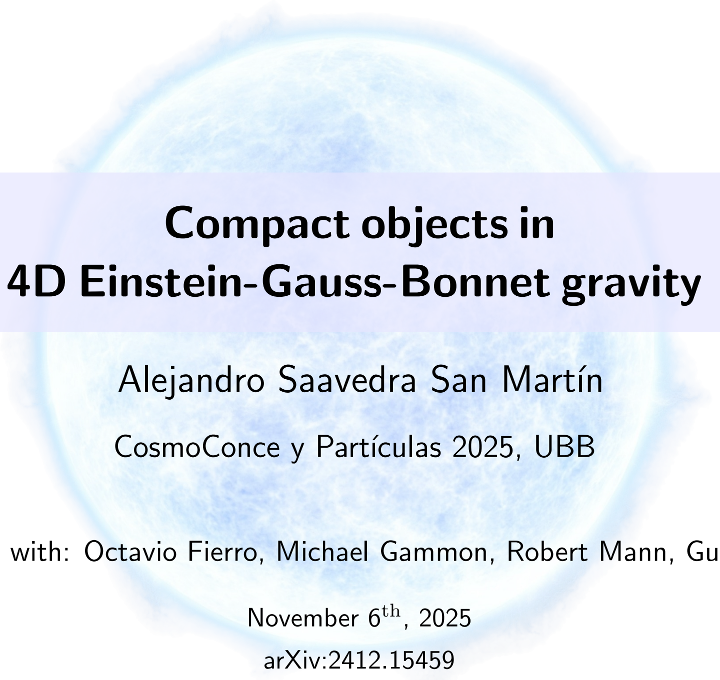

Compact objects in 4D Einstein-Gauss-Bonnet gravity

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CosmoConce y Partículas 2025, UBB

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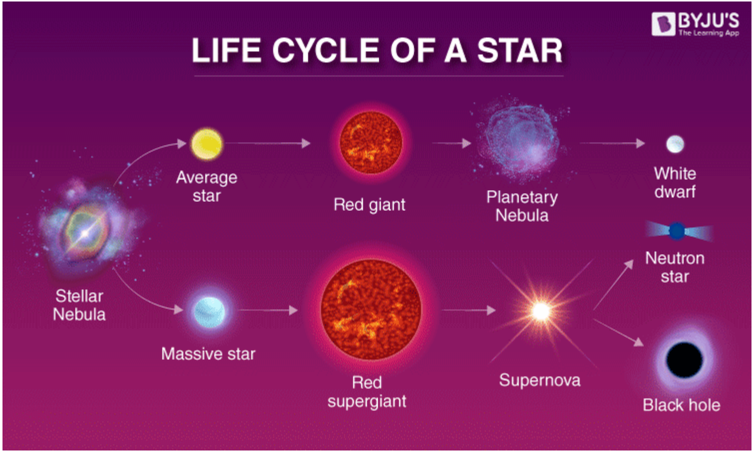
arXiv:2412.15459

Outline

Introduction: Compact Objects

Compact objects - white dwarfs, neutron stars, and black holes - are "born" when normal stars "die", that is, when most of their nuclear fuel has been consumed.



Introduction: Compact Objects

How do they prevent gravitational collapse?

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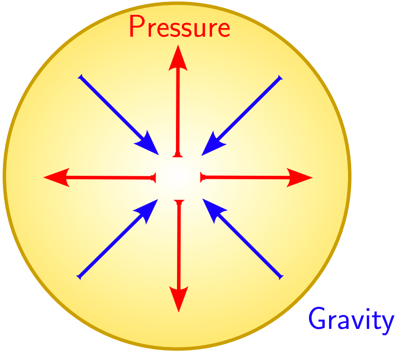
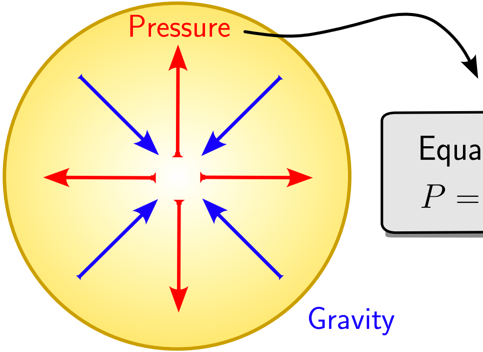


