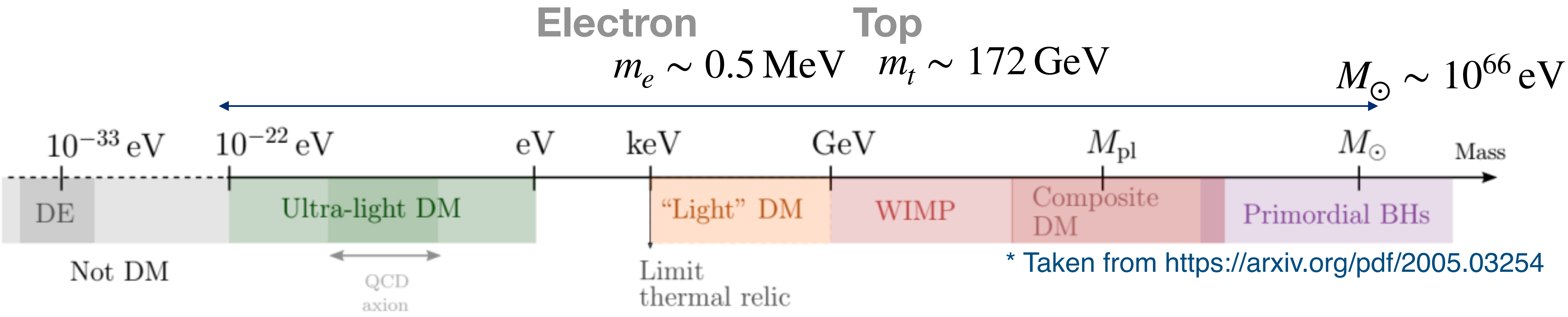
Probing Ultra-Light Dark Matter models with pulsars and gravitational waves interferometers

Diana López Nacir

Encuentro CosmoConce y Partículas 2025

Universidad del Bío-Bío, Concepción, Chile - November 5, 2025

Sketch (not to scale) of the huge range of possible DM models that have been conceived *



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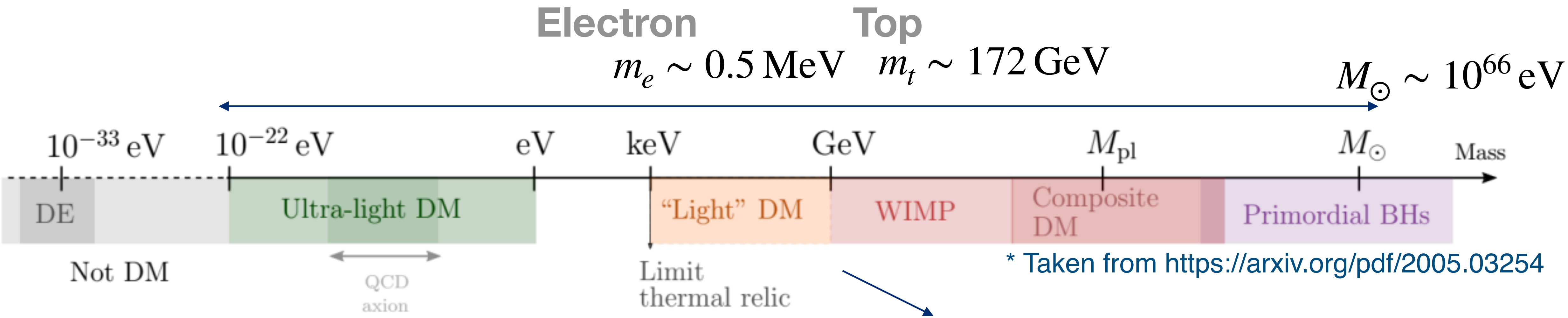

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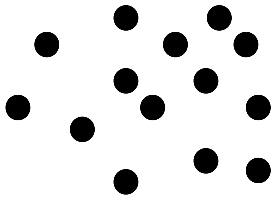
- Standard scenario: distribution of "cold" (small velocity dispersion) non-relativistic massive particles in an expanding FLRW universe



$$N = nV = na^3 V_0 = \text{cte}$$

$$E \simeq m$$

$$\rho_{DM} \simeq mn \propto a^{-3}$$

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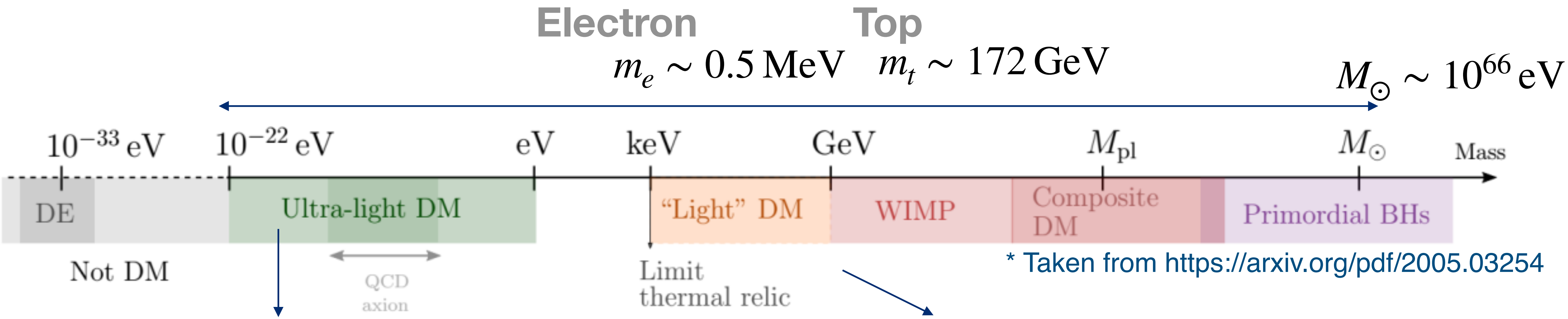

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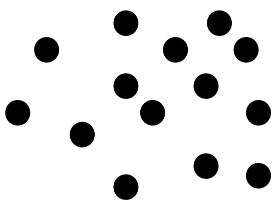
- <u>An alternative scenario</u>: ultralight DM (standard candidates are axion- like particles and dilatons, but can also be vectors or spin 2 tensors).

Very light DM with large $n = \rho_{DM}/m$

Classical field approximation

- <u>Standard scenario</u>: distribution of "cold" (small velocity dispersion) non-relativistic massive particles in an expanding FLRW universe



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`Small-scale' properties scalar DM as a classical field

$\hbar = 1$

$$S_{DM} = \frac{1}{2}\int d^4x\sqrt{-g}\left[-\partial_\mu\Phi\partial^\mu\Phi - m^2\Phi^2\right]$$

$$\Phi = e^{-imt}\,\Psi + e^{imt}\,\Psi*$$

Non-relarivivistic limit →

$(a\dot{\rho}_{DM} \ll c\,|\nabla\rho_{DM}|)$

Schrödinger-Poisson equations

$$\frac{i}{a^{3/2}}\frac{\partial}{\partial t}(a^{3/2}\Psi) = \left[\frac{-\nabla^2}{2a^2m} + m\phi_N\right]\Psi$$

$$\frac{\nabla^2}{a^2}\phi_N = 4\pi G\rho_{\rm DM}$$

$$\rho_{\rm DM} = 2m^2\,|\,\Psi\,|^2$$

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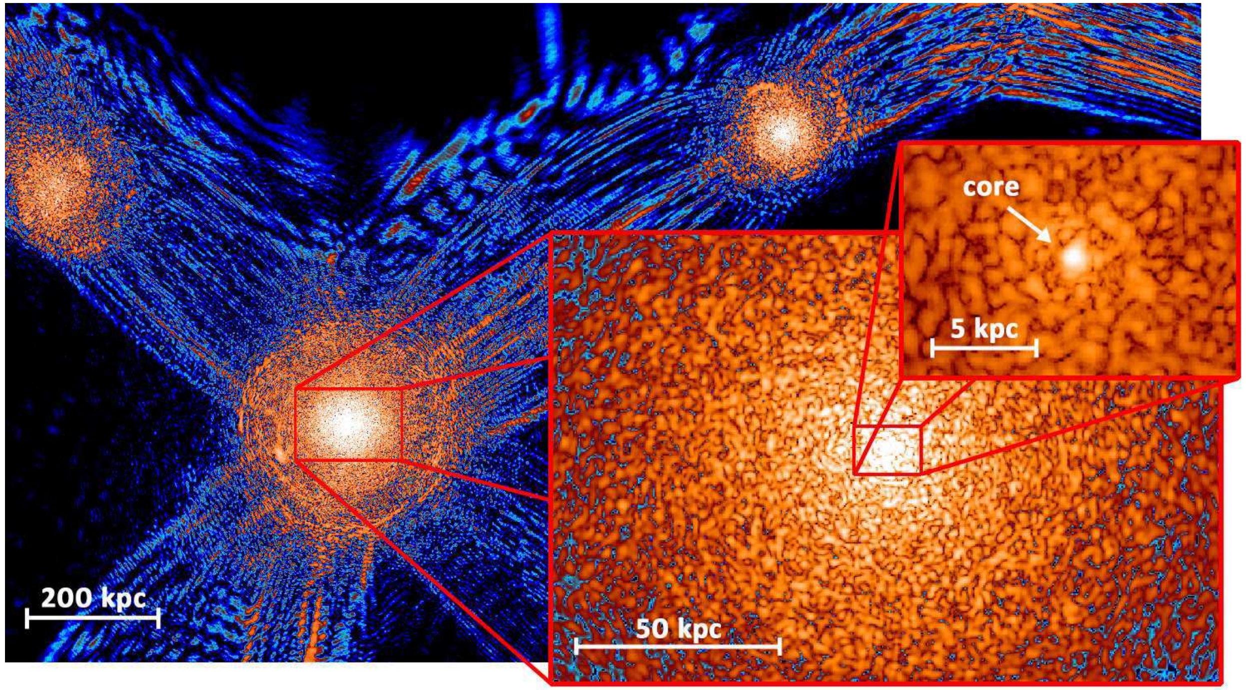
- **Halos structure (simulations)**

Schive, Chiueh, Broadhurst (2014) $m \sim 10^{-22}{\rm eV}$



Nature Phys., vol. 10, pp. 496–499, 2014.

$$\lambda_{dB} = \frac{1}{mv} \sim 10^{16}\,{\rm km}\left(\frac{10^{-3}{\rm c}}{v}\right)\left(\frac{10^{-22}{\rm eV}}{m}\right)$$

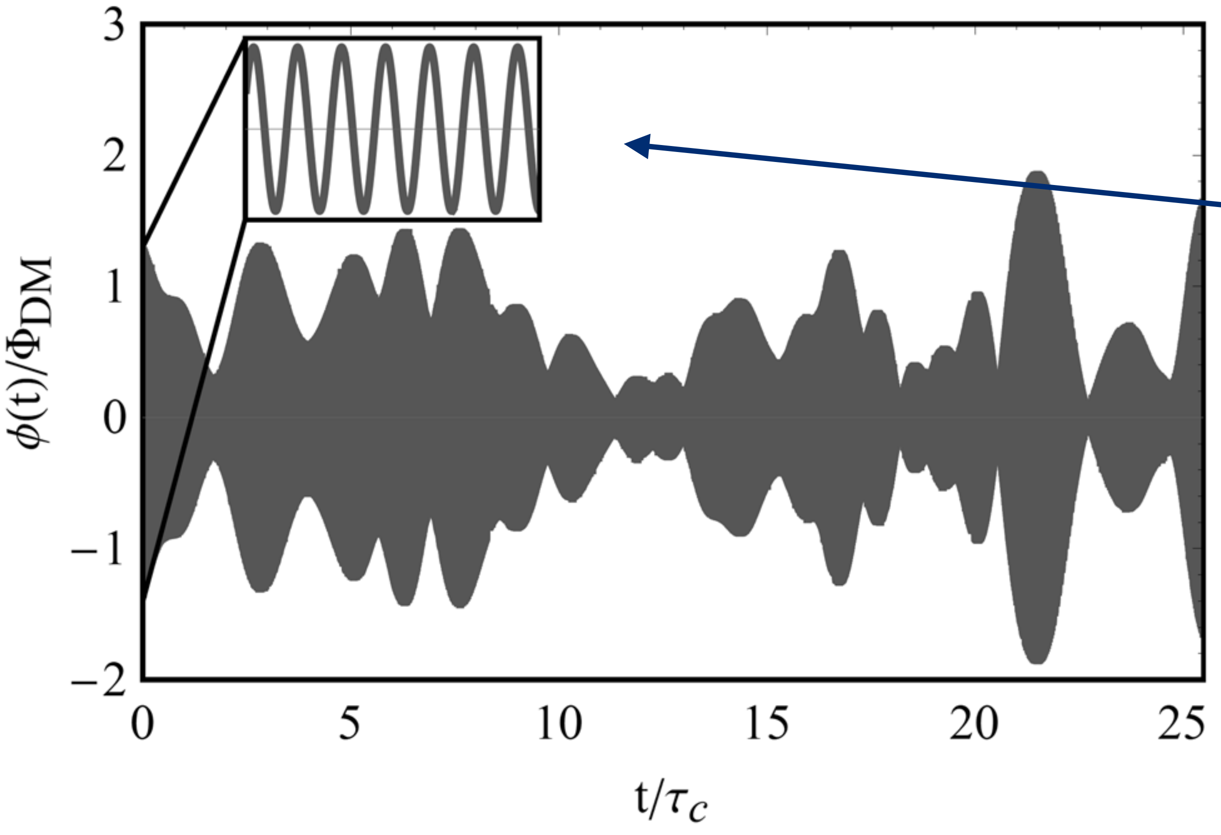
$$\tau_{coh} \sim \frac{\lambda_{dB}}{2v} \sim 6.5\times 10^5\,{\rm years}\left(\frac{10^{-3}\,c}{v}\right)^2\left(\frac{10^{-22}{\rm eV}}{m}\right)$$

$$\tau_{osc} \simeq \frac{2\pi}{m} \sim 1.3\,{\rm years}\left(\frac{10^{-22}{\rm eV}}{m}\right)$$

`Small-scale' stochastic model (spin 0)

Simulated VULF (Virialized UltraLight Field)

[Centers et al Nature Comm. **12**, 7321 (2021)]



Local homogenous model

$$\Phi(t) = \frac{\sqrt{2\rho_{DM}}}{m} r \cos(mt + \Upsilon)$$

- **Coherent on** $t \ll \tau_c$
- **Local** (r, Υ) **with:**

$$P(r) = 2re^{-r^2} \text{ \& } \Upsilon \in [0, 2\pi)$$

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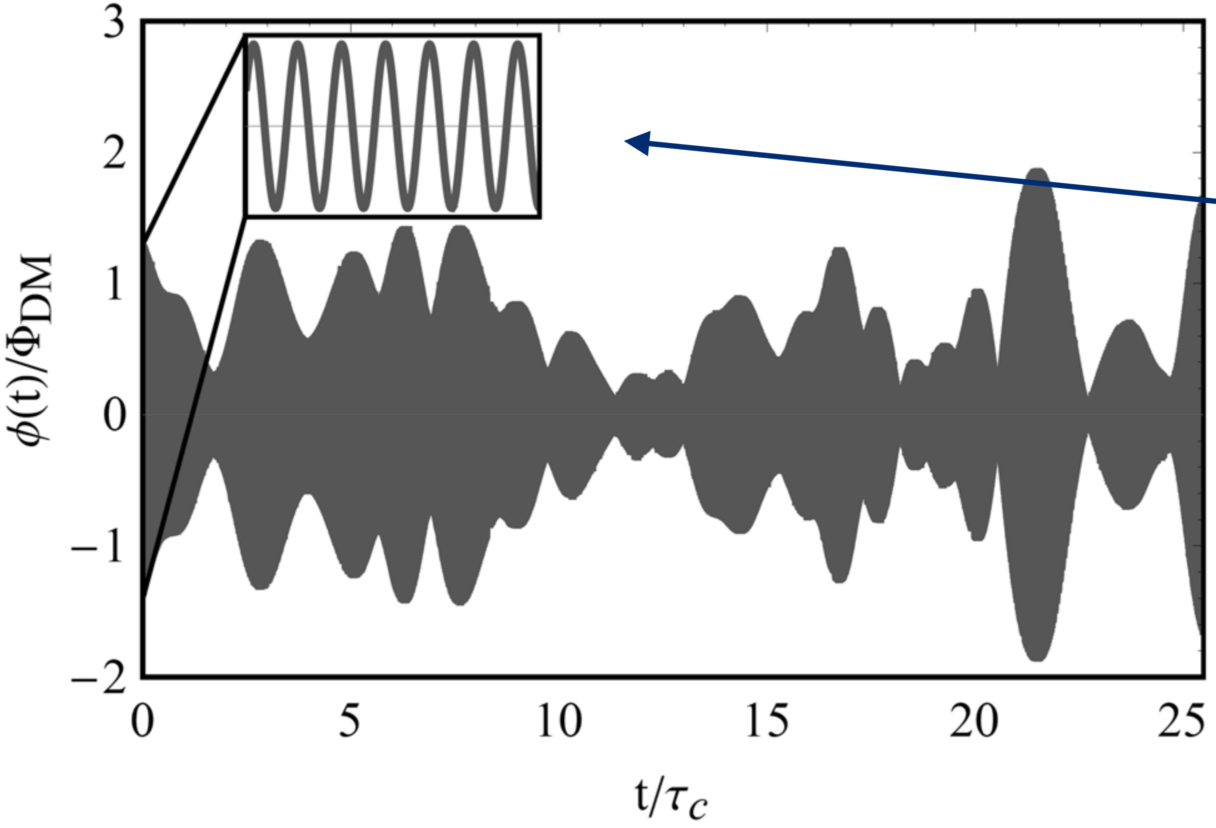

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- **Analog phenomenological descriptions for spin 1 and 2 are under development. See for instance [Amaral et al JCAP 06 (2024) 050, López-Sánchez, et al arXiv:2502.03561]**

Observation on diverse scales can probe different mass ranges

Linear dynamics: **see T. Ferreira Chase's talk**

Cosmic Microwave Background, Large Scale Structure

- T. Ferreira Chase and DLN, PRD (2024)
- T. Ferreira Chase, M. Leizerovich, DLN, and S. Landau, PRD (2025)

$m\,[\mathrm{eV}]$

$\sim 10^{-23}$

$\sim 10^{-21}$

$\sim 10^{-20}$

$\sim 10^{-18}$

Lyman-alpha forest,
Galaxy formation,
halos structure, 21cm,
Dynamics of stellar clusters,
Black Hole physics

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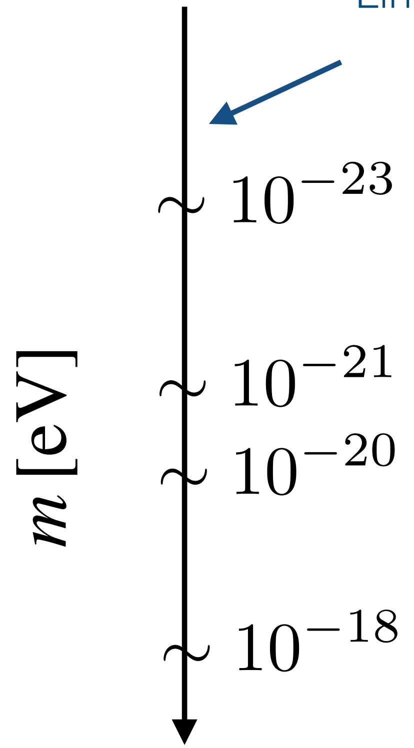

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- These observations can probe ULDM interacting only through gravity
- If ULDM is directly coupled to the Standard Model -> many other (but model-dependent) possibilities: atomic clocks, accelerometers, resonant-mass detectors, laser and atom interferometry…
- Robustness depends on the observable: theoretical uncertainties, modeling…

