CosmoConce 2025

06.11.2025

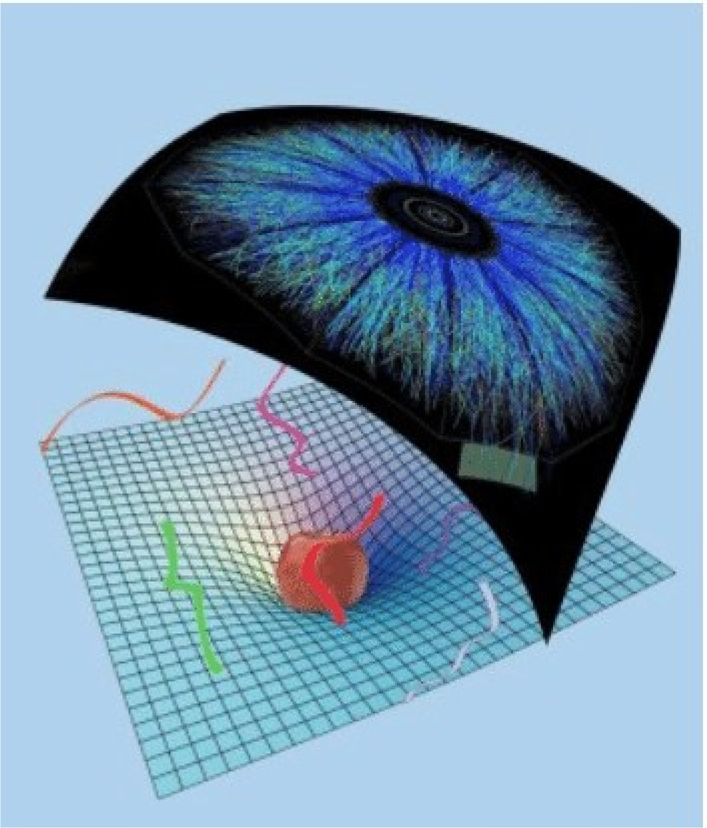
Marco S. Bianchi

Universidad San Sebastián, Santiago, Chile

Framing a string with a B-field

The main ingredients

This talk is about:



Wilson loops

Holography

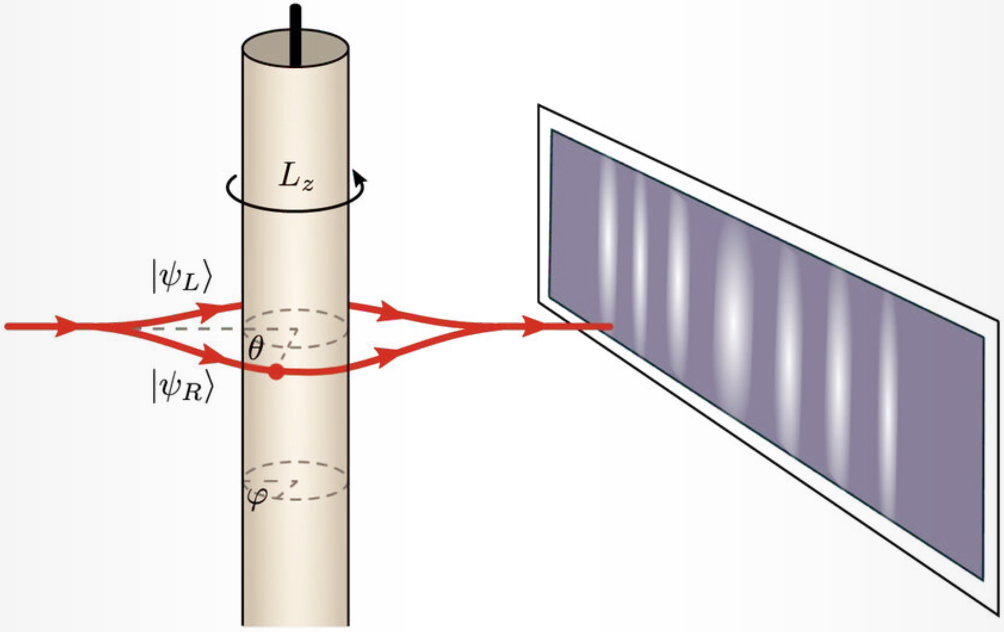
based on **2508.21068**
with Luigi Castiglioni, Silvia Penati, Marcia Tenser and Diego Trancanelli

What are Wilson loops?