Introduction: Compact Objects

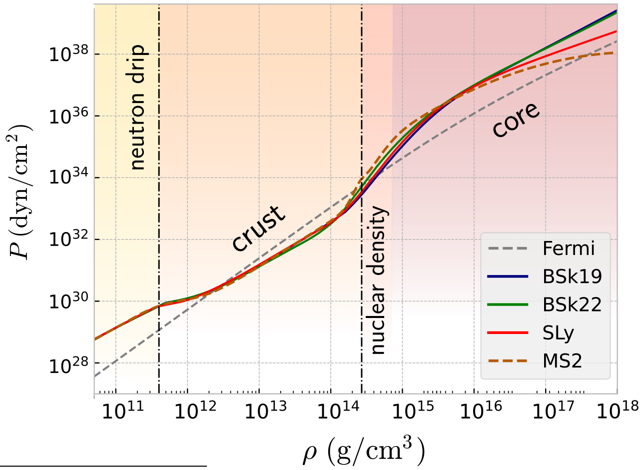
How do they prevent gravitational collapse?



Equation of State

$$P = P(\rho, T, \dots)$$

EOS's for a Neutron Star



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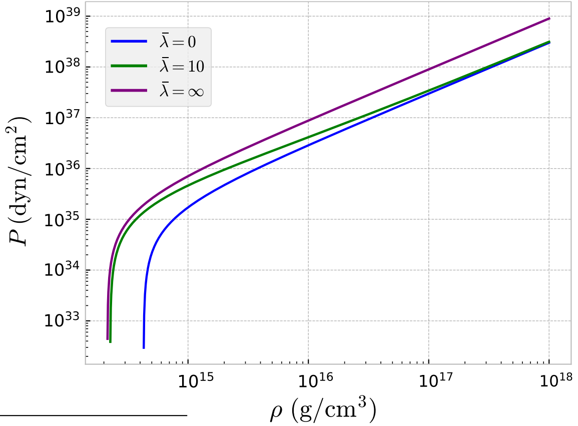
1 A. Y. Potekhin et al. Astronomy and Astrophysics **560** (2013).

2 F. Douchin and P. Haensel. Astronomy and Astrophysics **380** (2001).

3 Horst Müller and Brian D. Serot. Nuclear Physics A **606** (1996).

EOS for a Quark Star

$$P = \frac{1}{3}\left(\rho c^2 - 4B_{\text{eff}}\right) + \frac{4\lambda^2}{9\pi^2}\left(-1 + \text{sgn}(\lambda)\sqrt{1 + 3\pi^2\frac{(\rho c^2 - B_{\text{eff}})}{\lambda^2}}\right)$$



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4 Chen Zhang and Robert B. Mann. Phys. Rev. D, **103**:063018 (2021).

4D Einstein-Gauss-Bonnet

The action of 4DEGB gravity is given by [1,2]

Acción de 4DEGB:

$$
\begin{aligned}
S_{\mathrm{EGB}} = \frac{c^4}{16\pi G}\int d^4x\sqrt{-g}\,\Big[R - 2\Lambda + \alpha\big(\phi\mathcal{G} + 4G_{\mu\nu}\nabla^{\mu}\phi\nabla^{\nu}\phi - 4\nabla_{\mu}\phi\nabla^{\mu}\phi\Box\phi \\
+2(\nabla_{\mu}\phi\nabla^{\mu}\phi)^2\big)\Big] + S_{\mathrm{matter}}.
\end{aligned}
$$

Horndeski' theory

- Scalar field ϕ
  - Shift-symmetry
- Second order equations

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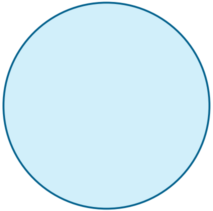
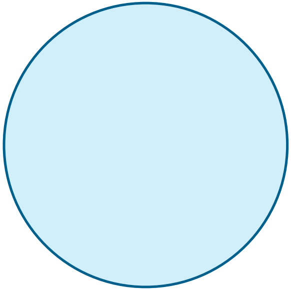
1 R. A. Hennigar, D. Kubizňák, R. B. Mann and C. Pollack. Journal of High Energy Physics **2020** (2020).