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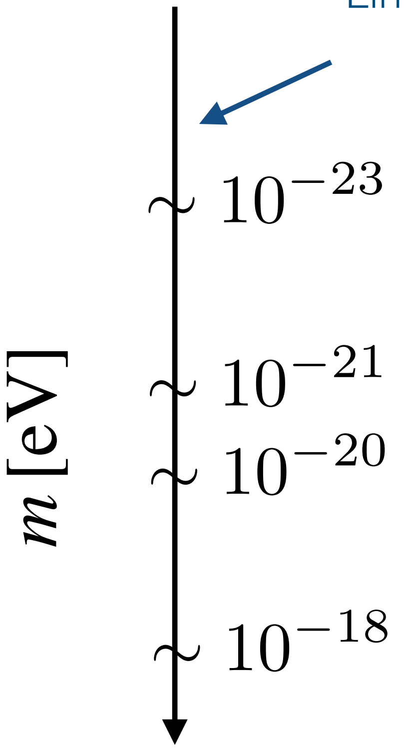

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Pulsar-Timing-Arrays (PTA)

- J.M. Armaleo, DLN and F. Urban, JCAP (2020)
- C. Smarra, A. Kuntz, E. Barausse, B. Goncharov, DLN et al, PRD (2024)

Binary Pulsars

- D. Blas, DLN and S. Sibiryakov, PRL (2017), PRD (2020)
- DLN and F. Urban, JCAP (2018)
- J.M. Armaleo, DLN and F. Urban, JCAP (2020)
- P. Kus, DLN and F. Urban, Astronomy&Astrophysics (2024)
- P. Kus, DLN and F. Urban, arXiv: 2506.04100

Gravitational waves from compact binaries

- T. Ferreira Chase, DLN and N. Yunes, arXiv: 2505.21383

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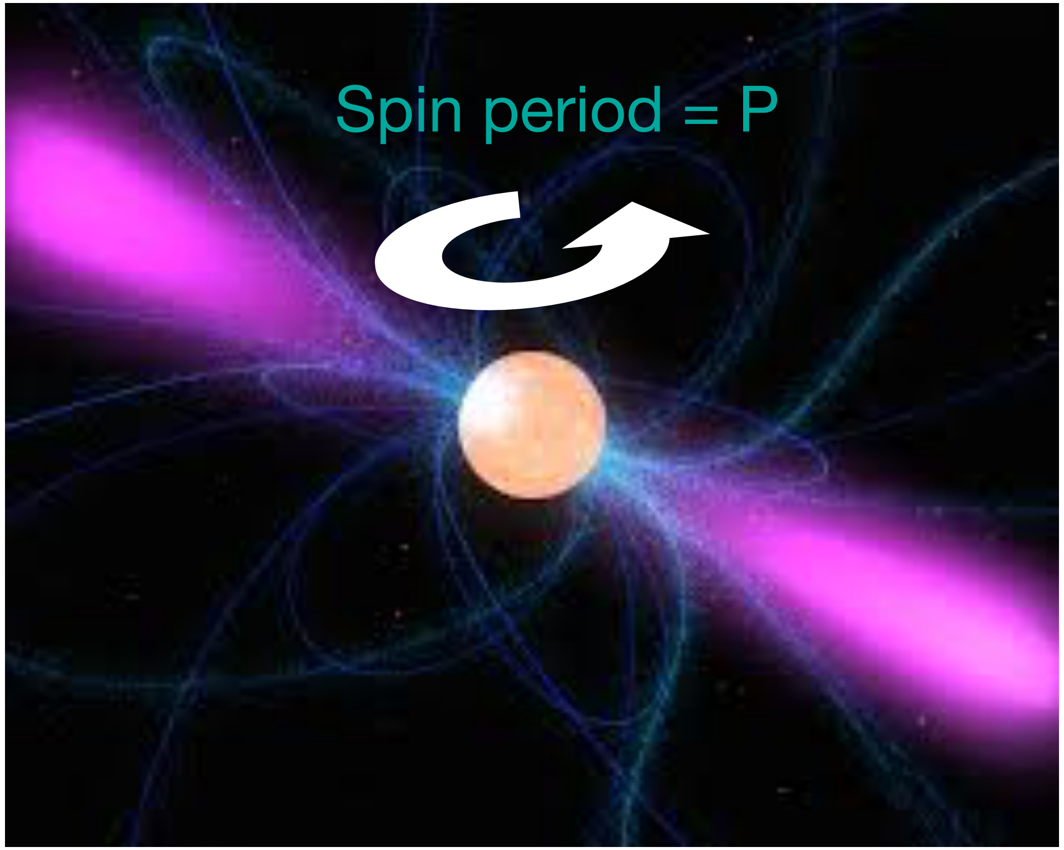
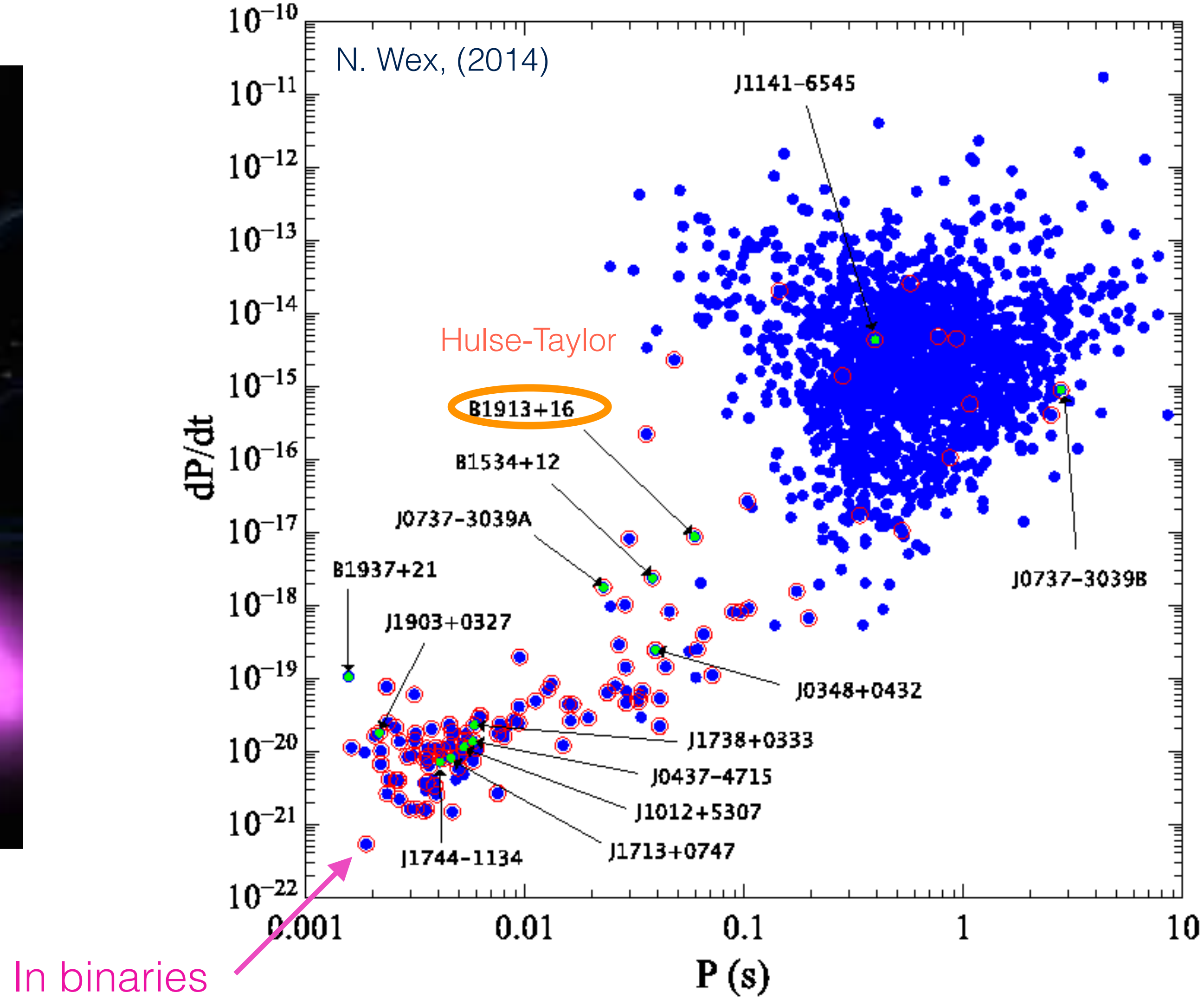
Why pulsars?



Spin period = P



N. Wex, (2014)
Hulse-Taylor
B1913+16
B1534+12
J0737-3039A
B1937+21
J1903+0327
J1141-6545
J0737-3039B
J0348+0432
J1738+0333
J0437-4715
J1012+5307
J1713+0747
J1744-1134
In binaries
dP/dt
P (s)
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-22}
0.001
0.01
0.1
1
10

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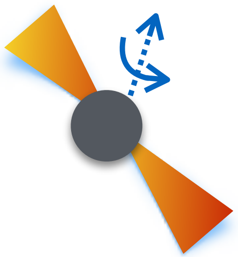

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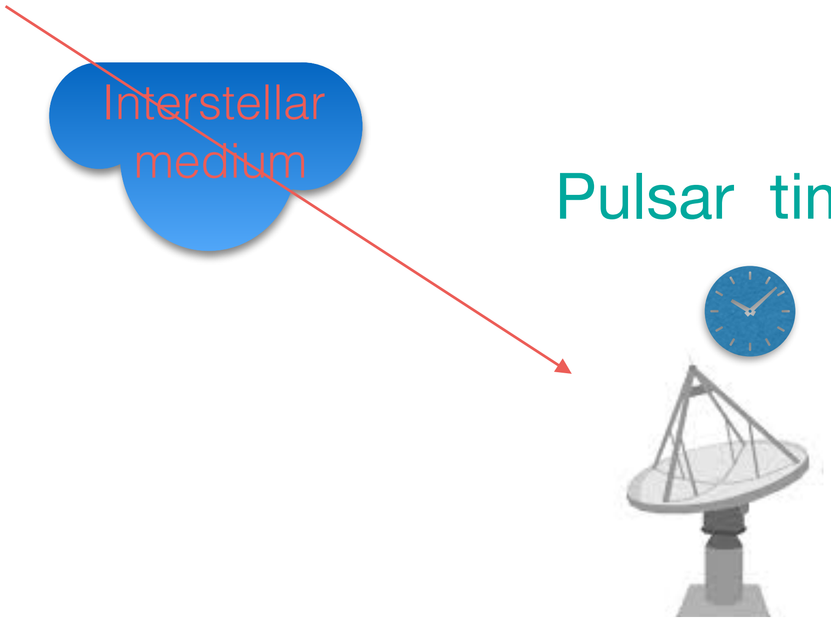
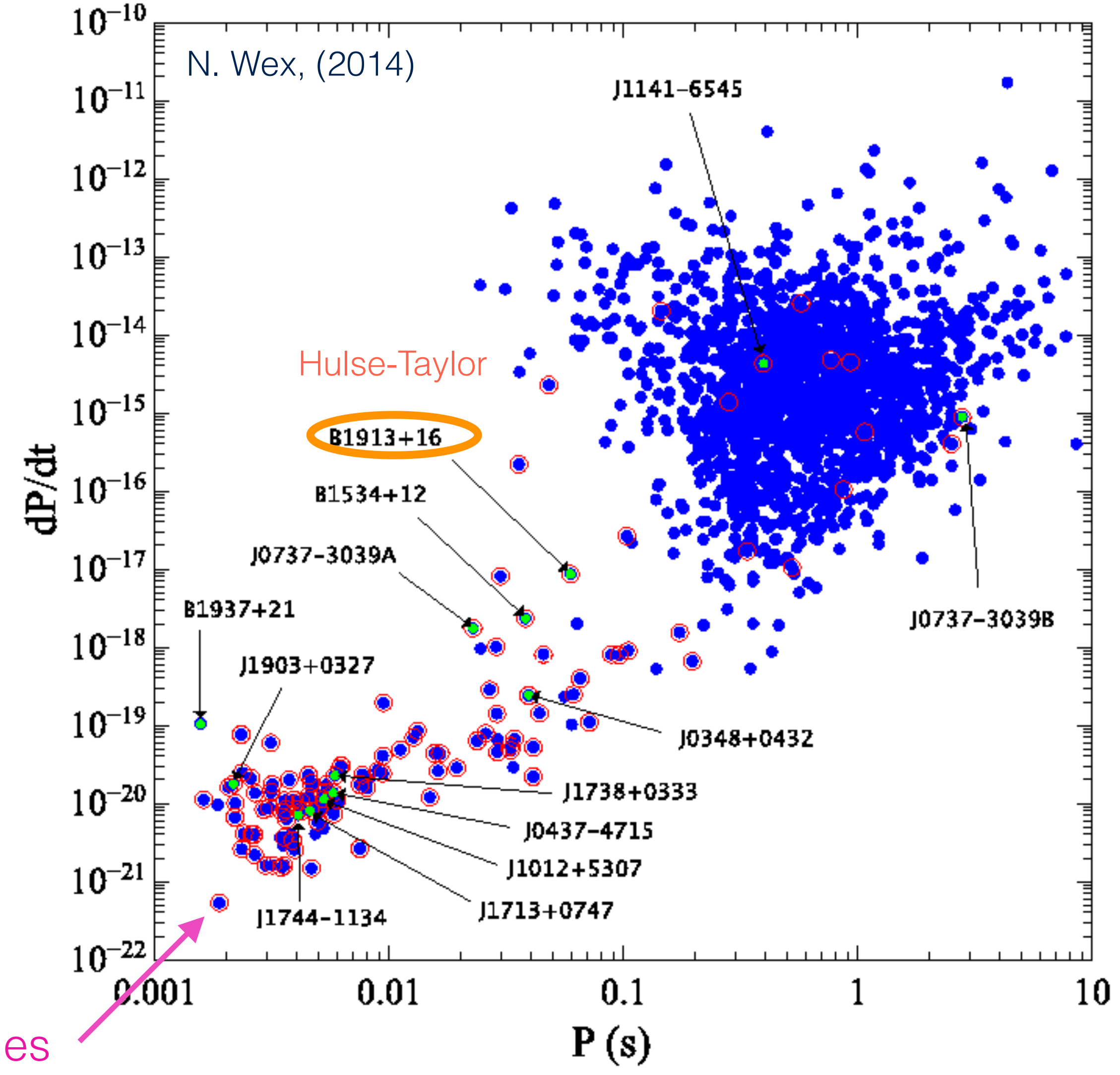


Pulsar timing



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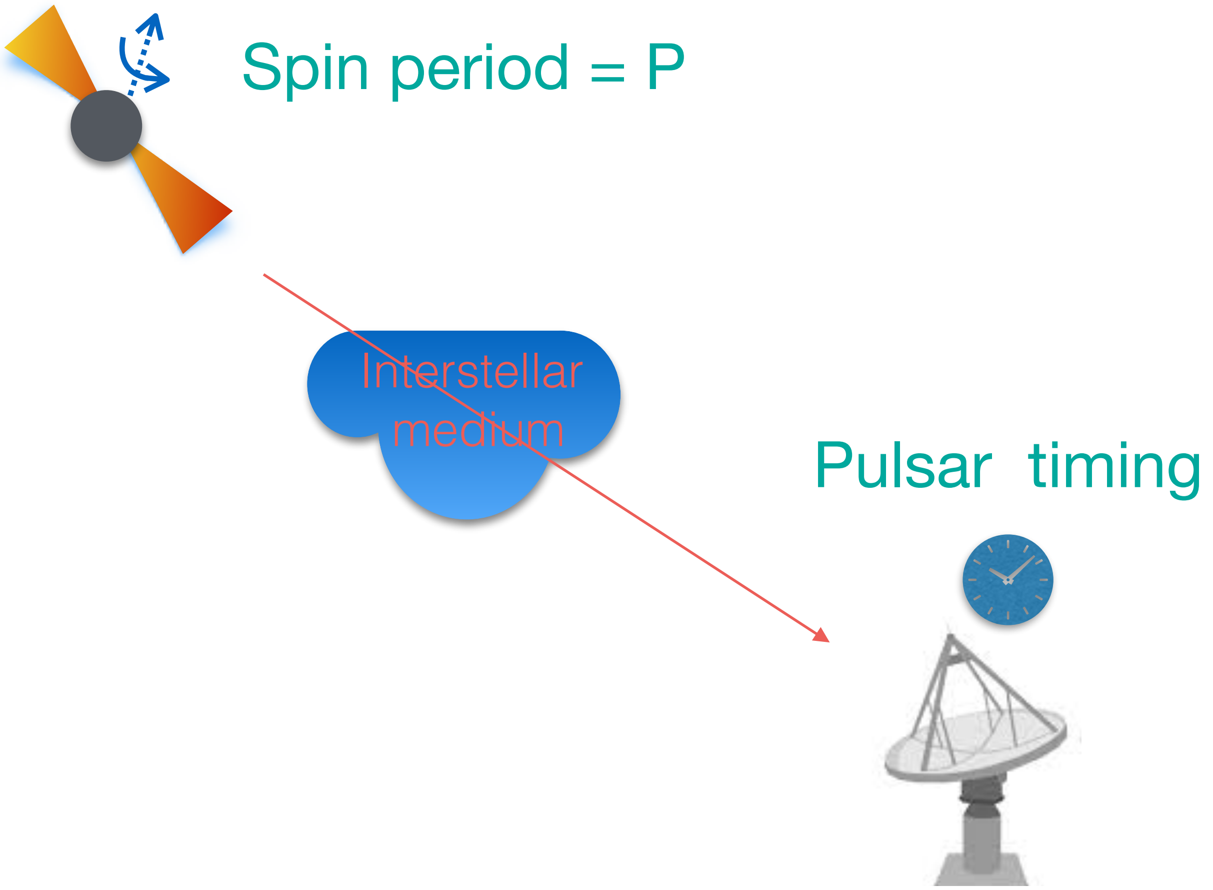
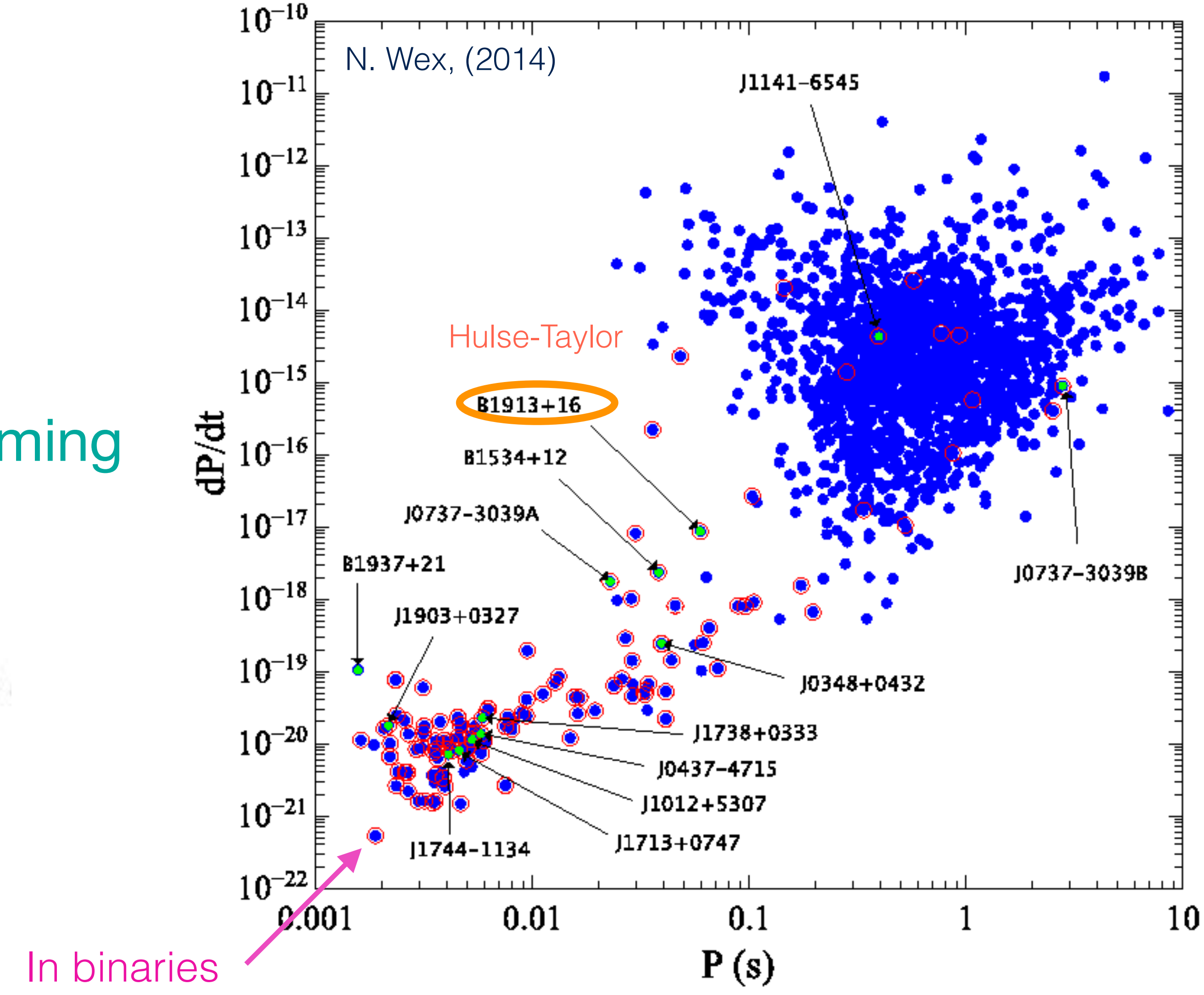




How would ULDM affect pulsar timing?