Quantum mechanical: Aharonov-Bohm phase due to magnetic flux

$$W(\mathcal{C}) = \exp\left(iq\oint_{\mathcal{C}} A\cdot dx\right) = \exp\left(iq\int_{\Sigma:\,\partial\Sigma=\mathcal{C}} B\cdot dS\right) = e^{iq\,\Phi_B}$$

$$\mathcal{C} : x^{\mu} = x^{\mu}(\tau)$$



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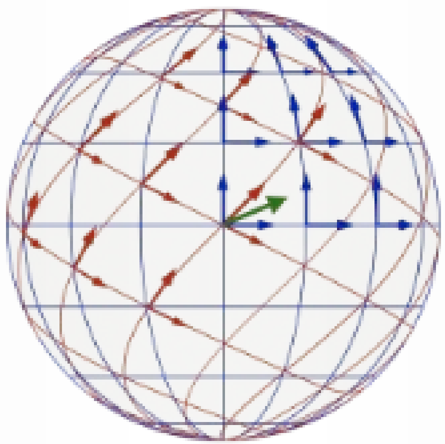
Gauge theoretic: non-local gauge invariant operators $\mathcal{C} : x^\mu = x^\mu(\tau)$

$$W[\mathcal{C}] = \mathrm{Tr}\,\mathcal{P}\,\exp\left(-i\,g\int_{\mathcal{C}} A_\mu dx^\mu\right)$$



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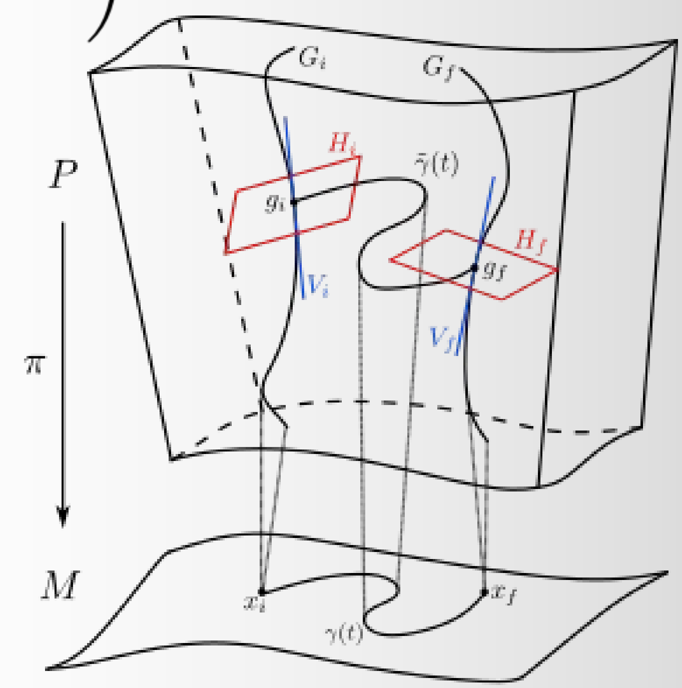
Geometric: *holonomy* of gauge field around closed curve → parallel transport



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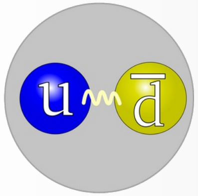
Effective: world-line of a *heavy charged particle*, e.g. massive quark



$$\langle W_{\square}(R\times T)\rangle \sim e^{-T\,V(R)} \quad (T\to\infty)$$

Diagnostic for confinement

What are Wilson loops?