2 P. G. S. Fernandes, P. Carrilho, T. Clifton et al. Phys. Rev. D **102** (2020).

Star solution



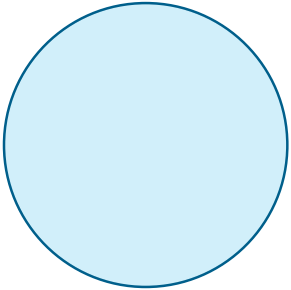
Star solution



$$\begin{aligned} ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\ &= -A(r)(cdt)^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2 \end{aligned}$$

$$T_{\mu\nu} = 0$$

Star solution

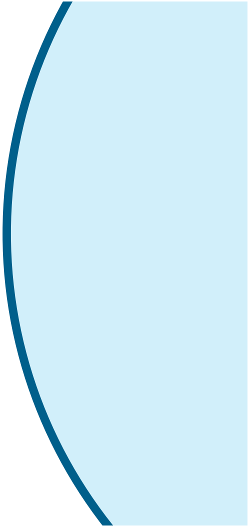


$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$
$$= -A(r)(cdt)^2 + \frac{dr^2}{A(r)} + r^2 d\Omega^2$$

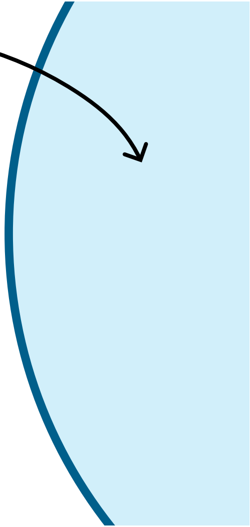
$$T_{\mu\nu} = 0$$

$$A(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha G M}{c^2 r^3}}\right)$$
$$\frac{d\phi}{dr} = \frac{\sqrt{A} - 1}{r\sqrt{A}}$$

Star solution



Star solution



$$ds^2 = -e^{\chi(r)} f(r)(cdt)^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$T_{\mu\nu} = \frac{1}{c^2}(\epsilon + P)u_\mu u_\nu + P g_{\mu\nu}$$

Star solution

IVP

$$\begin{cases} \dfrac{d\phi}{dr} = \dfrac{\sqrt{f}-1}{r\sqrt{f}} \\[2ex] \dfrac{df}{dr} = -\dfrac{(8\pi G/c^4)\epsilon r^4 + f^2\alpha + fr^2 - 2f\alpha - r^2 + \alpha}{r\left(r^2 - 2f\alpha + 2\alpha\right)} \\[2ex] \dfrac{d\chi}{dr} = \dfrac{8\pi G}{c^4}\dfrac{r^3\left(\epsilon + P\right)}{f\left(r^2 - 2f\alpha + 2\alpha\right)} \\[2ex] \dfrac{dP}{dr} = -\dfrac{\left(\epsilon + P\right)\left[-\alpha f^2 - \left(r^2 - 2\alpha\right)f + (8\pi G/c^4)r^4 P + r^2 - \alpha\right]}{2rf\left(r^2 - 2\alpha f + 2\alpha\right)} \\[2ex] P = \rho(r) \end{cases}$$

Initial conditions

$$f(0) = 1, \;\; \chi(0) = 0, \;\; P(0) = P_c$$

Star solution

How do we determine the mass of the star?

Star solution

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$$f_{\text{num}}(R) = 1 + \frac{R^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha GM}{c^2R^3}}\right)$$