ULDM interacting only through gravity (e.g spin=0)

[Khmelnitsky & Rubakov (2014)]

$$\Phi(t,\vec{x}) = \Phi_0 \cos(mt + \Upsilon(\vec{x})) \qquad (a \simeq 1)$$

$T^{\mu\nu}_{DM}$

$$\rho_{DM} = \frac{m^2\Phi_0^2}{2}$$

$$P_{DM} = -\rho_{DM}\cos(2mt + 2\Upsilon)$$

Einstein's Eqns: $G^{\mu\nu} = 8\pi G_N T^{\mu\nu}_{DM}$ with $g_{\mu\nu} \simeq \eta_{\mu\nu} + h_{\mu\nu}$

Oscillations in the geometry:

(at leading order in gradients)

$$h_{0\mu} \sim 0$$

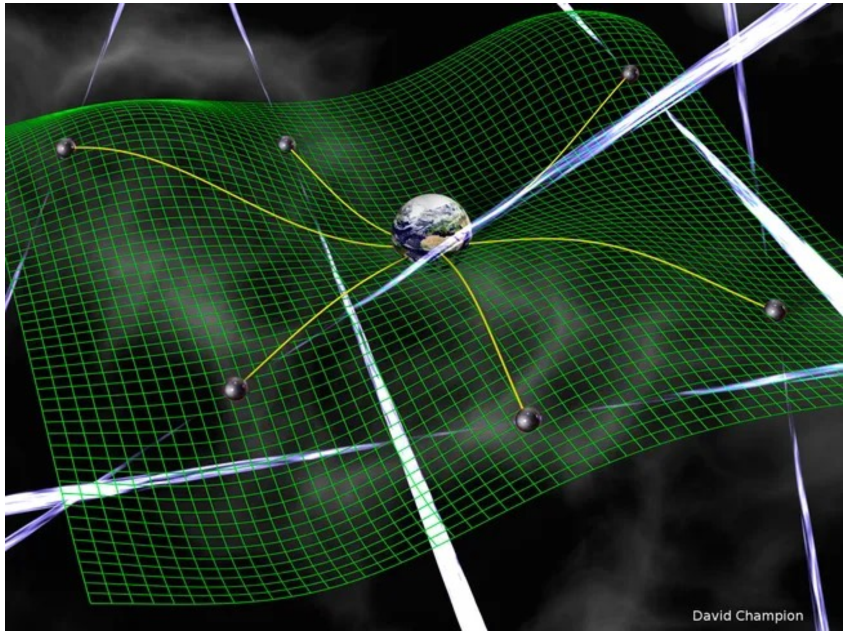
$$h_{ij} \sim -\frac{2\pi G\rho_{DM}}{m^2}\delta_{ij}\cos(2mt + 2\Upsilon)$$

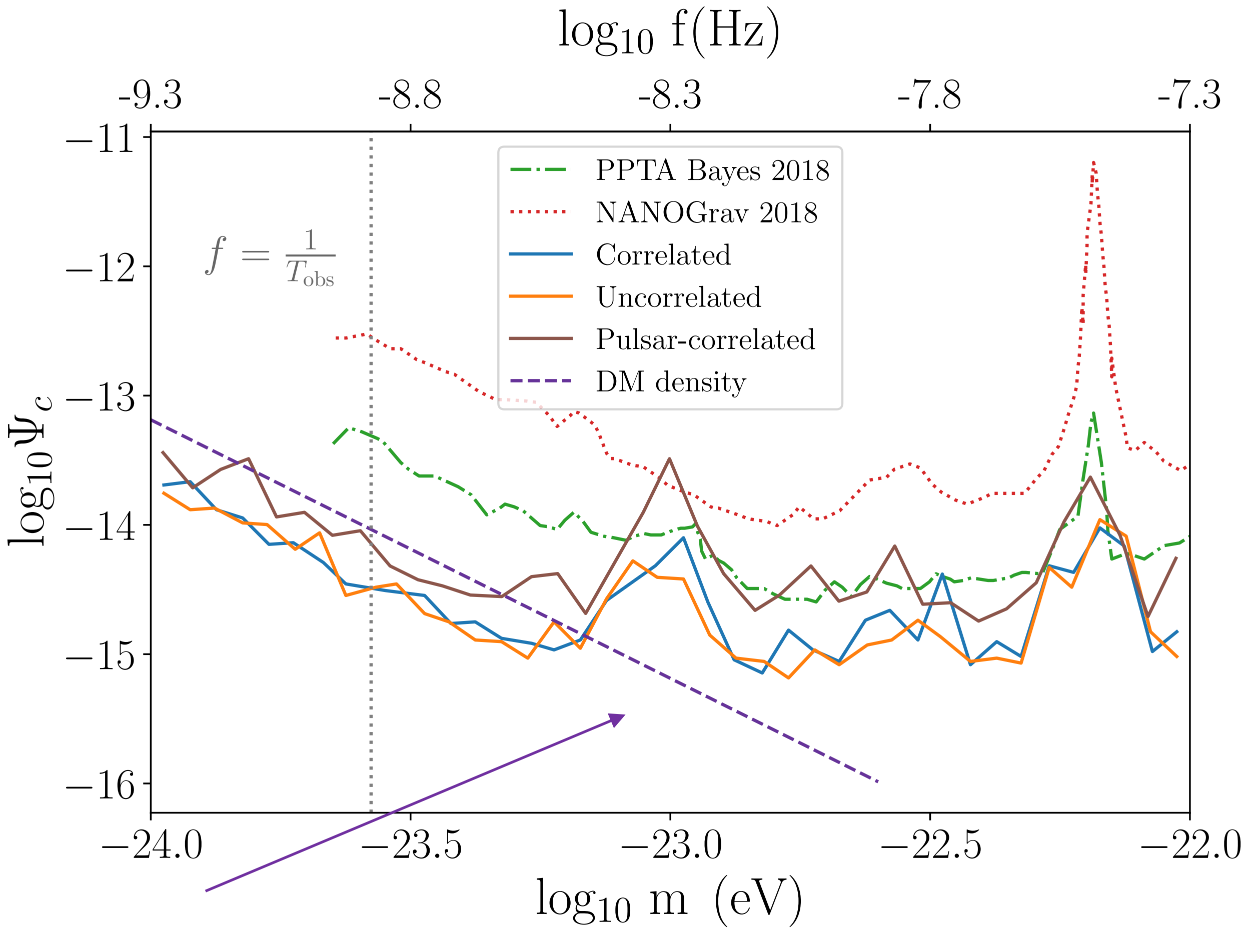
$:= \Psi_c \rightarrow$ **decreases with** m

Only gravity!

PTA- recent bounds [Smarra, C., Goncharov, B., Barausse, E., et al. 2023, PRL (2023)]

From the 2nd data release of the European Pulsar Timing Array





ULDM (spin=0) signal, assuming

$$\Psi_c = \frac{2\pi G \rho_{DM}}{m^2}$$

$$\rho_\phi = \rho_{DM} = 0.4\,\frac{\text{GeV}}{\text{cm}^3}$$

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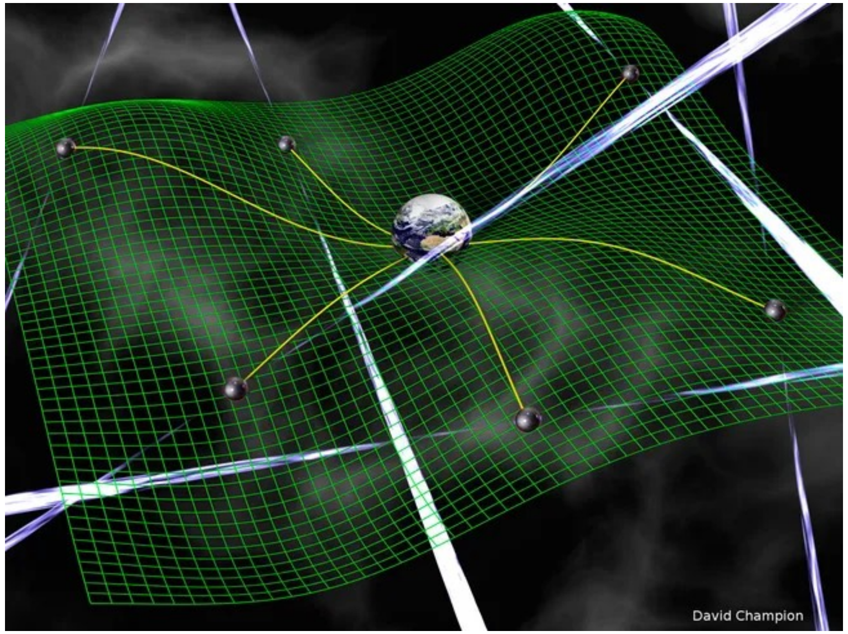

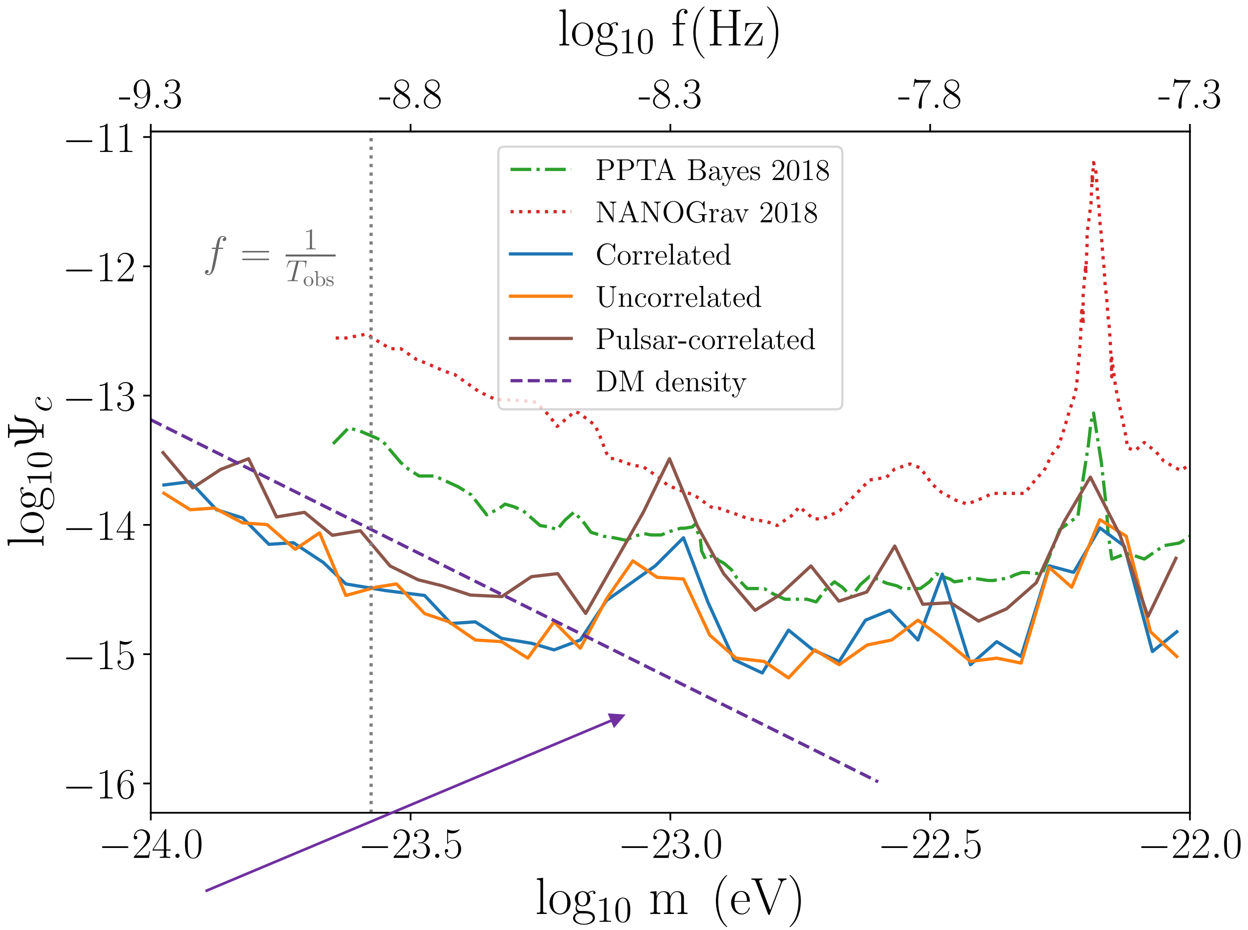

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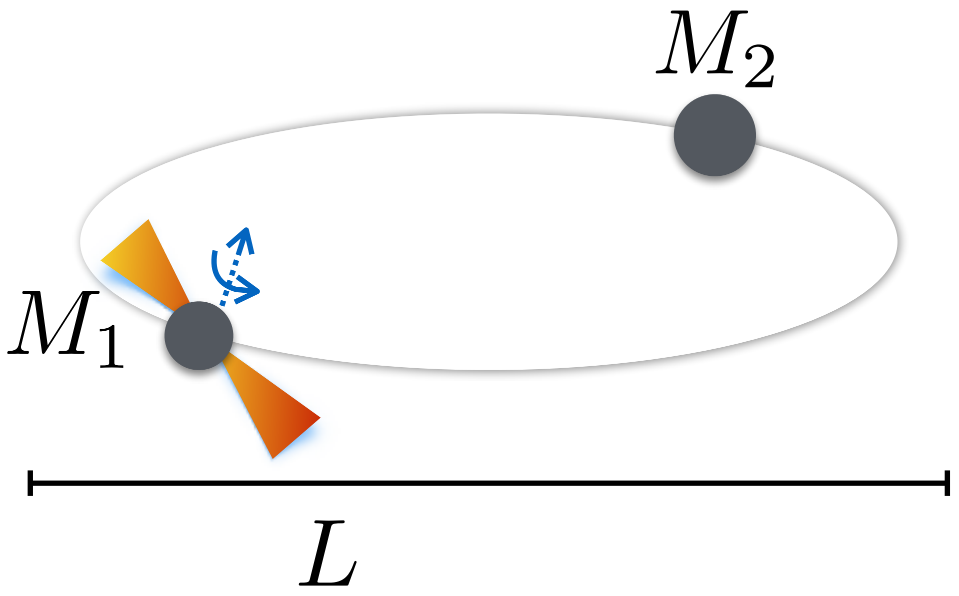
Also present for spin 1 [K. Nomura et al (2020)] and spin 2 [Armaleo, DLN & Urban (2020)]

Numbers:

Why Binary Pulsars (BPs)?



$$L \sim 10^8\,\mathrm{km}\left(\frac{P_b}{100\mathrm{days}}\right)^{2/3}\left(\frac{M_1+M_2}{M_\odot}\right)^{1/3}$$

$$m \sim 10^{-18} \div 10^{-22}\mathrm{eV}$$

$$\lambda_{dB} \sim 1.3 \times 10^{12}\mathrm{km}\left(\frac{10^{-3}c}{v}\right)\left(\frac{10^{-18}\mathrm{eV}}{m}\right)$$

$$\tau_{coh} \sim \frac{\lambda_{dB}}{2v} \sim 65 \times \text{ years}\left(\frac{10^{-3}\,c}{v}\right)^2\left(\frac{10^{-18}\mathrm{eV}}{m}\right)$$

$$t_{osc} \simeq 100\text{ days}\left(\frac{10^{-22}\mathrm{eV}}{m}\right)$$

- Homogeneous: $\lambda_{dB} \gg L$ ✓
- Coherent: $\tau_{coh} \gg T_{obs}$ ✓
- Osc. are relevant!

Secular effects of DM on BPs

- Oscillations of the DM field produce periodic perturbations to the BP orbits:

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- We studied constrains from Binary pulsars assuming resonance for different couplings and spins in:
  - Spin=0: D. Blas, DLN and S. Sibiryakov, PRL (2017), PRD (2020)
  - Spin=1: DLN and F. Urban, JCAP (2018)
  - Spin=2: JM Armaleo, DLN and F. Urban, JCAP01 (2020)

Recent Progress:

Sensitivity curves for ULDM couplings on Binary Pulsars vs m

- Bayesian sensitivity of binary pulsars to ultra-light dark matter
[Kus, DLN, F. Urban, A&A 690, A51 (2024)]

- Deep Neural Networks Hunting Ultra-Light Dark Matter
[Kus, DLN, F. Urban, arXiv:2506.04100]

We can now go beyond resonance & combine data from different systems!

Bounds on models beyond resonance?