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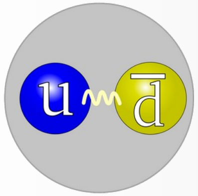

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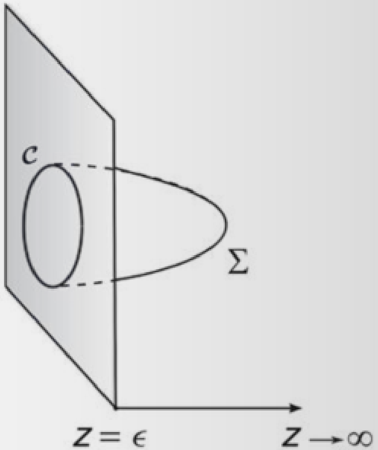
Holographic: *open strings* ending on a contour in *AdS*



What is holography?

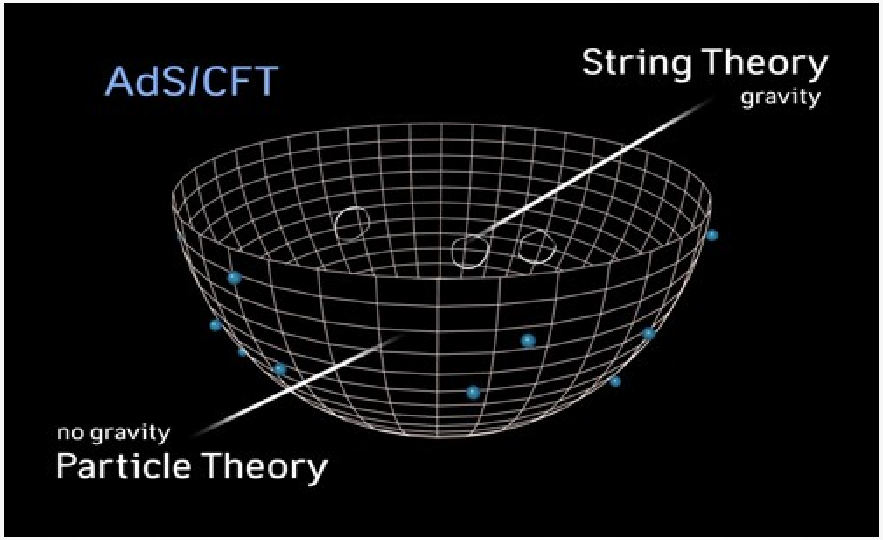
understand strongly coupled QFT

<u>conjecture</u>
Maldacena, 1997

understand quantum gravity

CFT_d — ***conformal* field theory in *d* dimensions** ⟺ ***dual*** ⟺ **gravity in *d+1* *anti-de Sitter* space-time** — **AdS_{d+1}**

$\lambda=0$ ⟶ $\lambda=\infty$



Precise AdS/CFT

understand strongly coupled QFT

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CFTd — ***super-conformal* field theory in *d* dimensions** ⟺ ***dual*** ⟺ **string theory in *d+1* *anti-de Sitter* space-time** — AdSd+1

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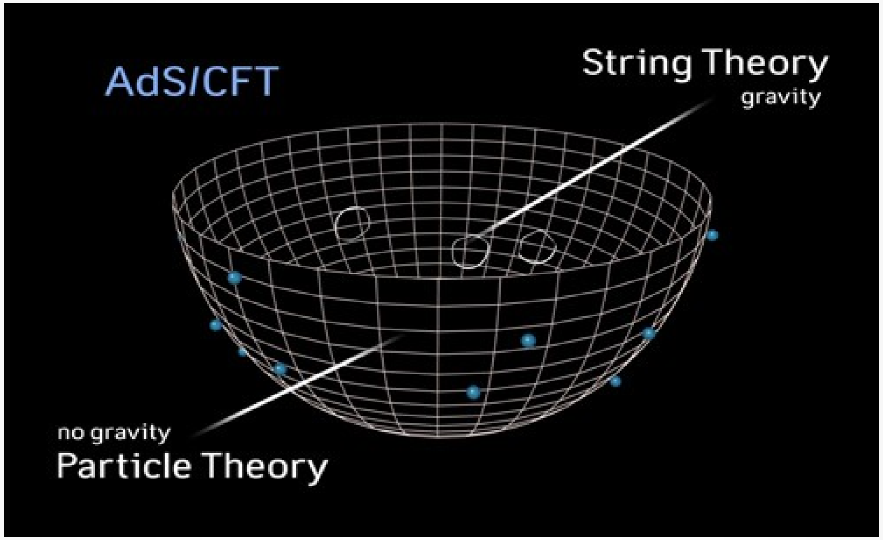