Neutron star solutions

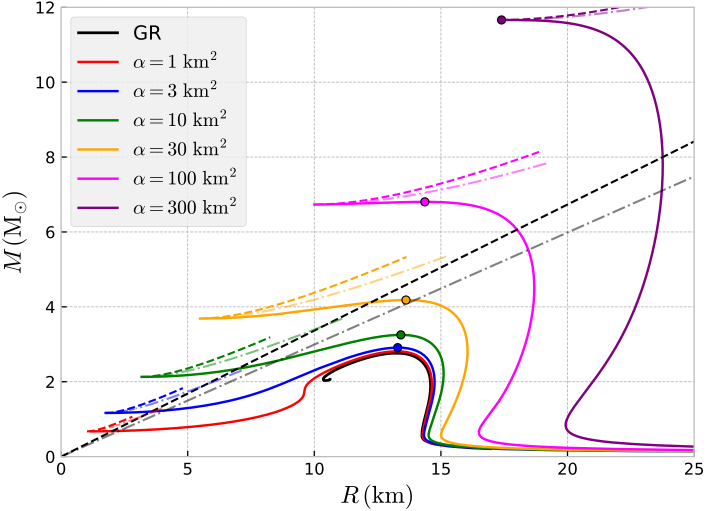


Figure 1: Mass versus radius for NS using the MS2 EOS in 4DEGB.

Quark star solutions

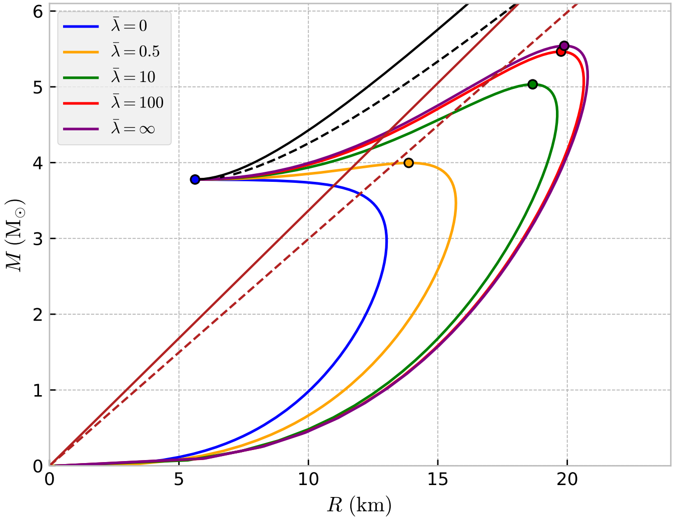


Figure 2: Mass versus radius for quark stars with positive λ in 4DEGB for $\bar{\alpha} = 0.01$.

Radial oscillations

Perturbed metric

$$ds^2 = -e^{\chi(t,r)} f(t,r)(cdt)^2 + \frac{dr^2}{f(t,r)} + r^2 d\Omega^2$$

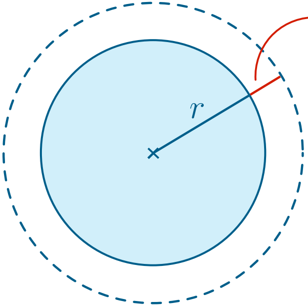
$$f(t,r) = \boxed{f_0(r)} + \boxed{\delta f(t.r)}$$

non-perturbed

perturbation

$$\delta\phi \sim \sum_n \varphi_n(r) e^{-i\omega_n t}$$

scalar field perturbation



$$\delta r \sim \sum_n u_n(r) e^{-i\omega_n t}$$

radial displacement

Radial oscillations

Eqs. radial oscillations:

$$\frac{d}{dr}\left(a_1\frac{du}{dr}\right)+(a_2+\omega^2 a_3)u+a_4\frac{d\varphi}{dr}=0$$

$$\frac{d}{dr}\left(b_1\frac{d\varphi}{dr}\right)+b_2\frac{du}{dr}+b_3u=0$$

Boundary conditions:

$$u\big|_{r=0}=0,\qquad a_1\frac{du}{dr}\bigg|_{r=0}\overset{!}{=}1,\qquad \frac{d\varphi}{dr}\bigg|_{r=0}=0,\qquad a_1\frac{du}{dr}\bigg|_{r=R}=0.$$