E.g. with s=0: Effective universal direct coupling on BPs

$$S = S_{Gravity}(g_{\mu\nu}) + S_{SM}(\bar{g}_{\mu\nu}, \Psi) - \frac{1}{2}\int dx^4 \sqrt{-g}\left[\partial_\nu \Phi \partial^\nu \Phi - m^2 \Phi^2\right]$$

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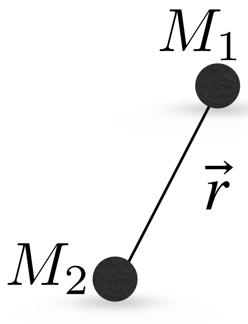
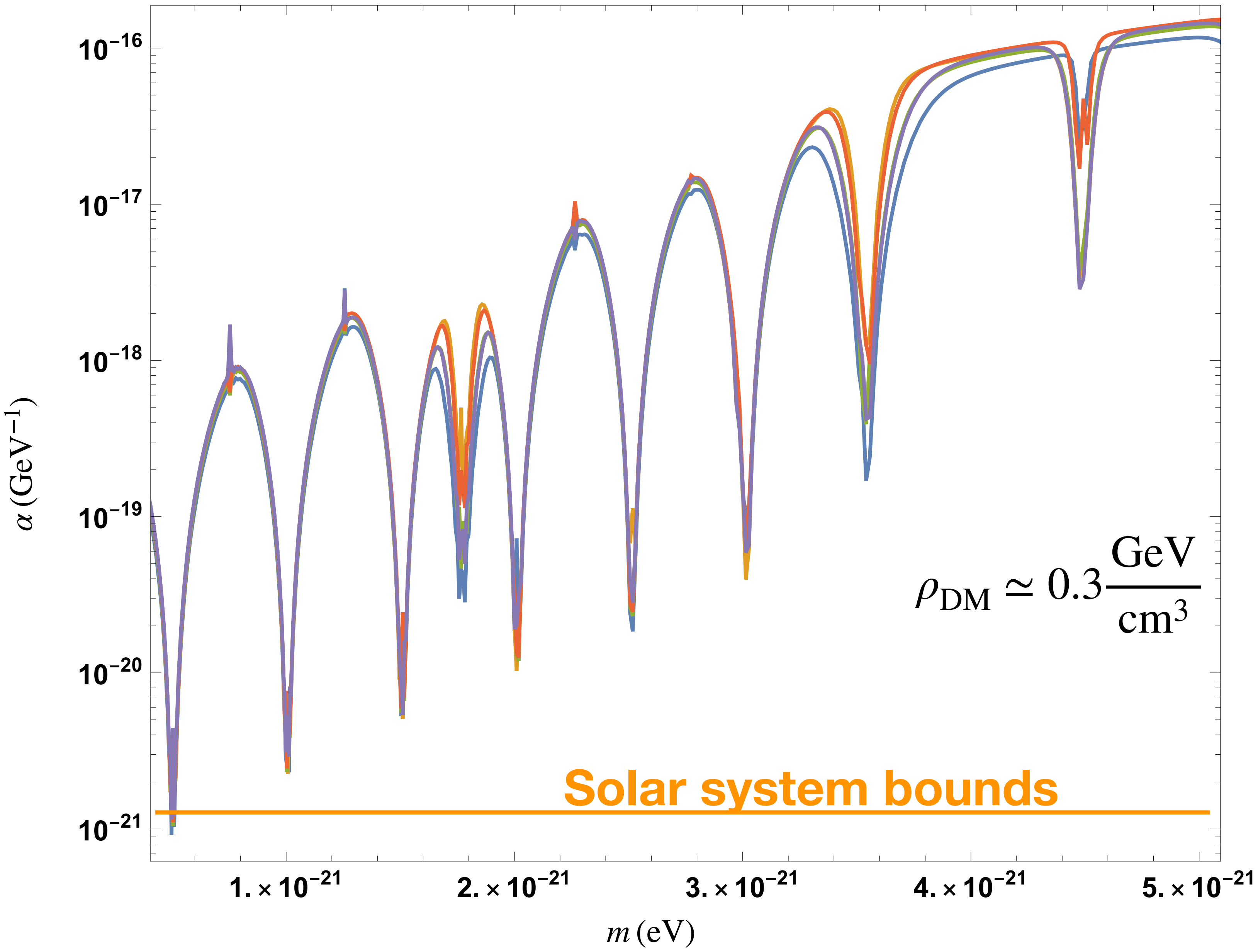


$$S_B = -\sum_{A=1,2} \int d\tau_A\, M_A(\Phi) \qquad M_A(\Phi) = \bar{M}_A(1 + \alpha\Phi)$$

$$\ddot{\vec{r}} = -(1 + \alpha\Phi)\frac{G\bar{M}_T \vec{r}}{r^3} - \alpha\dot{\Phi}\,\dot{\vec{r}} = -\frac{G\bar{M}_T \vec{r}}{r^3} + \vec{F}$$

Characterizes the perturbation

Using only the effect on δP_b: + systems from NANOGrav 15-year



Combined systems:

-PSRJ19463417

-PSRJ22340611

-PSRJ1903+0327

Six realizations of (r, Υ)

[Kus, DLN, F. Urban, A&A 690, A51 (2024)]

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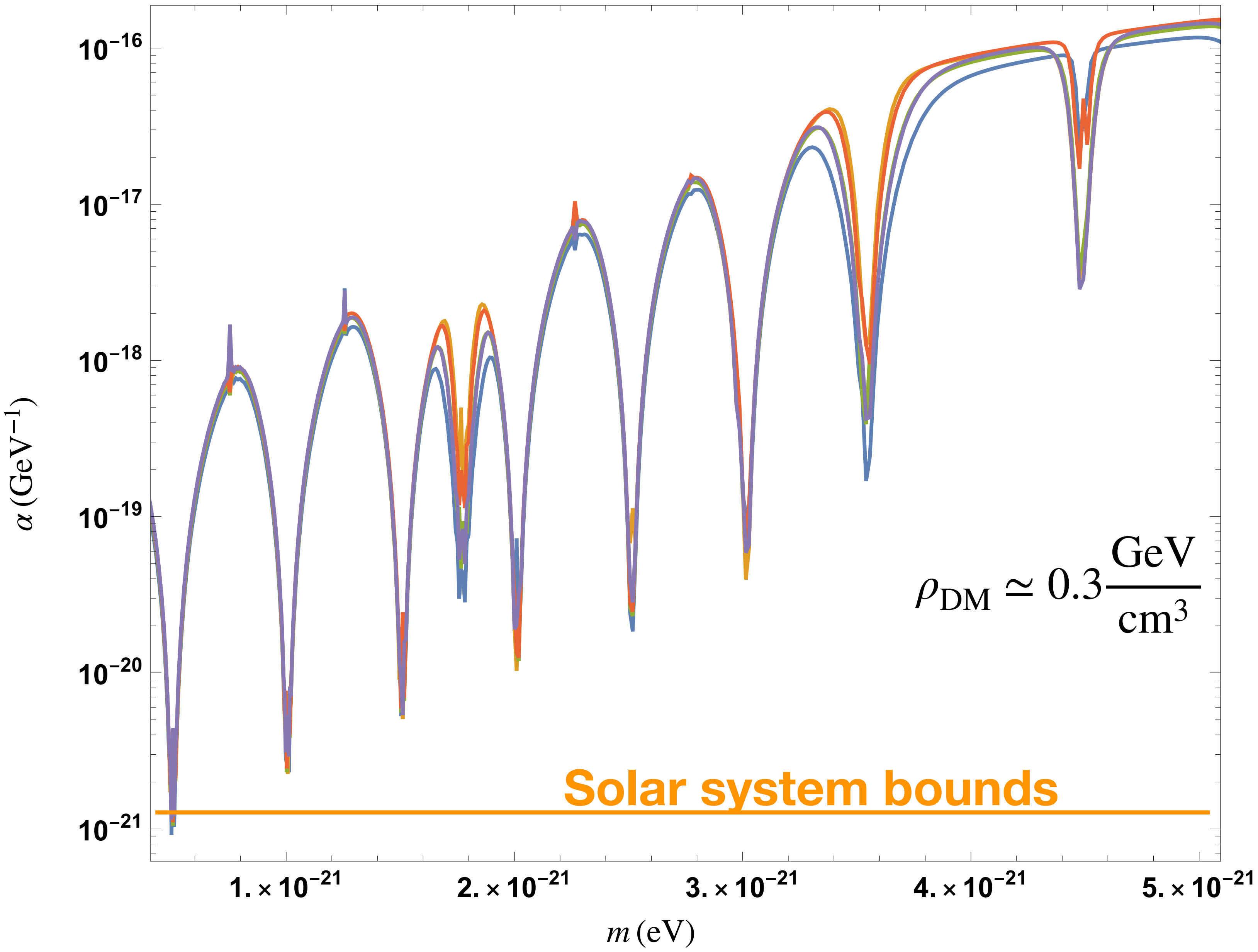

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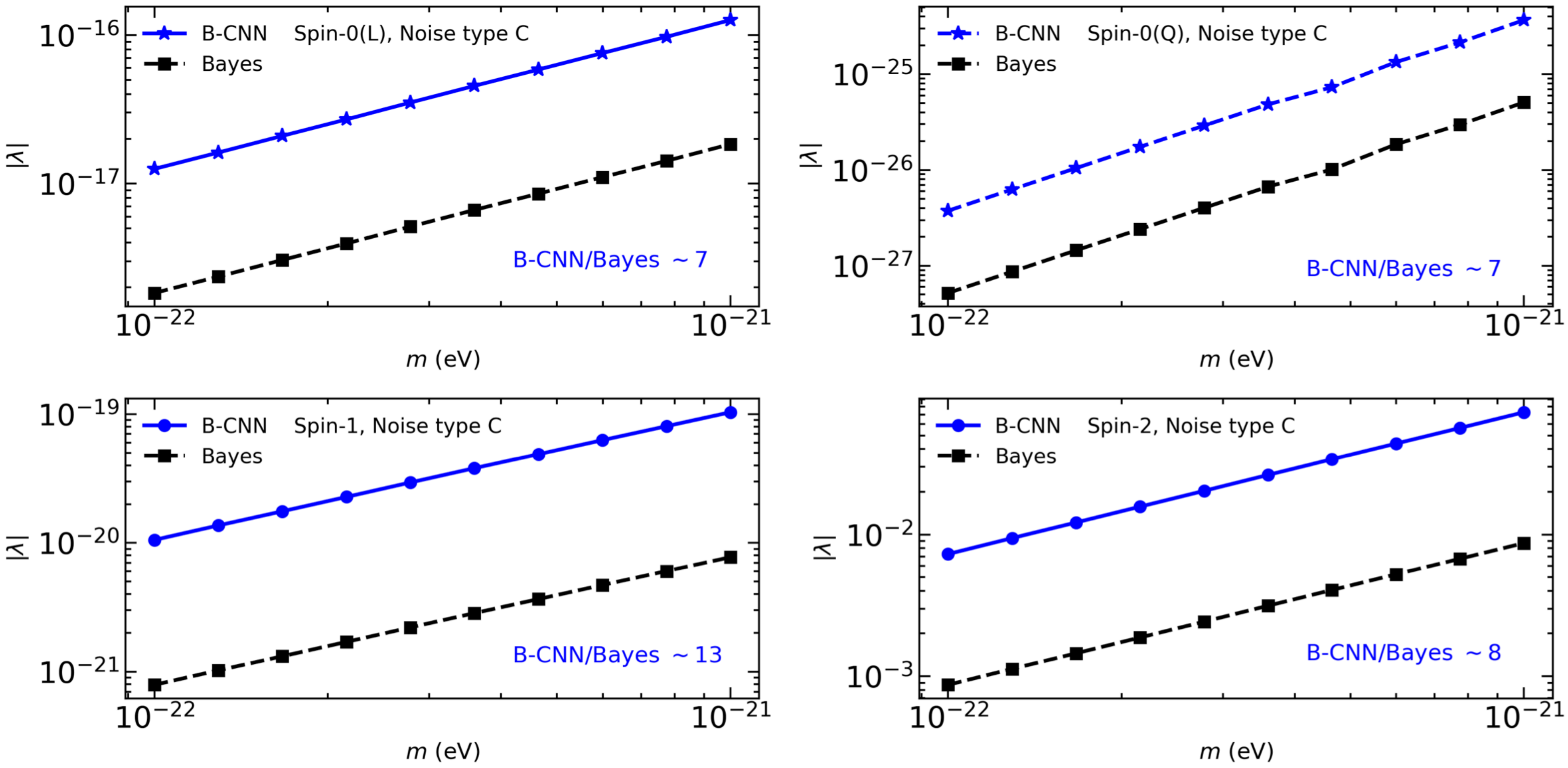
New (~1000) Binary Pulsars are expected to be discovered by the Square Kilometre Array!

- Using Deep Neural Networks

$\lambda =$ Normalized coupling constant



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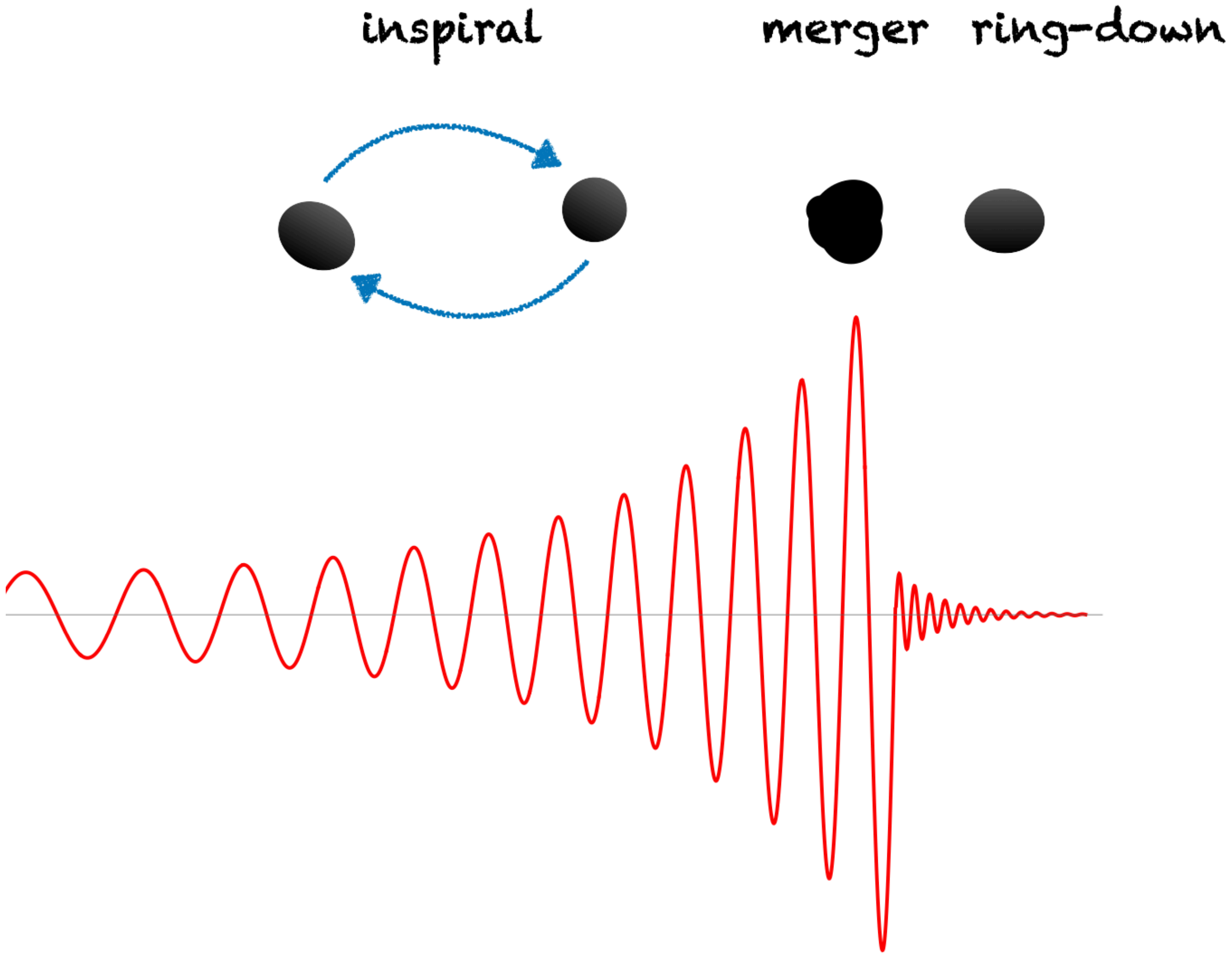

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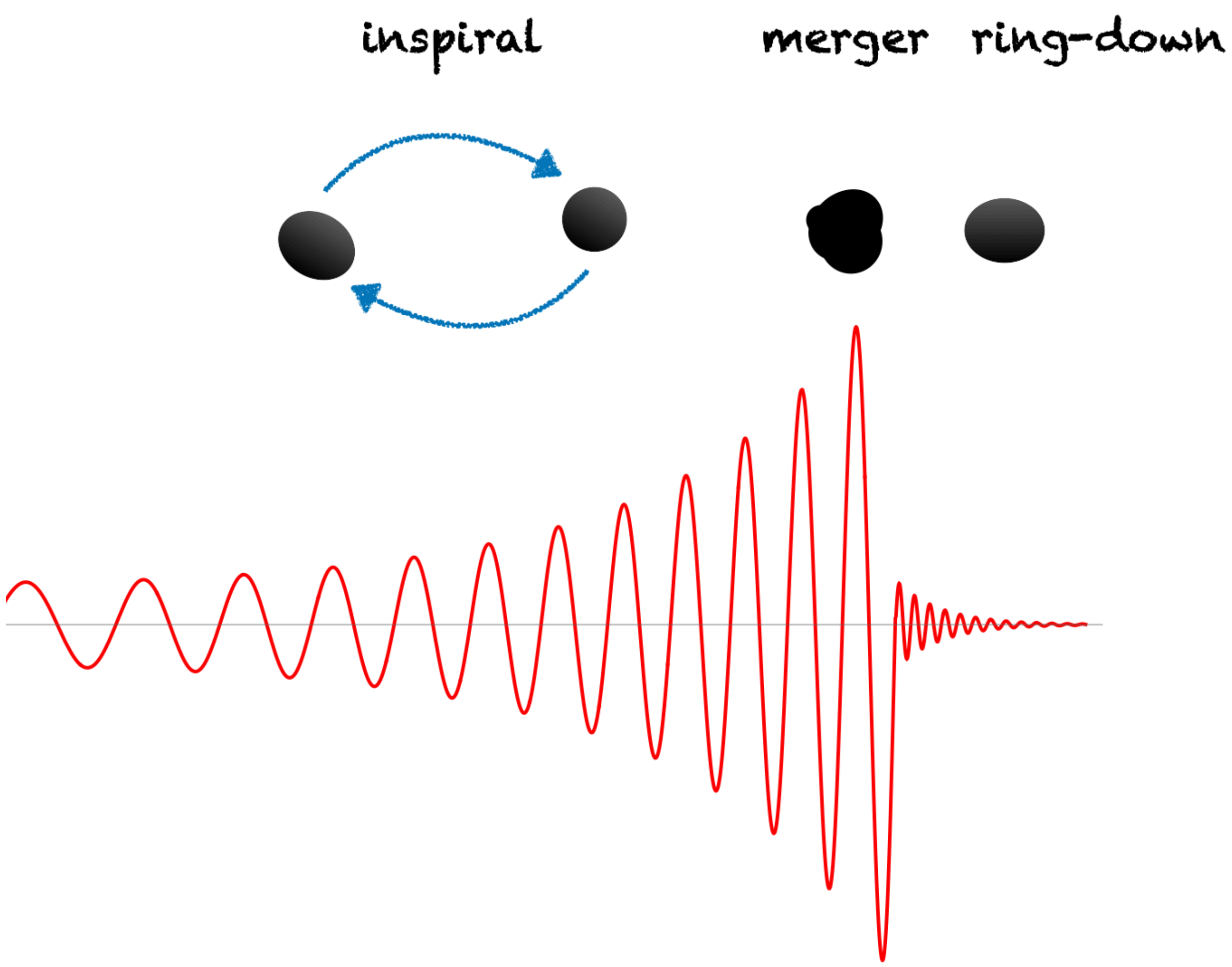

Main assumptions

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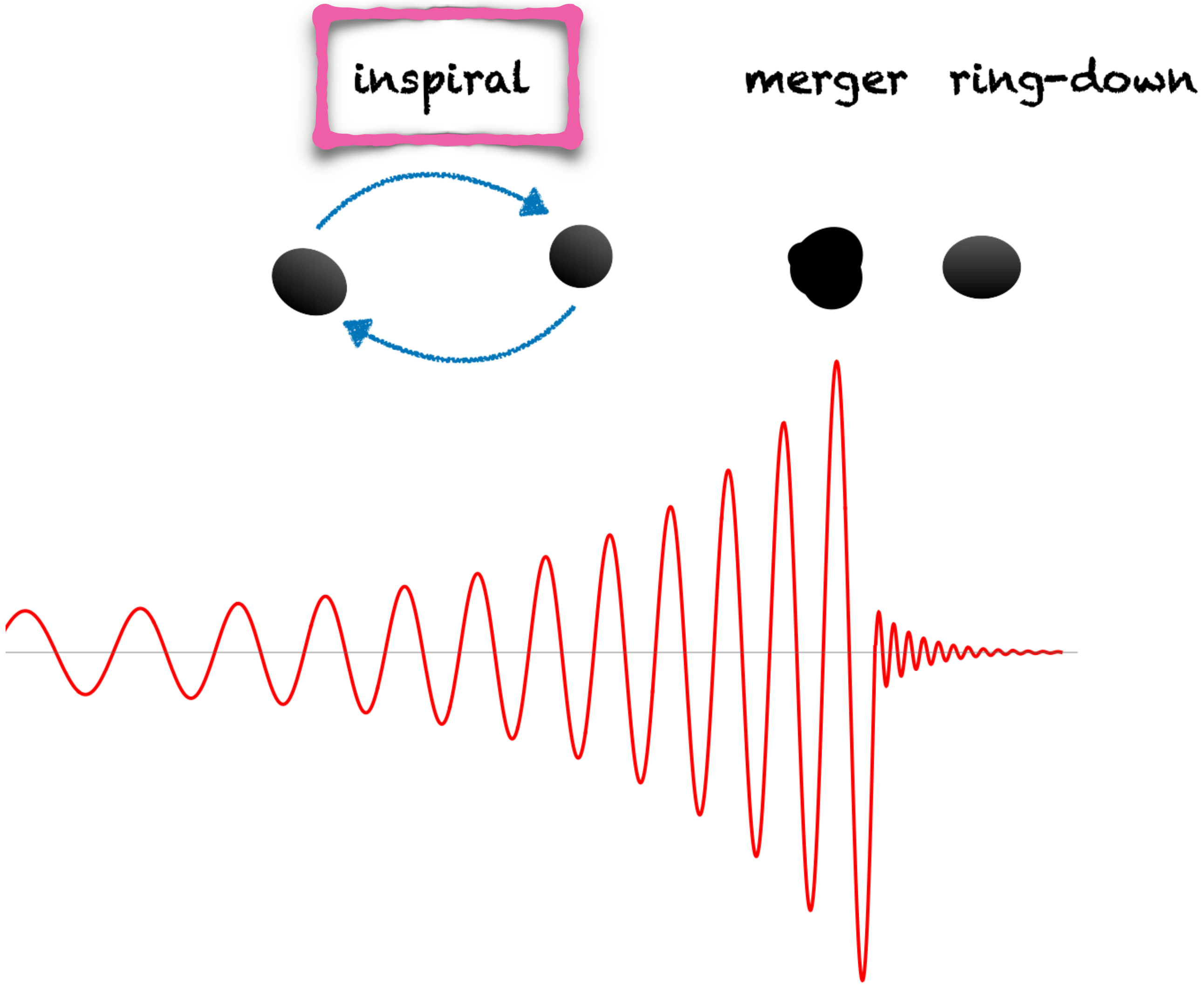