In this talk: AdS_4/CFT_3

understand strongly coupled QFT

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understand quantum gravity

CFT_3 — ***super-conformal* field theory in *3* dimensions** ⟺ ***dual*** ⟺ **string theory in 4 *anti-de Sitter* space-time** — **AdS_4**

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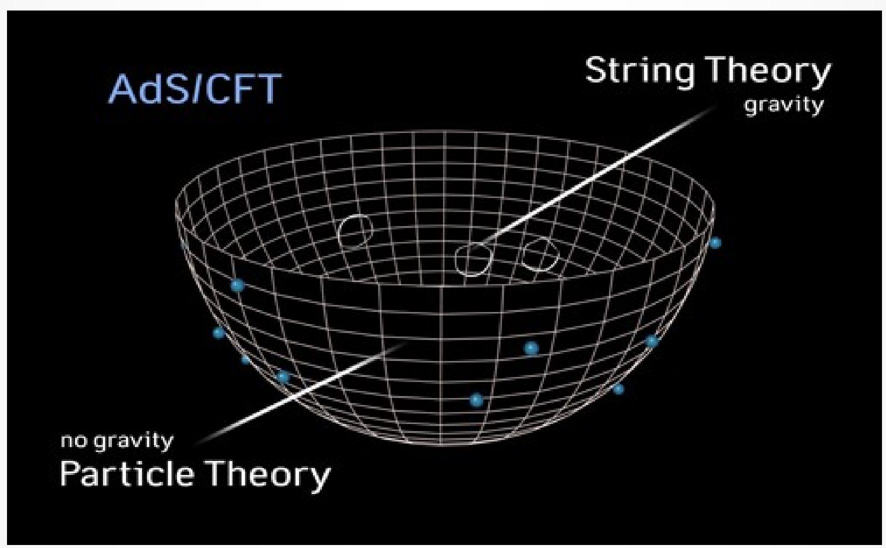

Wilson loops and holography

- In AdS/CFT Wilson loops are dual to **open strings** with worldsheet in AdS and ending on a contour *C*

Maldacena, Lee 1998

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- Supersymmetric operators required: coupling to matter

$$W_{1/2} = \frac{1}{N}\,\mathrm{Tr}\,\mathcal{P}\,\exp\left[-i\,g\int_{\Gamma} d\tau\,\left(A_\mu\,\dot{x}^\mu(\tau) + i\,n_I(\tau)\,|\dot{x}|\,\Phi^I\right)\right] \qquad I = 1,\ldots 6$$

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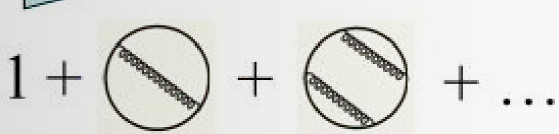
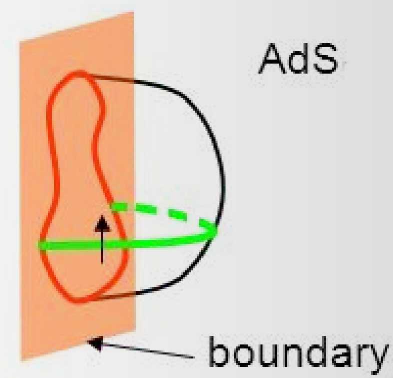
Supersymmetric Wilson loops