Radial oscillations: numerical solutions

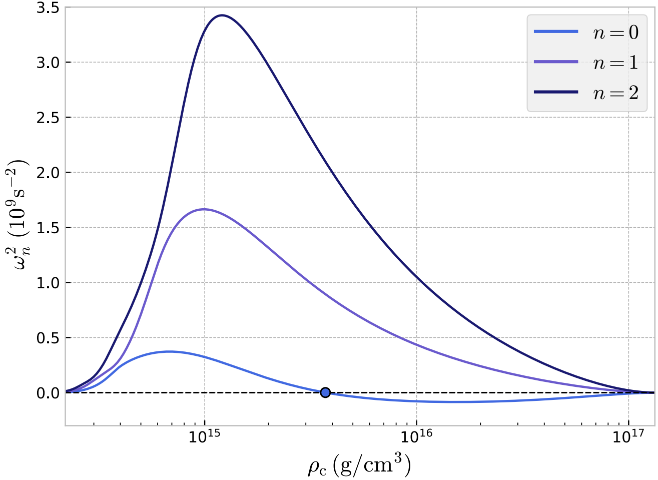
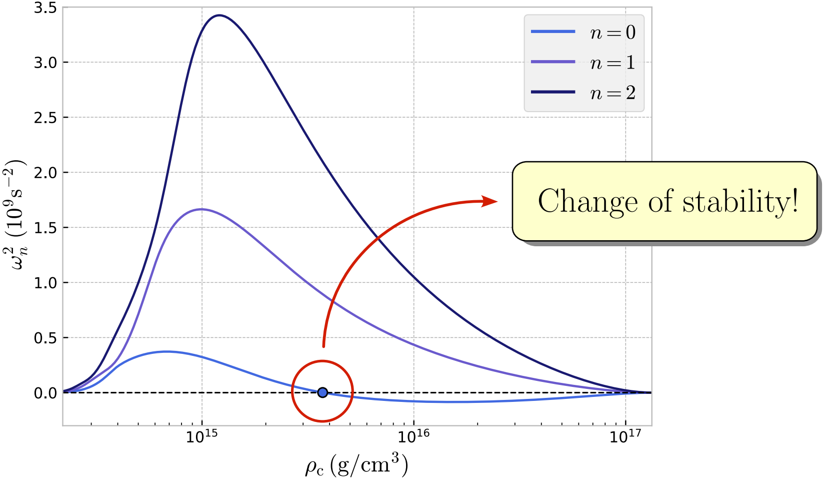


Figure 3: Eigenfrequencies of the first three oscillation modes for $\alpha = 10\ \mathrm{km}^2$ using the SLy EOS.

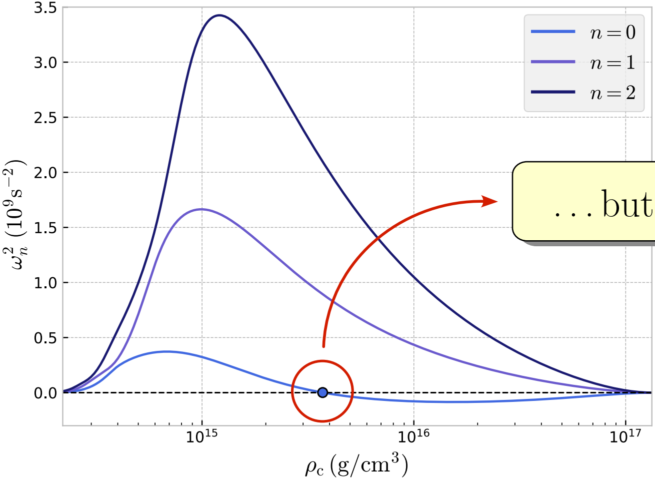
Radial oscillations: numerical solutions



Change of stability!

Figure 4: Eigenfrequencies of the first three oscillation modes for $\alpha = 10\ \mathrm{km}^2$ using the SLy EOS.

Radial oscillations: numerical solutions



...but where?

Figure 5: Eigenfrequencies of the first three oscillation modes for $\alpha = 10\ \mathrm{km}^2$ using the SLy EOS.