Main assumptions

- Inspiral phase

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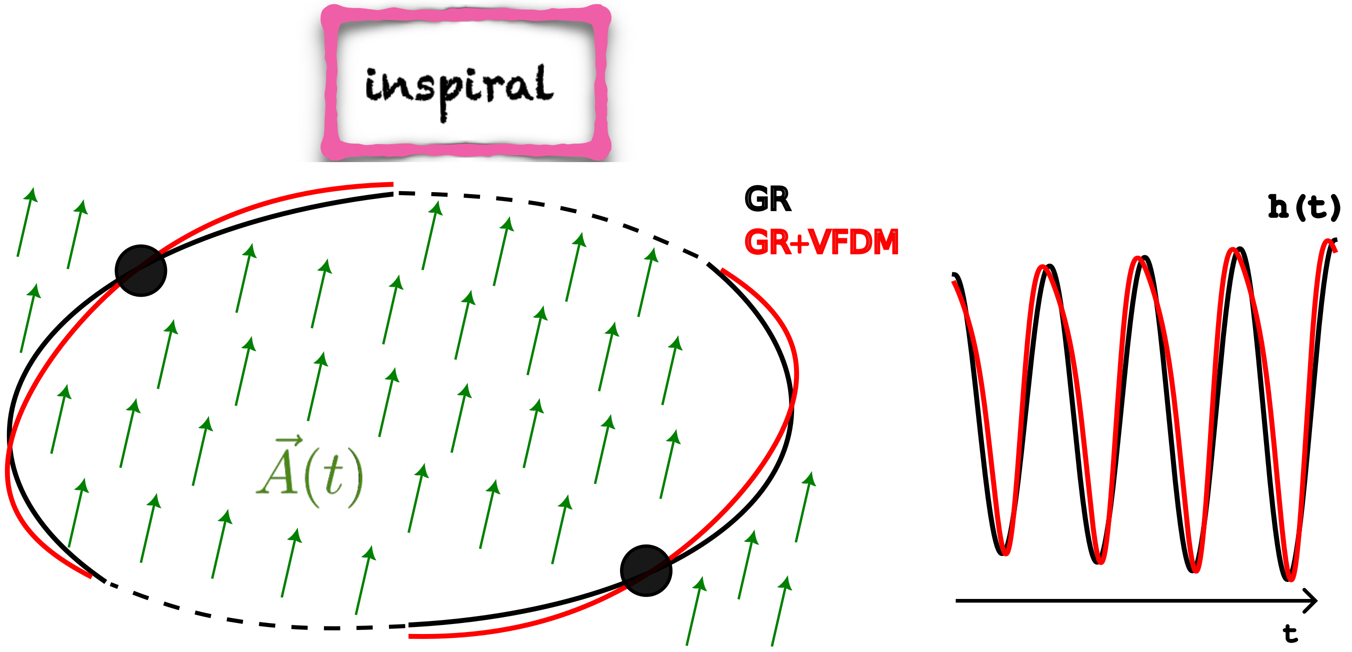
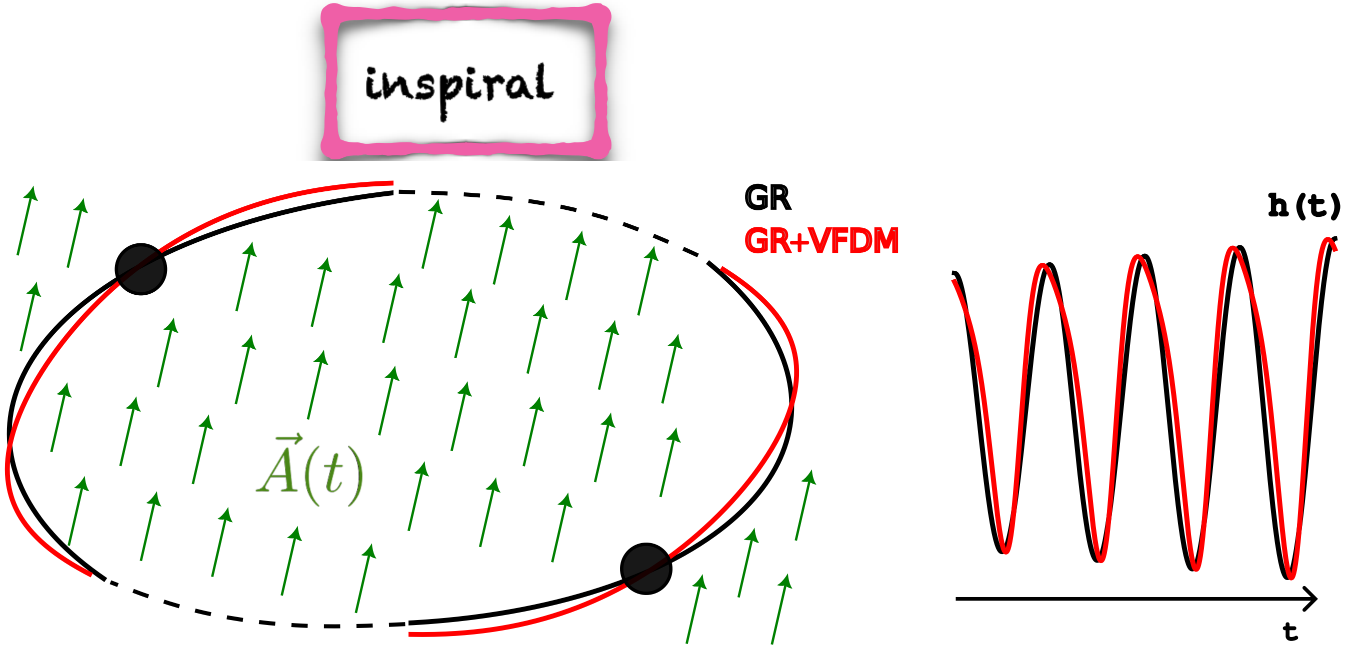


Main assumptions

- Inspiral phase
- Quasi-circular orbits

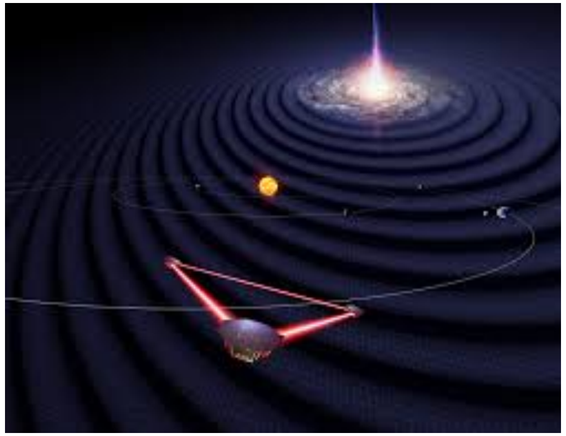
— Vector Field Dark Matter (VFDM)

Could an ultralight dark-matter environment affect the gravitational waves emitted by compact binaries? Could the effect be detectable?



Laser
Interferometer
Space
Antenna

(scheduled for launch in 2035)



Main assumptions

- **Inspiral phase**
- **Quasi-circular orbits**

— Vector Field Dark Matter (VFDM)

- **Black Hole (BH) mass range**

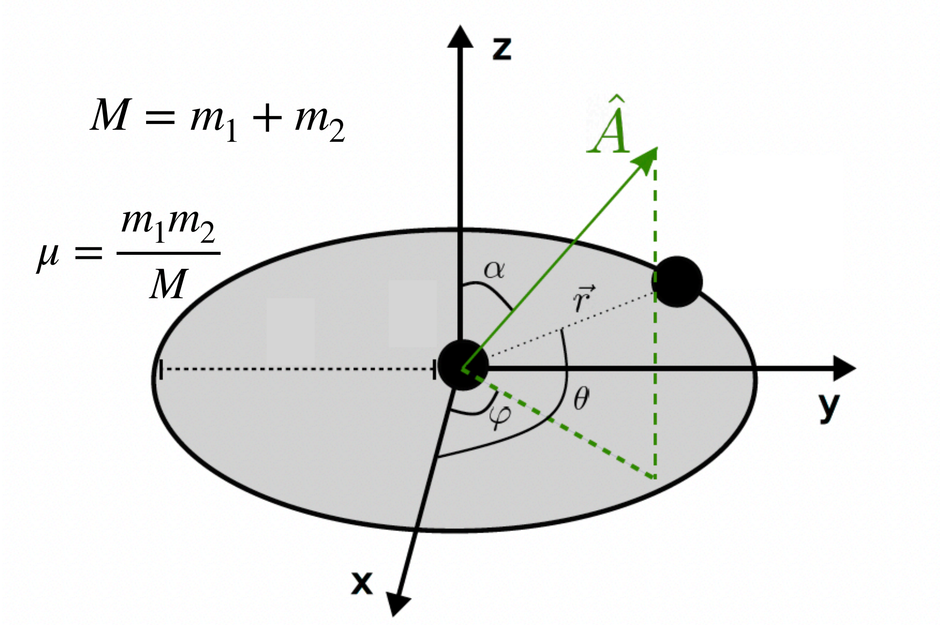
$M_{\rm BH} \in (10^2, 10^6)\, M_\odot$

- **Laser Interferometer Space Antenna (LISA)**

NUMBERS for GW emission of quasi-circular BH binaries

$\hbar = 1$

Effective one-body description



$M = m_1 + m_2$

$\mu = \frac{m_1 m_2}{M}$

- **GW emission:** $g_{\mu\nu} = \eta_{\mu\nu} + h^M_{\mu\nu}, \quad |h^M_{\mu\nu}| \ll 1$

$$h^{M,TT}_{ij} = \frac{2G}{c^4 D} \ddot{M}^{TT}_{ij}$$

Quadrupole moment:

$$\ddot{M}_{11} = 2\mu b^2 \omega_b^2 \cos(4\pi f_b t)$$
$$\ddot{M}_{22} = 2\mu b^2 \omega_b^2 \sin(4\pi f_b t)$$

D: Distance source-detector in source frame

Orbital frequency:

Kepler $\longrightarrow$ $\omega_b = 2\pi f_b = \sqrt{\frac{GM}{a^3}}$

GW frequency:

$$f = 2 f_b = \frac{\omega_b}{\pi}$$

LISA band: $f \sim (10^{-5} - 1)\,\mathrm{Hz}$ or $\omega \sim (10^{-18} - 10^{-14})\,\mathrm{eV}$

Total mass: $M \sim (10^2 - 10^6)\ M_\odot$

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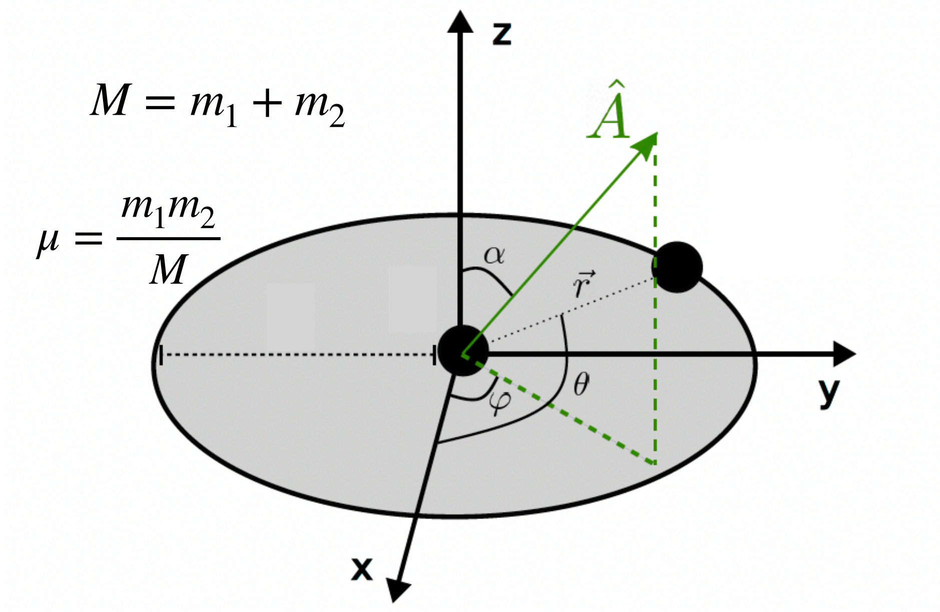

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Orbital radius: $r_{12} \sim 3.2 \times 10^8\ \text{km} \left(\dfrac{\text{M}}{10^6 \text{M}_\odot}\right)^{\frac{1}{3}} \left(\dfrac{\text{f}}{10^{-5}\text{Hz}}\right)^{-\frac{2}{3}}$

De Broglie wavelength: $\lambda_{dB} = \dfrac{1}{mv} \sim 10^{12}\,\text{km} \left(\dfrac{10^{-3}\text{c}}{v}\right)\left(\dfrac{10^{-18}\text{eV}}{m_A}\right)$

$\lambda_{dB} \gg r_{12} \rightarrow$ **Homogeneus**

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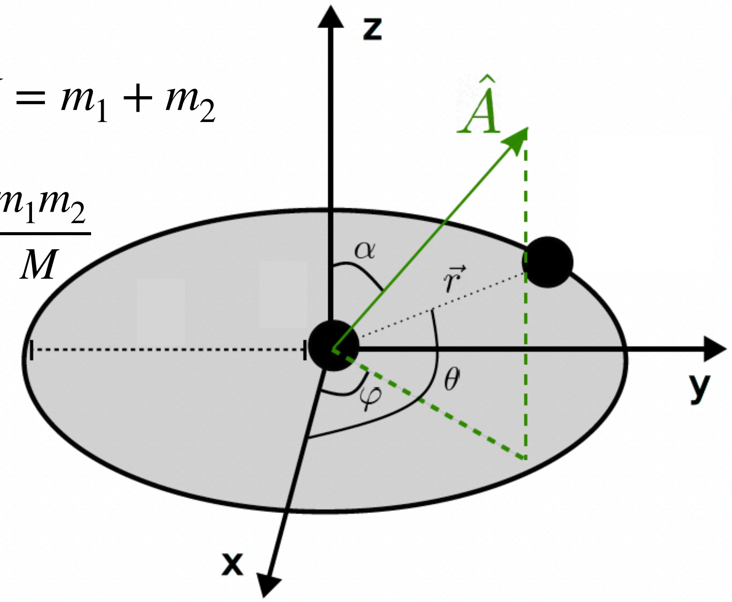

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Observation time: $\tau_{obs} \sim 4$ years

Coherence time: $\tau_{coh} \sim \dfrac{\lambda_{dB}}{2v} \sim 65 \times \text{ years} \left(\dfrac{10^{-3}\,c}{v}\right)^2 \left(\dfrac{10^{-18}\mathrm{eV}}{m_A}\right)$

$\tau_{coh} \gg \tau_{obs} \rightarrow$ **Coherent field**

Perturbed chirp signal for quasi-circular BH binaries

- **GW emission:** $h_{ij}^{M,TT} = h_+ \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + h_\times \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Define $h \equiv h_+ - i\, h_\times \rightarrow h(t) = \mathscr{A}(t)\, e^{2i\Phi(t)}$

- **GW backreaction:** $\dot{E} = -\mathscr{L}$

GW frequency

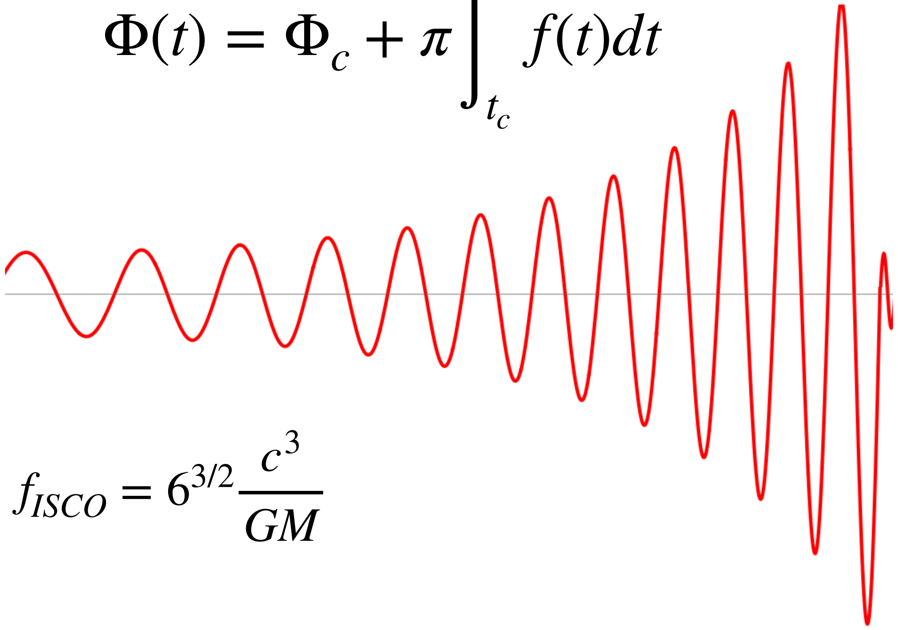
$$\Phi(t) = \Phi_c + \pi \int_{t_c}^{t} f(t) dt$$

Total radiated power*

$$f(t) = \frac{1}{\pi} \left(\frac{G\mathscr{M}}{c^3} \right)^{-5/8} \left(\frac{5}{256(t - t_c)} \right)^{3/8}$$