QFT perturbation theory



minimal surface



string PT

λ

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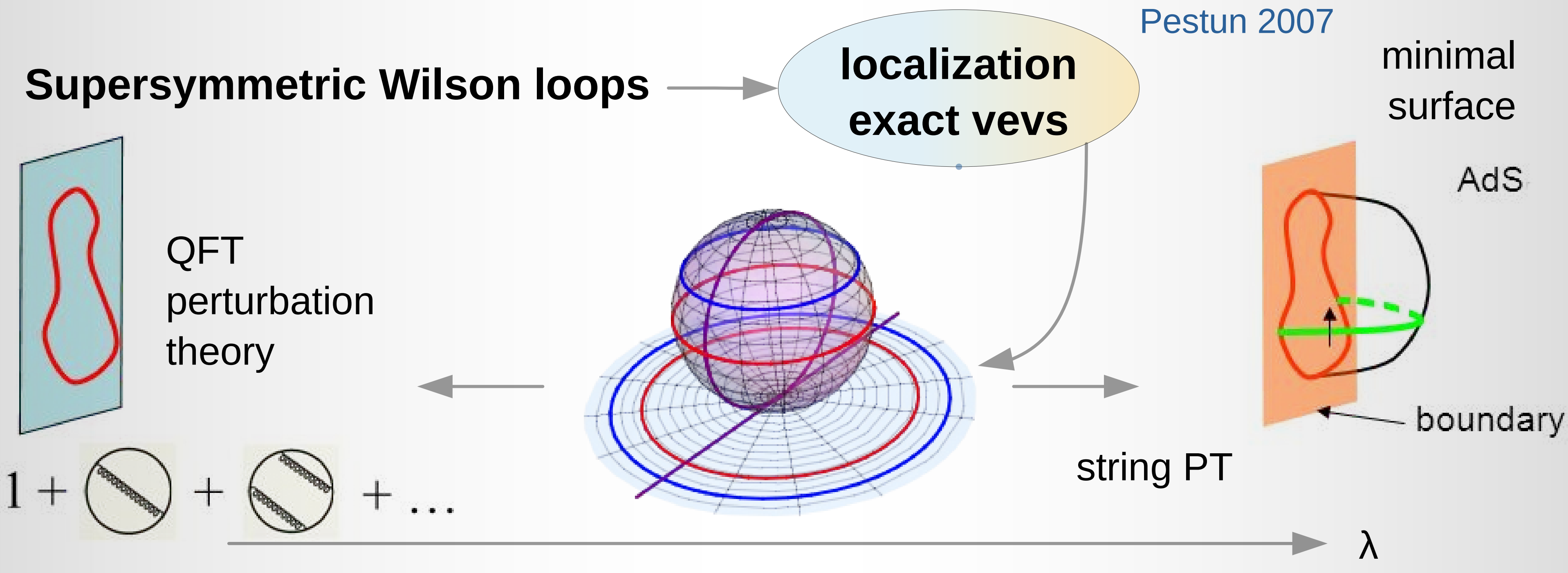

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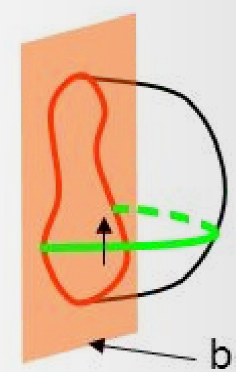
Pestun 2007