Radial oscillations: numerical solutions

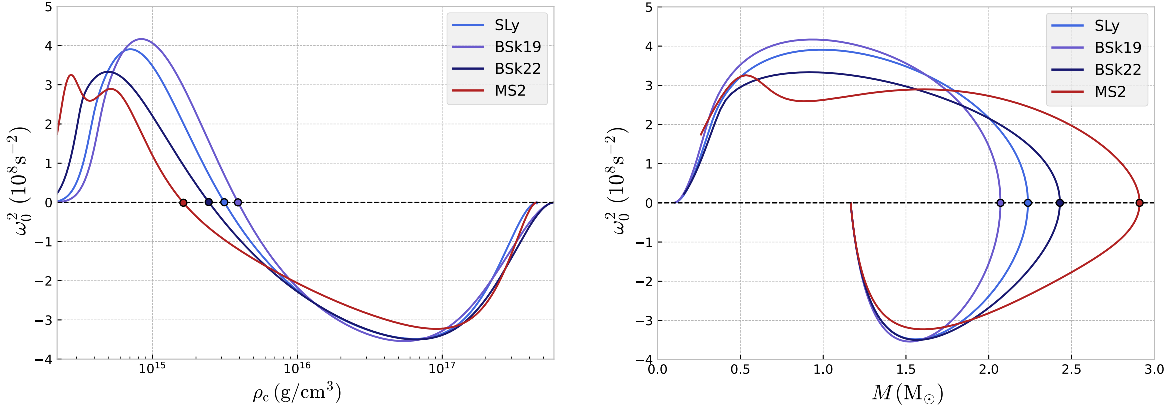


Figure 6: Fundamental eigenfrequency versus central density and mass for NS in 4DEGB for $\alpha = 3\ \mathrm{km}^2$.

Radial oscillations: numerical solutions

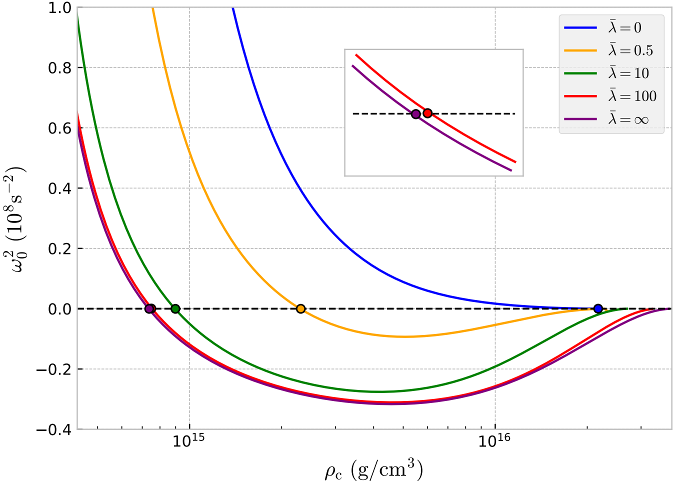


Figure 7: Fundamental eigenfrequency versus central density for QS with positive λ in 4DEGB for $\bar{\alpha} = 0.01$.

Radial oscillations: numerical solutions

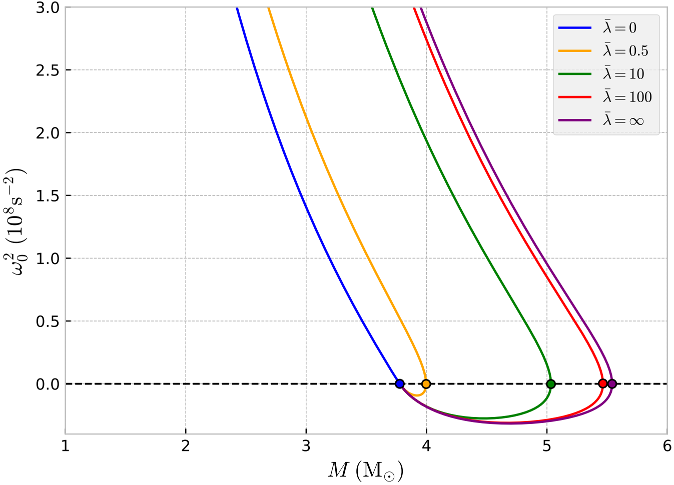


Figure 8: Fundamental eigenfrequency versus mass for QS with positive λ in 4DEGB for $\bar{\alpha} = 0.01$.

Conclusion

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  - ◇ **Improvements:** Use other equations of state, analyze gravitational perturbations (gravitational waves/tidal love numbers), etc.

Thank you for your attention :)