Chirp mass: $\mathscr{M} = \mu^{3/5} M^{2/5}$ Coalescence time

$$f \lesssim f_{ISCO} = 6^{3/2} \frac{c^3}{GM}$$



** P. C. Peters, Phys. Rev., 136, B1224 (1964)*

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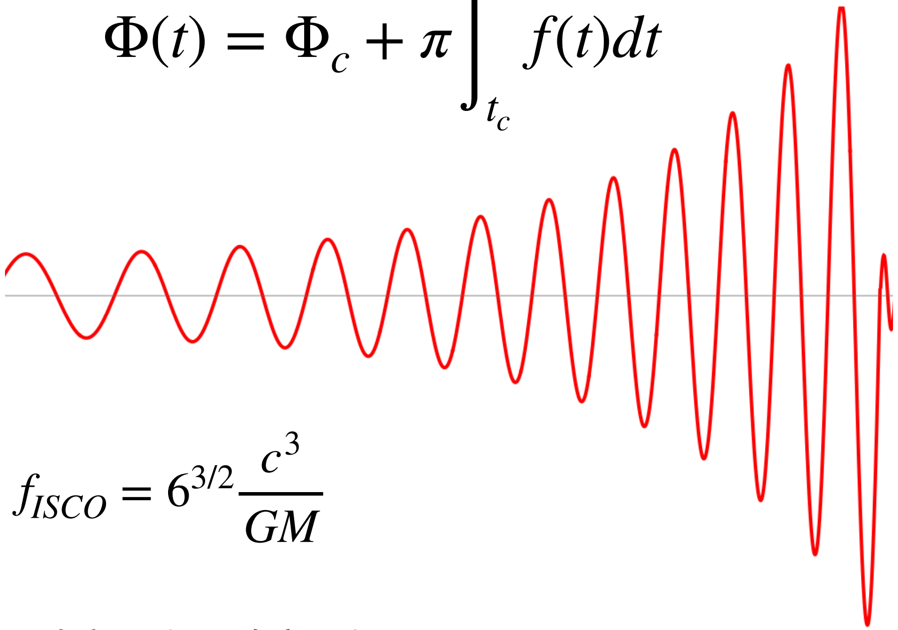

- **Frequency domain:**

$$\tilde{h}(f) = \int \mathscr{A}(t) e^{i(2\Phi(t) - 2\pi f t)} dt \;\equiv \tilde{\mathscr{A}}(f)\, e^{i\Psi(f)}$$

$$\Psi(f) = \Psi_M(f) + \Psi_A(f)$$

$$\Psi_M(f) = 2\Phi_c - 2\pi f t_c + \text{GR}$$

$$f_{\rm stp} = \frac{8 m_A}{5\pi}$$

$$\Psi_A(f) \propto \rho_A \mathscr{M}^{-5/2} m_A{}^{-9/2} \left(1 - \frac{f}{f_{\rm stp}} \right) \theta(f_{\rm stp} - f)$$

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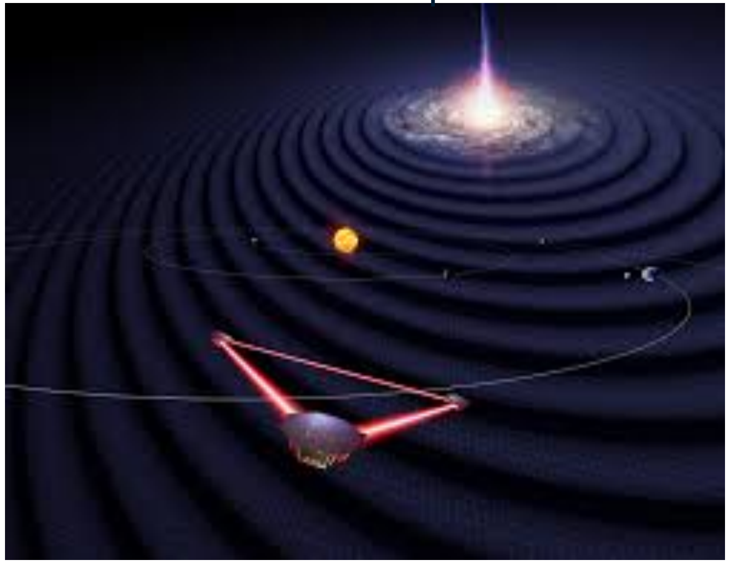
LISA's sensitivity

Laser **I**nterferometer **S**pace **A**ntenna



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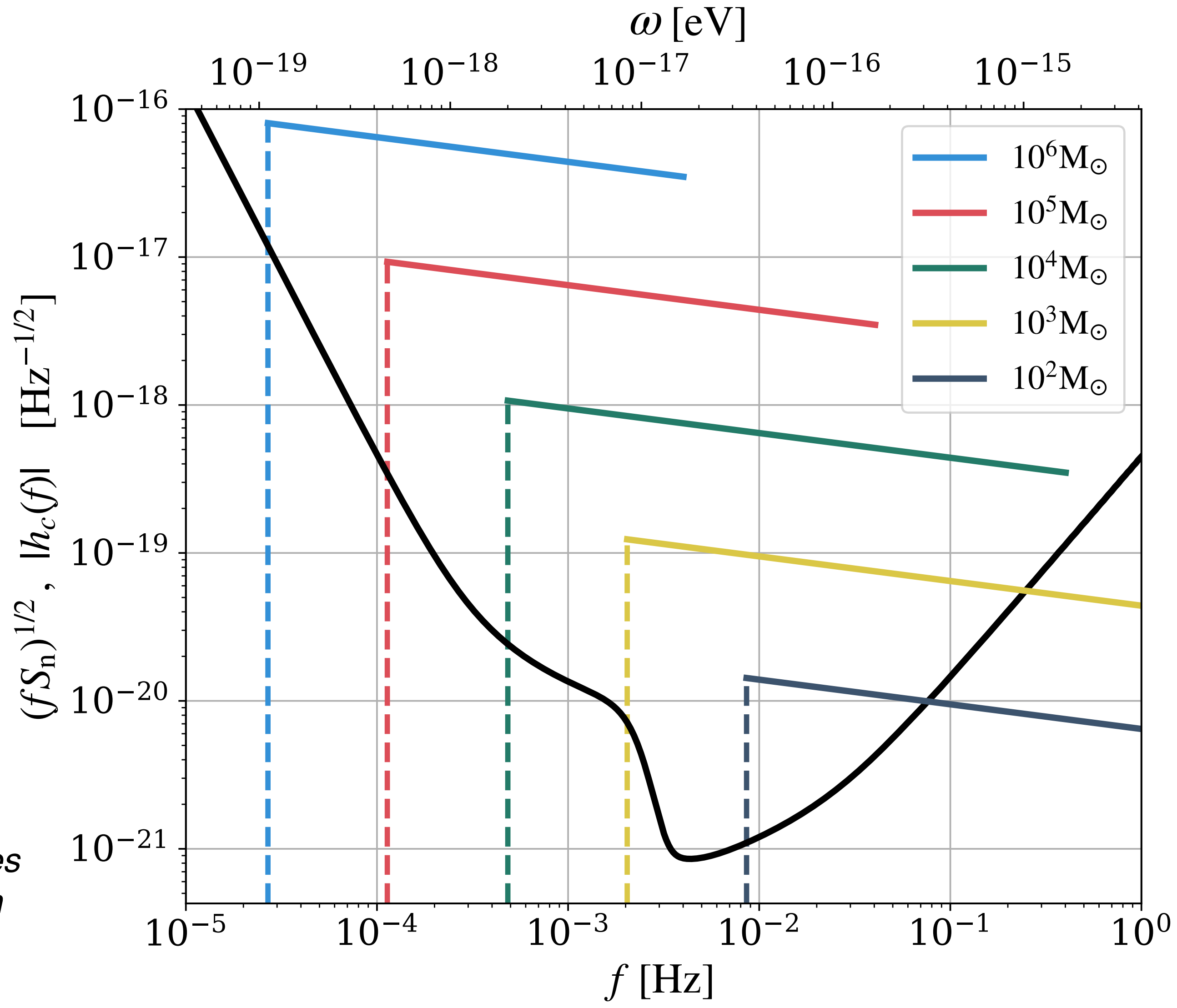
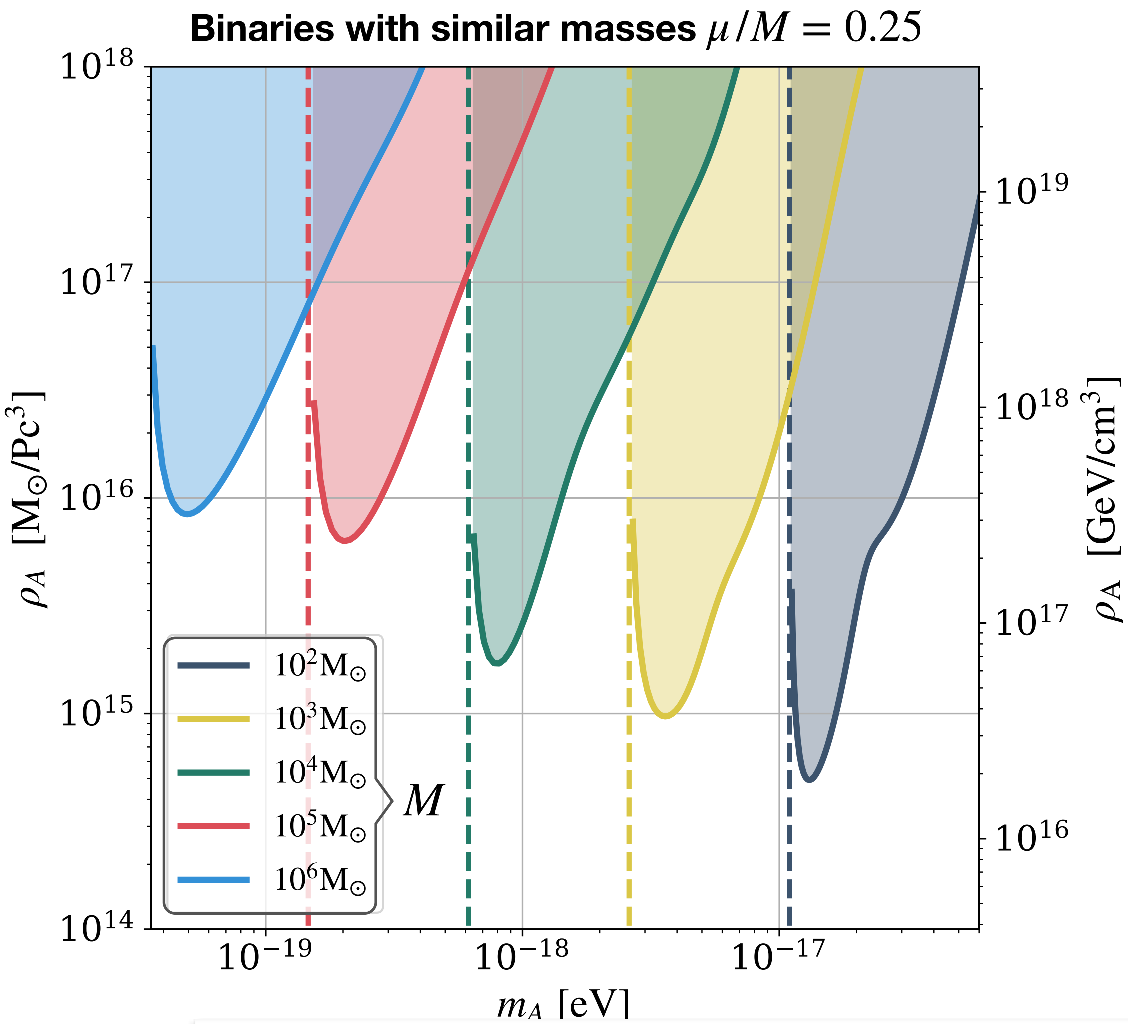
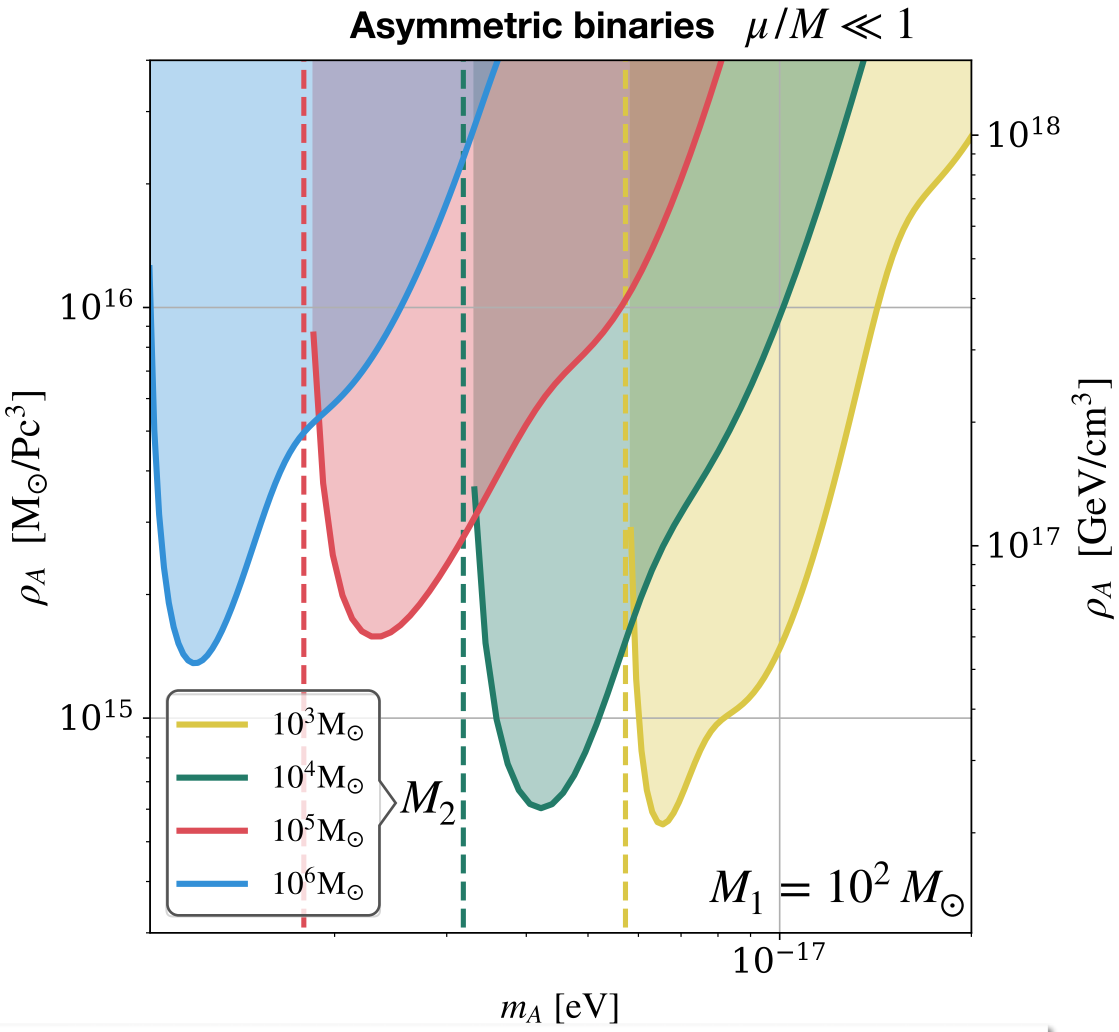


BLACK: LISA's sensitivity strain $(fS_n(f))^{1/2}$

COLOR: GW characteristic strain $|h_c(f)| = 2f|\mathscr{A}(f)|$ for each binary. We assumed symmetric-mass inspiraling binaries at D = 1Gpc during 4 years before ISCO. The masses of each binary member is indicated in the label.

Fisher forecast (4 years of LISA): 1σ accuracy





If a binary system is immersed in a VFDM environment with ρ_A in the shaded regions, then the GW emitted can carry an imprint of the VFDM that could be detectable with LISA for a 4-year observation period.

Conclusions and Perspectives

Observational constrains are sensitive to uncertainties in the modeling of small scale structure $\rightarrow$ crucial to have complementary independent probes

Phenomenological descriptions on small scales: necessary and under development

On Pulsar timing measurements: useful to probe ULDM models!

- Limits from Binary pulsars (BPs) beyond resonance are being studied for different coupling and for spin 0, spin 1 and spin 2.

Main references: - P. Kus, DLN and F. Urban, Astronomy&Astrophysics 690, A51 (2024)
- P. Kus, DLN and F. Urban, arXiv: 2506.04100

On Binary Black Hole Emission:

- VFDM with $m_A \sim (10^{-19}, 10^{-15})\,\mathrm{eV}$ could be probed by LISA if $\rho_A > 10^{14} M_\odot/pc^3$
- Future work to extend this to eccentric orbits and other ULDM spins

Main reference: T. Ferreira Chase, DLN and N. Yunes, arXiv: 2505.21383

THANKS !

Extra Slides

ULDM as a classical field with spin 0,1,2

`Small-scale' properties

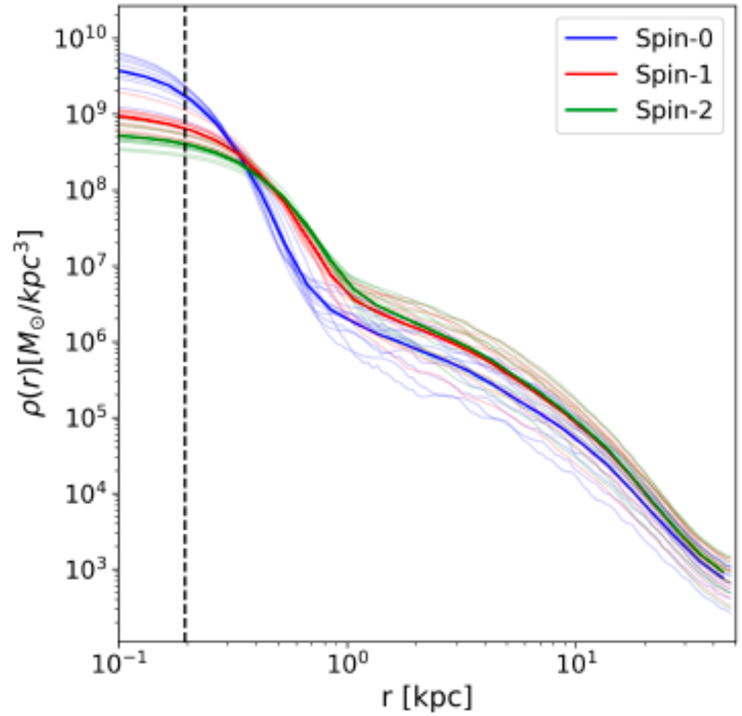
- **Halos simulations** $m \sim 10^{-22}\text{eV}$

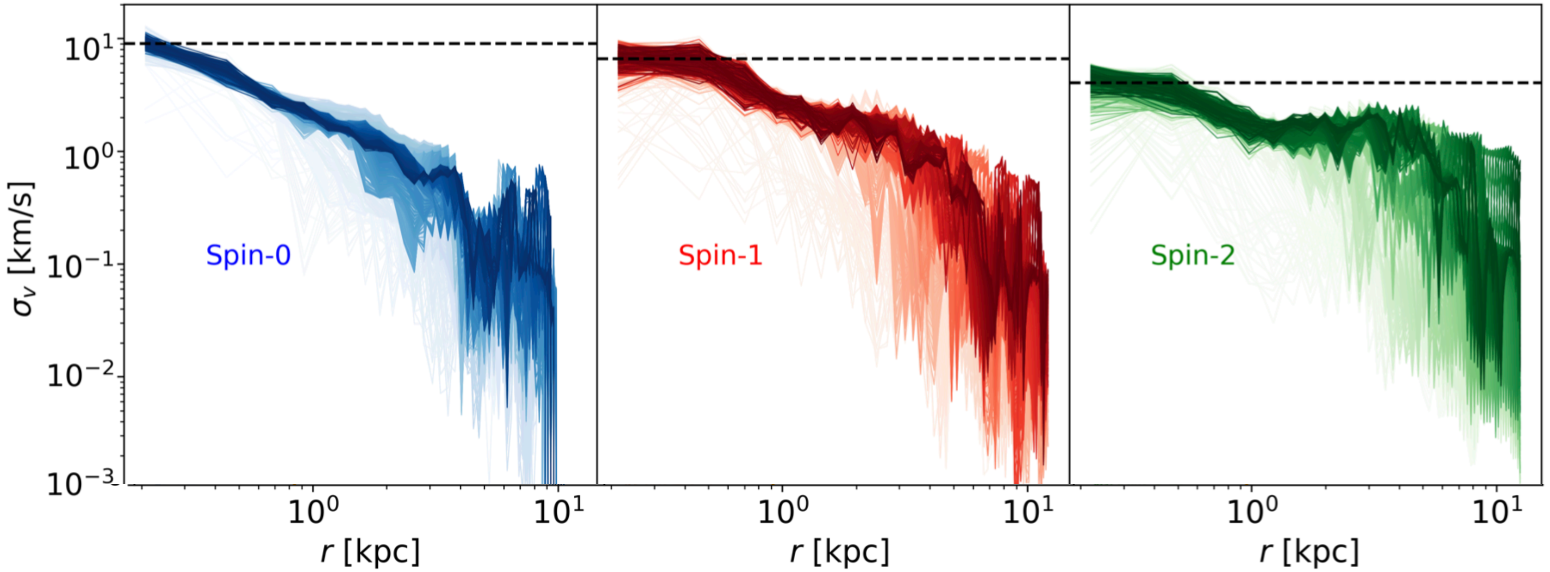
$$\rho_{\rm halo}(r) = \begin{cases} \rho_{\rm sol}(r) & r < r_\epsilon \\ \rho_{\rm NFW}(r) & r > r_\epsilon \end{cases}$$