Supersymmetric Wilson loops → **localization exact vevs**

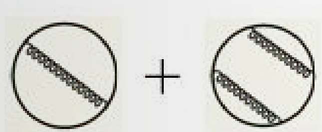
QFT perturbation theory

Precision AdS/CFT tests

minimal surface



string PT

$1 +$  $+ \ldots$

→ λ

AdS_4/CFT_3

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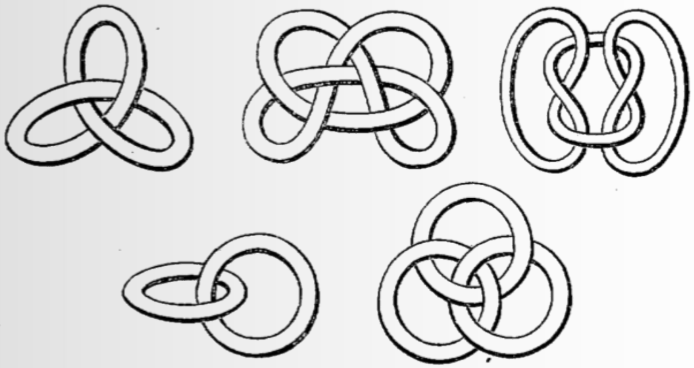
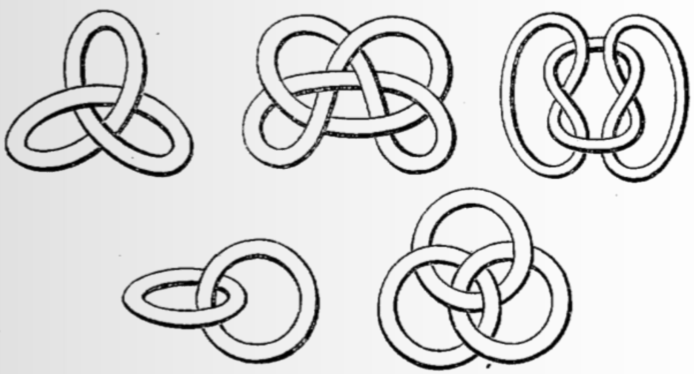


3
dimensions

$U(N)_k$ x $U(N)_{-k}$ **Chern-Simons**

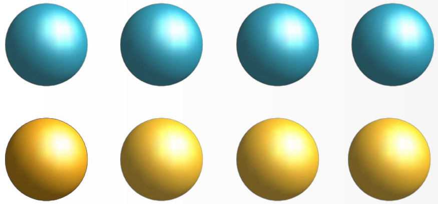
ABJM theory



$U(N)_k$ x $U(N)_{-k}$ Chern-Simons

+

bi-fundamental matter



supersymmetry

3
dimensions

Aharony et al 2008

$\lambda = \frac{N}{k}$ only one coupling k^{-1} and one parameter N.

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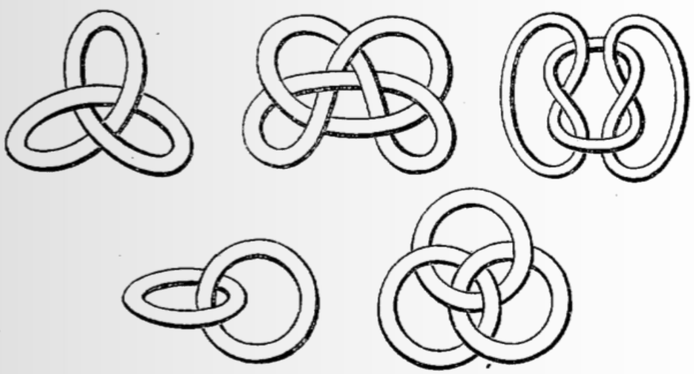

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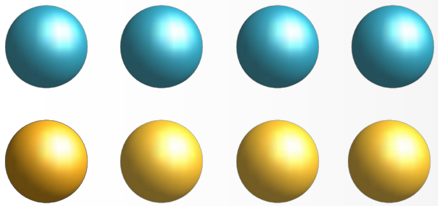