Spin 0:

$$\rho_{\rm sol}(r) = \frac{\rho_c}{[1+\alpha(r/r_c)^2]^8} \qquad \rho_{\rm NFW}(r) = \frac{\rho_s}{(r/r_s)\,(1+r/r_s)^2}$$

$\alpha = 0.091$





J. López-Sánchez, et al arXiv:2502.03561

See also Amaral et al JCAP 08 (2022) 014

Binary systems in ultralight dark matter halos: discussion and order of magnitudes

- Extrapolate a soliton profile*

$$\rho_{\rm sol} \sim \rho_{c,0}[1 + 0.091(r/r_c)]^{-8}$$

$$\rho_{c,0} \sim 2 \times 10^{12} \left(\frac{m_A}{10^{-18}\mathrm{eV}}\right)^2 \left(\frac{M_{\rm halo}}{10^{12} M_\odot}\right)^{4/3} \frac{\mathrm{M}_\odot}{\mathrm{pc}^3}$$

$$r_c = 10^{11} \left(\frac{10^{-18}\mathrm{eV}}{m_A}\right)\left(\frac{M_{\rm halo}}{10^{12}\mathrm{M}_\odot}\right)^{-1/3} \mathrm{km}$$

- Assume the halo hosts a galaxy with a supermassive black at its center, and the SMBH-halo mass relation**:

$$M \sim 10^7 \mathrm{M}_\odot \left(\frac{\mathrm{M}_{\rm halo}}{10^{12}\mathrm{M}_\odot}\right)^{1.6}$$

- For SMBH binaries:

E.g. $M \sim 10^6\, M_\odot$, $M_{\rm halo} \sim 10^{11}\mathrm{M}_\odot$, for $m_A \sim 10^{-18}\mathrm{eV}$, $\rho_{c,0} \sim 10^{11}\mathrm{M}_\odot/\mathrm{pc}^3$, $r_c \sim 2.5 \times 10^{11}\mathrm{km} \gg a \sim 3.2 \times 10^8$ km

E.g. **Milky Way,** $M_{\rm Sag\ A^\star} \sim 4 \times\ 10^6\, \mathrm{M}_\odot$, $\mathrm{M}_{\rm halo} \sim 7 \times 10^{11}\mathrm{M}_\odot$

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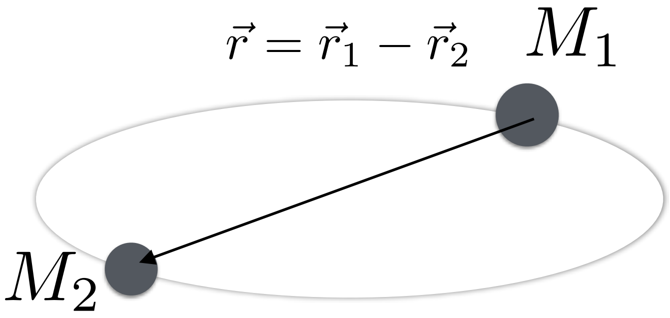
- For low/intermediate mass BH binaries: the orbit might be embedded in the core or in an interference granule of size λ_{dB}

*See Schive et al. Nature Phys. (2014)

**See Booth and Schaye, MNRA (2010)

ULDM perturbations to BH binaries



Metric perturbations: $g_{\mu\nu} \simeq \bar{g}_{\mu\nu} + h^M_{\mu\nu} + h^A_{\mu\nu}$

Sourced by BH binary **Sourced by VFDM**

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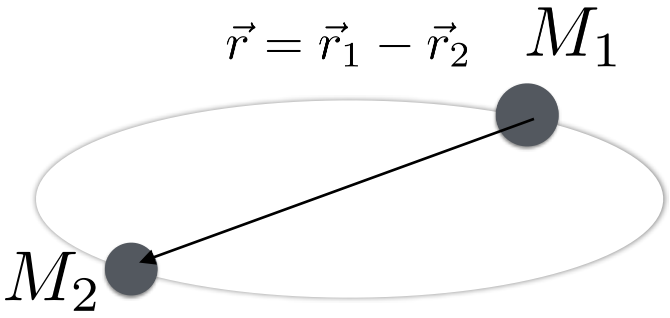

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$$\bar{A}_0 = 0, \;\; \ddot{\bar{A}}_i + m_A^2 \bar{A}_i = 0\,, \qquad \bar{A}^i(t) \sim \tilde{A}\, \cos(m_A t + \gamma)\, \hat{A}^i$$

$$T_A{}^0{}_0 = -\frac{m_A^2 \tilde{A}^2}{2} \equiv -\rho_A$$

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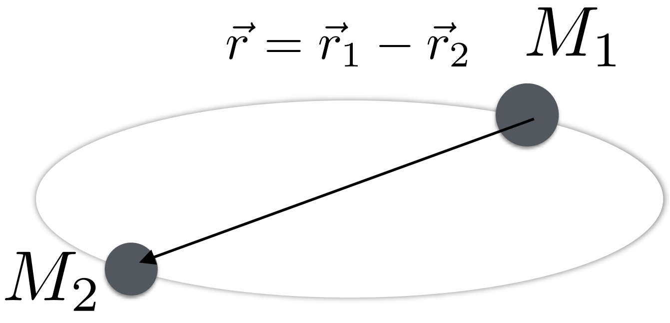

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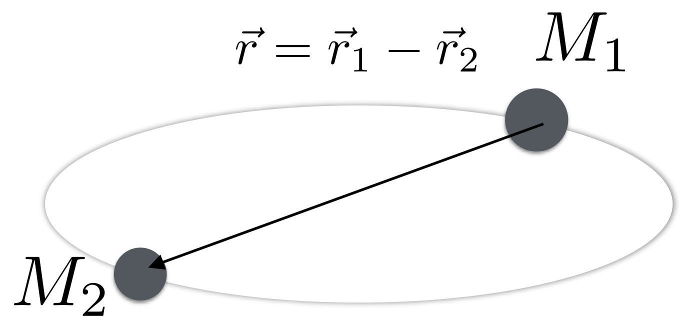

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$$\frac{d^2 r^i}{dt^2} = \left(R^i{}_{0j0}[h^A_{\mu\nu}] + R^i{}_{0j0}[h^M_{\mu\nu}]\right) r^j$$

$$R^i{}_{0j0}[h^A_{\mu\nu}] r^j := F^i \qquad R^i{}_{0j0}[h^M_{\mu\nu}] r^j = -\frac{GM}{r^2} r^i$$

$$F_i = -\partial_i V_A$$

$$V_A = 2\pi\rho_A \cos(2m_A t + \gamma)\left[\,|\vec{r}\,|^2 - 4(\vec{r}\cdot\hat{A})^2\right]$$

Perturbed Kepler problem

$$\frac{d^2 r^i}{dt^2} = -\frac{GM}{r^2} r^i + F^i$$

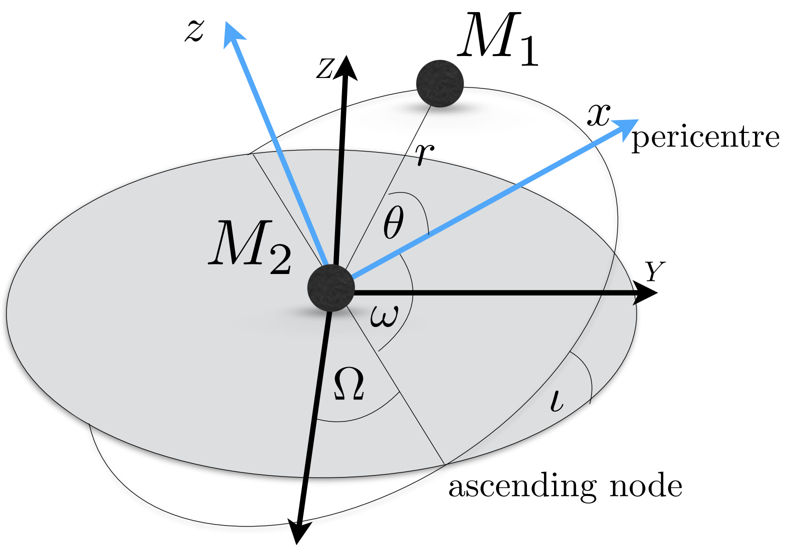
$$\vec{F} = F_r \hat{r} + F_\theta \hat{\theta} + F_z \hat{z}$$

Osculating Keplerian orbits

Six orbital elements: $\{a, e, \Omega, \iota, \varpi = \omega + \Omega, T\}$

Exact Keplerian solution

$$r(t) = \frac{a(1-e^2)}{1+e\cos\theta(t)}, \quad \dot{\theta}(t) = \frac{\omega_0(1+e\cos\theta(t))^2}{(1-e^2)^{3/2}}.$$



$$\hat{r}(t) = (c_\Omega c_{\omega+\theta(t)} - c_\iota s_\Omega s_{\omega+\theta(t)})\hat{X} + s_\Omega c_{\omega+\theta(t)} - c_\iota c_\Omega s_{\omega+\theta(t)}\hat{Y} + s_\iota s_{\omega+\theta(t)}\hat{Z}.$$

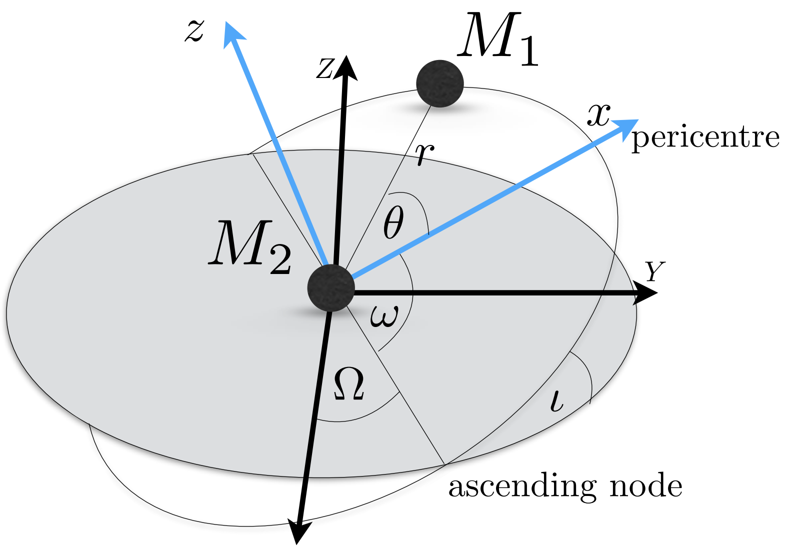
$$\hat{\theta} = \frac{\partial \hat{r}}{\partial \theta}, \qquad \hat{z} = \hat{r} \times \hat{\theta}.$$

Osculating Keplerian orbits

Six orbital elements: $\{a, e, \Omega, \iota, \varpi = \omega + \Omega, T\}$

Exact Keplerian solution

$$r(t) = \frac{a(1-e^2)}{1+e\cos\theta(t)}, \quad \dot{\theta}(t) = \frac{\omega_0(1+e\cos\theta(t))^2}{(1-e^2)^{3/2}}.$$



$$\hat{r}(t) = (c_\Omega c_{\omega+\theta(t)} - c_\iota s_\Omega s_{\omega+\theta(t)})\hat{X} + s_\Omega c_{\omega+\theta(t)} - c_\iota c_\Omega s_{\omega+\theta(t)}\hat{Y} + s_\iota s_{\omega+\theta(t)}\hat{Z}.$$

$$\hat{\theta} = \frac{\partial\hat{r}}{\partial\theta}, \qquad \hat{z} = \hat{r}\times\hat{\theta}.$$

- **Perturbed Kepler problem**

$$M = M_1 + M_2$$

$$\ddot{\vec{r}} = -\frac{GM\vec{r}}{r^3} + \vec{F},$$

$$\vec{F} = F_r\hat{r} + F_\theta\hat{\theta} + F_z\hat{z}$$

$$\omega_0 = \sqrt{\frac{GM}{a^3}} = \frac{2\pi}{P_b}$$

Lagrange planetary equations

$$\frac{da}{dt} = \frac{2}{\omega_0}\left\{\frac{F_\theta}{r}a\sqrt{1-e^2} + \frac{F_r e}{\sqrt{1-e^2}}\sin\theta\right\}$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{a\omega_0}\{F_\theta(\cos\theta + \cos E) + F_r\sin\theta\}$$

$$\omega_0(t-T) = E - e\sin E$$

$$\frac{d\Omega}{dt} = \frac{r}{a}\frac{F_z\sin(\theta+\omega)}{a\omega_0\sqrt{1-e^2}\sin\iota}$$

$$\frac{d\iota}{dt} = \frac{r}{a}\frac{F_z\cos(\theta+\omega)}{a\omega_0\sqrt{1-e^2}}$$

$$\frac{d\varpi}{dt} = \frac{\sqrt{1-e^2}}{ae\omega_0}\left\{F_\theta\sin\theta\left[1+\frac{r}{a(1-e^2)}\right] - F_r\cos\theta\right\} + 2\sin^2\left(\frac{\iota}{2}\right)\frac{d\Omega}{dt}$$

$$\frac{d\epsilon_1}{dt} = -\frac{2}{\omega_0}\frac{r}{a}\frac{F_r}{a} + \left[1-\sqrt{1-e^2}\right]\frac{d\varpi}{dt} + 2\sqrt{1-e^2}\sin^2\left(\frac{\iota}{2}\right)\frac{d\Omega}{dt}$$

$$\epsilon_1 = \omega_0(t-T) + \omega - \int\omega_0 dt$$

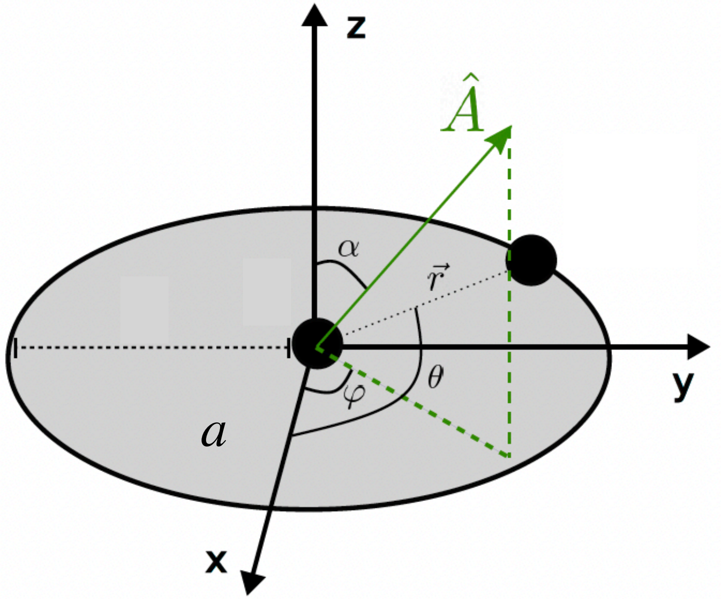
Perturbations to the orbital motion

- **VFDM perturbation:**

$$\hat{A} = \sin(\alpha)\cos(\varphi)\,\hat{x} + \sin(\alpha)\sin(\varphi)\,\hat{y} + \cos(\alpha)\,\hat{z}$$

$$\vec{r} = b\left[\cos(\theta)\,\hat{x} + \sin(\theta)\,\hat{y}\right] + z\hat{z}$$

$$\dot{a} = \frac{2}{\omega_b} f_\theta \;\rightarrow\; \left(\frac{\dot{a}}{a}\right)_A = \frac{16\pi G \rho_A}{c^2 \omega_b}\cos(2m_A t + \gamma)\sin(2\varphi - 2\omega_b t)\sin(\alpha)^2$$



- **Gravitational-wave backreaction:**

$$\left(\frac{\dot{a}}{a}\right)_M = -\frac{64}{5}\frac{G^3\eta}{c^5 a}\left(\frac{M}{a}\right)^3, \quad \eta = \frac{\mu}{M} = \textit{the symmetric mass ratio}$$

$$\frac{(\dot{a}/a)_A}{(\dot{a}/a)_M} \sim \frac{c^3 a^4}{G^2 \eta M^3 \omega_b}\rho_A \sim 1.5 \times 10^{-16}\frac{\rho_A \eta^{-1}}{0.3\,\frac{\text{GeV}}{\text{cm}^3}}\left(\frac{10^6 \text{M}_\odot}{M}\right)^{5/3}\left(\frac{10^{-5}\text{Hz}}{f}\right)^{8/3}$$

The waveform for quasi-circular BH binaries

GW frequency

$$\Phi(t) = \Phi_c + \pi \int_{t_c}^{t} f(t) dt$$

The waveform for quasi-circular BH binaries

- **Frequency domain:** $$\tilde{h}(f) = \int \mathscr{A}(t) e^{i(2\Phi(t) - 2\pi f t)} dt \ , \qquad \Phi(t) = \Phi_c + \pi \int_{t_c}^{t} f(t) dt$$

GW frequency

The waveform for quasi-circular BH binaries

- **Frequency domain:** $$\tilde{h}(f) = \int \mathcal{A}(t) e^{i(2\Phi(t) - 2\pi f t)} dt \ ,$$ $$\Phi(t) = \Phi_c + \pi \int_{t_c}^{t} f(t) dt$$ (GW frequency)

- **Stationary-phase approximation:** Taylor expansion around a ``stationary point'' t_* where $\dot{\Phi}(t_*) = \pi f$

$$(2\Phi(t) - 2\pi f t) \sim 2\Phi(t_*) - 2\pi f t_* + \ddot{\Phi}\Big|_{t_*} (t - t_*)^2$$

$$\mathcal{A}(t) \sim \mathcal{A}(t_*)$$

$$\tilde{h}(f) = i\tilde{\mathcal{A}}(t_*) \sqrt{\frac{\pi}{\ddot{\Phi}_*}} e^{i(2\Phi_* - 2\pi f t_*)} \equiv \tilde{\mathcal{A}}(f)\, e^{i\Psi(f)}$$

The waveform for quasi-circular BH binaries

- **Frequency domain:** $$\tilde{h}(f) = \int \mathscr{A}(t) e^{i(2\Phi(t) - 2\pi f t)} dt \,,$$ $$\Phi(t) = \Phi_c + \pi \int_{t_c}^{t} f(t) dt$$ GW frequency

- **Stationary-phase approximation:** Taylor expansion around a ``stationary point'' t_* where $\dot{\Phi}(t_*) = \pi f$ $(F = f/2)$

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$$\mathscr{A}(t) \sim \mathscr{A}(t_*)$$

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$$\Phi(t_*) = \Phi_c - 2\pi \int_{f/2}^{\infty} \frac{F(F')}{\dot{F}(F')} dF' \,,$$

$$t_* = t_c - \int_{f/2}^{\infty} \frac{1}{\dot{F}(F')} dF',$$

The waveform for quasi-circular BH binaries

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$$\Psi(f) = 2\Phi_c - 2\pi f t_c + 2\pi \int_{\frac{f}{2}}^{\infty} (f - 2F) \frac{dt}{dF} dF$$

The waveform for quasi-circular BH binaries

- **Frequency domain:**

$$\tilde{h}(f) = \int \mathscr{A}(t) e^{i(2\Phi(t) - 2\pi f t)}\, dt \,,$$

$$\Phi(t) = \Phi_c + \pi \int_{t_c}^{t} f(t) dt$$

GW frequency

- **Stationary-phase approximation:** Taylor expansion around a ``stationary point'' t_* where $\dot{\Phi}(t_*) = \pi f$

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$$\Psi(f) = 2\Phi_c - 2\pi f t_c + 2\pi \int_{\frac{f}{2}}^{\infty} (f - 2F) \frac{dt}{dF}\, dF$$

At leading post-Newtonian order

$$\frac{dF}{dt} \rightarrow \frac{\dot{f}}{2} = \frac{3}{5\pi} 2^{1/3} \left(\frac{G\mathscr{M}}{c^3} \right)^{5/3} (4\pi F)^{11/3}$$

Waveform for quasi-circular BBH in an ultralight VFDM environment

$$\Psi(f) = 2\Phi_c - 2\pi f t_c + 2\pi \int_{\frac{f}{2}}^{\infty} (f - 2F) \frac{dt}{dF} \, dF = \Psi_M(f) + \Psi_A(f)$$

$$\tilde{h}(f) = \tilde{h}_M(f) e^{i\Psi_A(f)}$$

GR+other effects

$$\frac{dF}{dt} = \dot{F}_M + \dot{F}_A, \quad |\dot{F}_M| \gg |\dot{F}_A|$$

$$\dot{F}_M = -\frac{3}{2} F \left(\frac{\dot{a}}{a}\right)_M + \cdots$$

$$\dot{F}_A = -\frac{3}{2} F \left(\frac{\dot{a}}{a}\right)_A$$

Waveform for quasi-circular BBH in an ultralight VFDM environment

$$\Psi(f) = 2\Phi_c - 2\pi f t_c + 2\pi \int_{\frac{f}{2}}^{\infty} (f - 2F)\frac{dt}{dF}\, dF = \Psi_M(f) + \Psi_A(f)$$

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$$\tilde{h}(f) = \tilde{h}_M(f) e^{i\Psi_A(f)}$$

GR+other effects

$$\dot{F}_M = -\frac{3}{2}F\left(\frac{\dot{a}}{a}\right)_M + \cdots \qquad \dot{F}_A = -\frac{3}{2}F\left(\frac{\dot{a}}{a}\right)_A$$

$$\Psi_A(f) = -2\pi \int^{\frac{f}{2}} (2F - f)\frac{\dot{F}_A}{\dot{F}_M^2} dF = \int_{\infty}^{x(f/2)} dx\, \mathscr{A}(x) e^{i\phi(x)},$$

$$x = (2\pi F \mathscr{M})^{\frac{1}{3}} \qquad \phi(x) = \frac{5}{128x^8}\left(x^3 - \mathscr{M} m_A\right) - 2\gamma$$

Waveform for quasi-circular BBH in an ultralight VFDM environment

$$\Psi(f) = 2\Phi_c - 2\pi f t_c + 2\pi \int_{\frac{f}{2}}^{\infty} (f - 2F)\frac{dt}{dF}\, dF = \Psi_M(f) + \Psi_A(f)$$

$$\frac{dF}{dt} = \dot{F}_M + \dot{F}_A, \quad |\dot{F}_M| \gg |\dot{F}_A|$$

$$\tilde{h}(f) = \tilde{h}_M(f) e^{i\Psi_A(f)}$$

GR+other effects

$$\dot{F}_M = -\frac{3}{2}F\left(\frac{\dot{a}}{a}\right)_M + \cdots \qquad \dot{F}_A = -\frac{3}{2}F\left(\frac{\dot{a}}{a}\right)_A$$

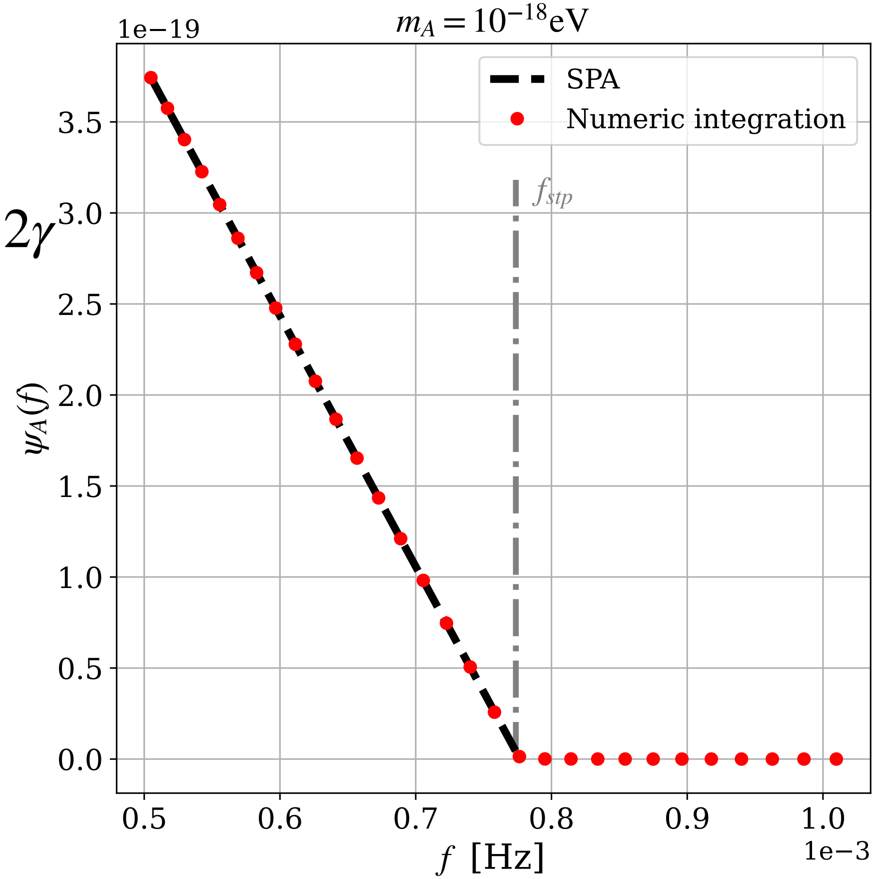
$$\Psi_A(f) = -2\pi \int^{\frac{f}{2}} (2F - f)\frac{\dot{F}_A}{\dot{F}_M^2} dF = \int_{\infty}^{x(f/2)} dx\, \mathscr{A}(x) e^{i\phi(x)},$$

$$x = (2\pi F \mathscr{M})^{\frac{1}{3}} \qquad \phi(x) = \frac{5}{128 x^8}\left(x^3 - \mathscr{M} m_A\right) - 2\gamma$$

- **Stationary-phase approximation:** $\phi'(x_{\rm stp}) = 0 \rightarrow \pi f_{\rm stp} = 8 m_A/5$

$$\Psi_A(f) = \beta\left(1 - \frac{f}{f_{\rm stp}}\right)\theta(f_{\rm stp} - f)$$

$$\beta \sim \sqrt{\frac{5}{6}}\frac{625\,\pi^{3/2} G \rho_A}{65536 \mathscr{M}^{5/2} m_A^{9/2}} \sin(\alpha)^2 \sim 3\times 10^{-12} \frac{\rho_A}{0.3\,\frac{\rm GeV}{\rm cm^3}} \left(\frac{10^4 M_\odot}{M}\right)^{5/2} \left(\frac{10^{-19}{\rm eV}}{m_A}\right)^{9/2}$$



Prospects for high-precision pulsar timing

[K. Liu et al MNRAS (2011)]

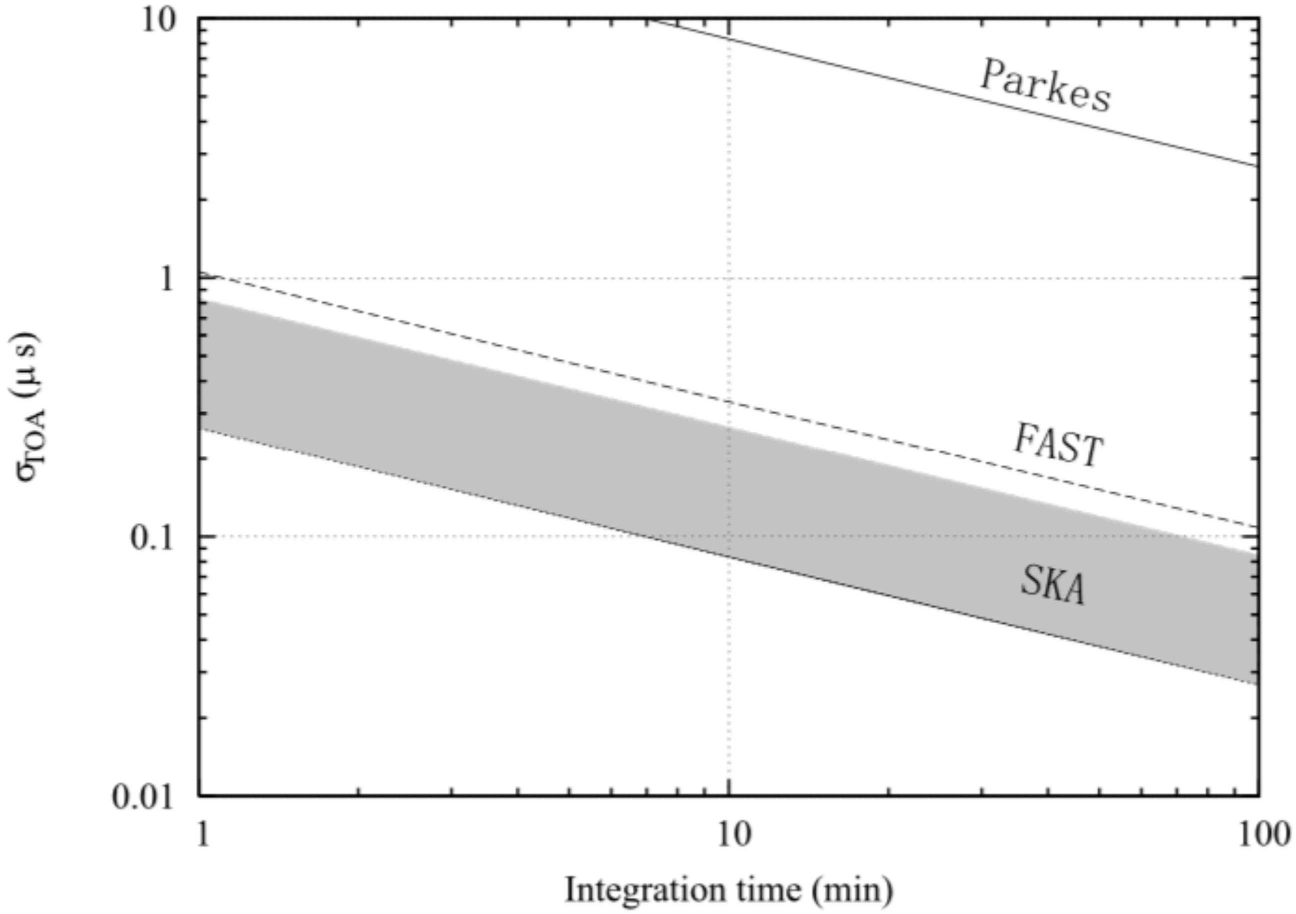
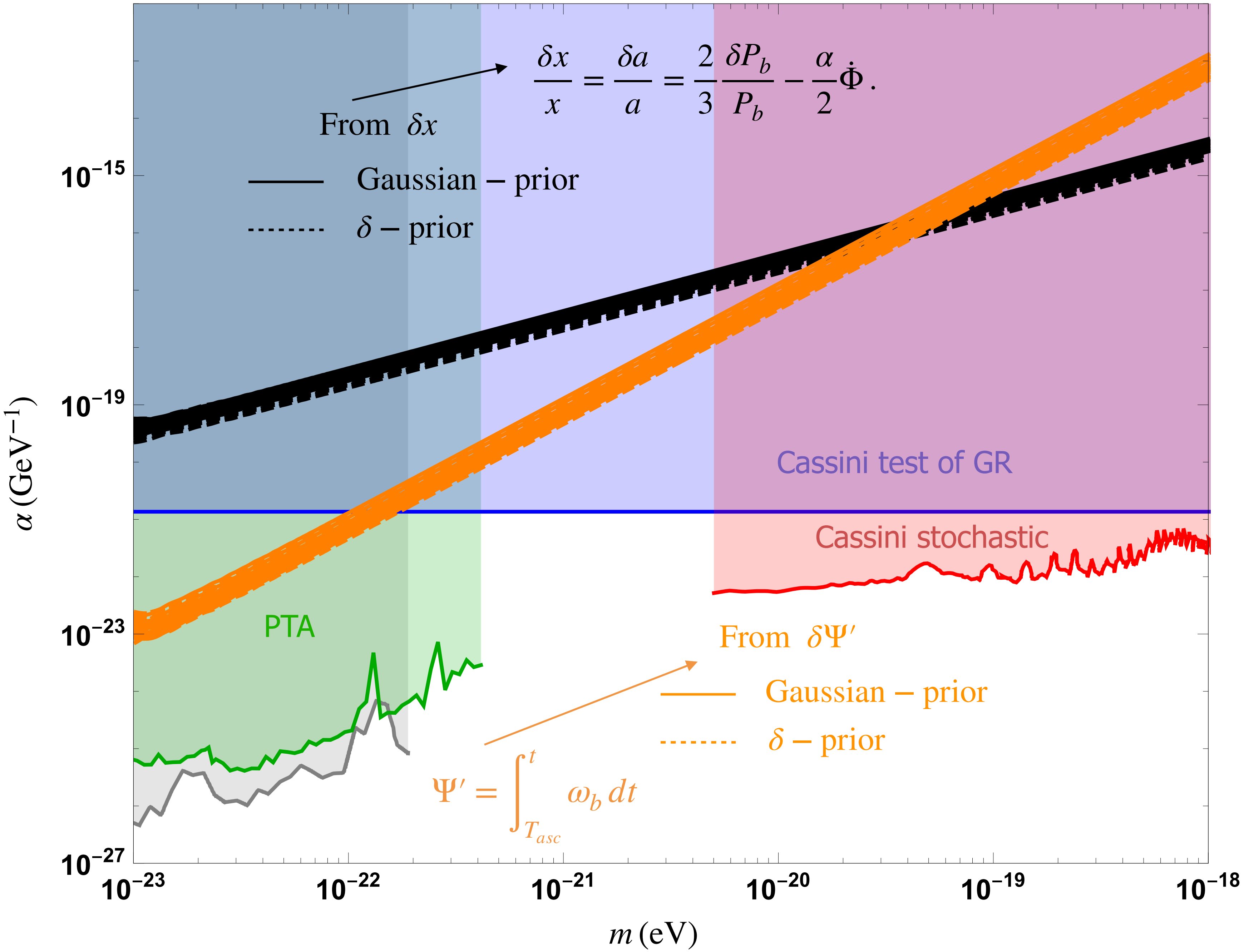


Figure 12. Expected average-brightness MSP TOA scatter versus integration time for different instruments. Only radiometre noise and pulse jitter are accounted for as influences on the measurement accuracy.

Systems with small e within ELL1 model: NANOGrav 15-year



- **BP & observational parameters**

Taken from [NANOGrav 2023]

- 20 BPs ELL1 combined

- **Assumed DM parameters**

- $\rho_{\mathrm{DM}} \simeq 0.3\,\mathrm{GeV/cm^3}$

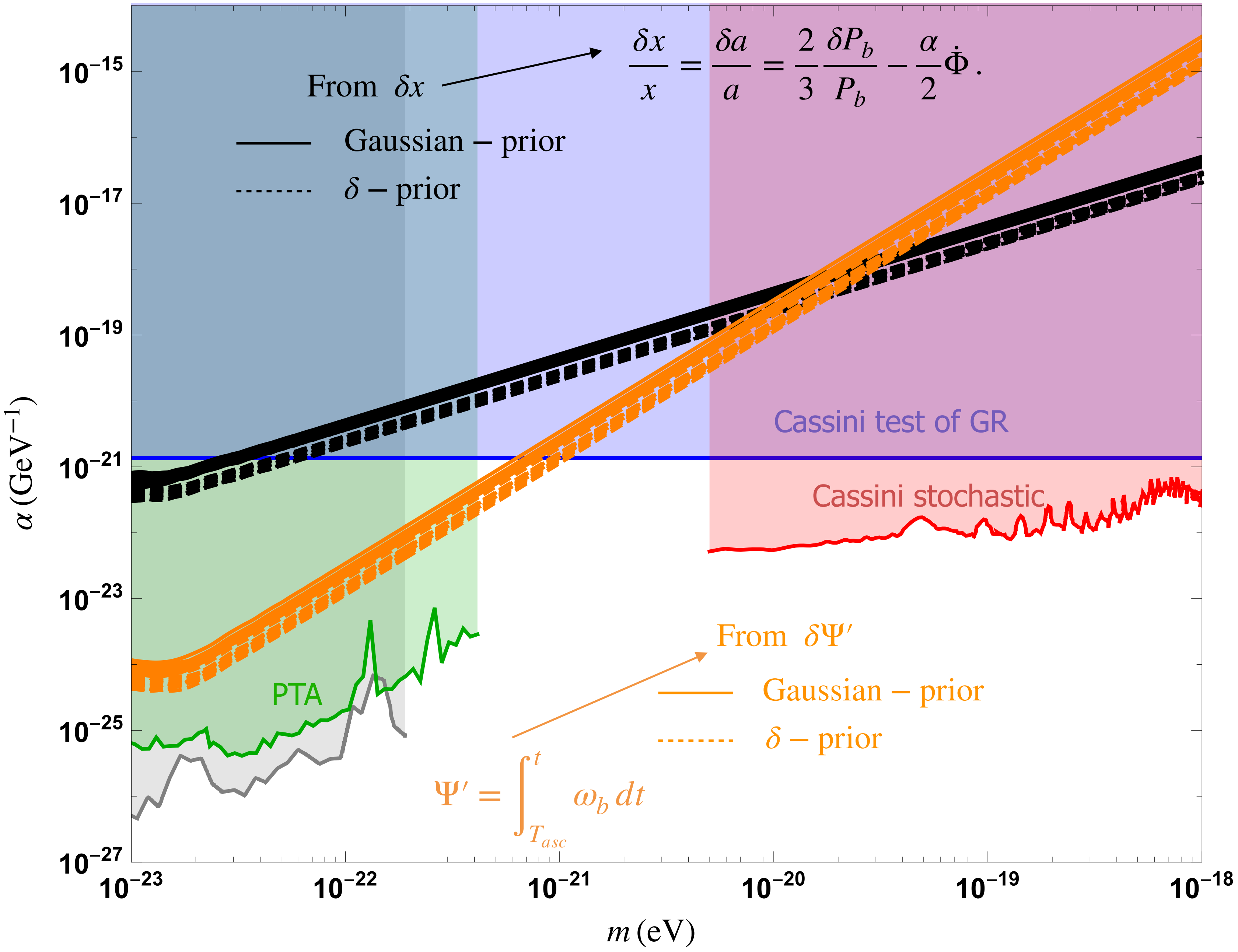
- r_f, Υ_f (20 random realization)

$X_f = \sqrt{2} r_f \cos\Upsilon_f$ & $Y = \sqrt{2} r_f \sin\Upsilon_f$

PTA: can be obtained from pure-gravity bounds by

$g_{\mu\nu} \to \bar{g}_{\mu\nu} = g_{\mu\nu}(1 + 2\alpha\Phi)$

Effect on systems with small e within ELL1 model: Forecasting



- Pulsars parameters are taken from the ATNF pulsar catalogue https://www.atnf.csiro.au/research/pulsar/psrcat/
- 111 BPs ELL1 combined
- 20 random realization of fiducial r & Υ. Recall: $X = \sqrt{2}r\cos\Upsilon$ & $Y = \sqrt{2}r\sin\Upsilon$
- $T_{obs} = 10\ y$
- $\epsilon = 0.1\,\mu s$
- $\dot{n} = 1/\mathrm{day}$
- $\rho_{\mathrm{DM}} = 0.3\,\mathrm{GeV/cm^3}$