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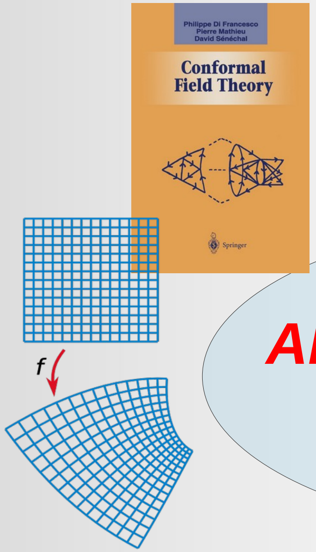
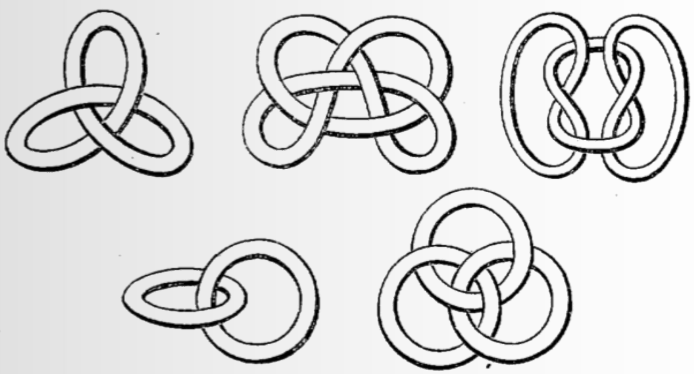


weak coupling

***ABJM* superconformal**
model: *OSp(6|4)*

ABJM theory



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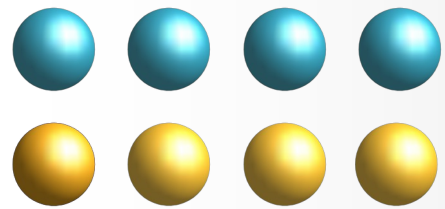

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3 dimensions

Aharony et al 2008

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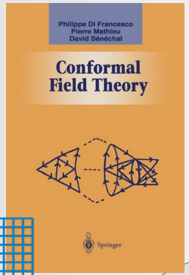
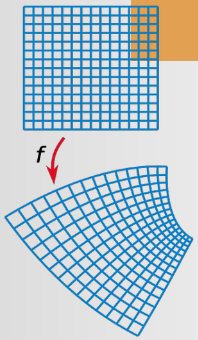




weak coupling

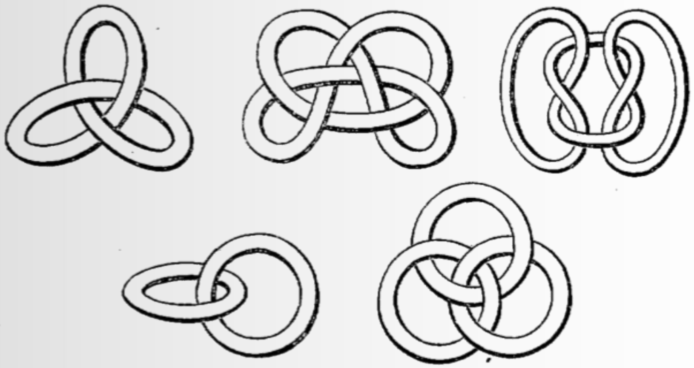
strong coupling

***ABJM* superconformal**
model: *OSp(6|4)*

dual *AdS*$_4$
IIA string / M-theory

holography

ABJM theory



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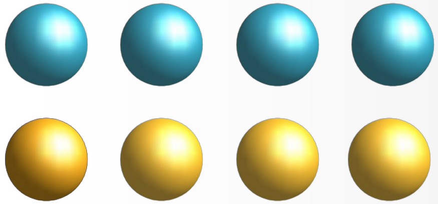

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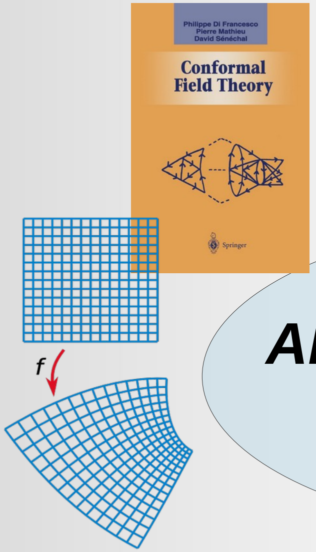

Holography in *d = 3* rests mostly on the **ABJM** theory

weak coupling

strong coupling

***ABJM* superconformal**
model*: OSp(6|4)*

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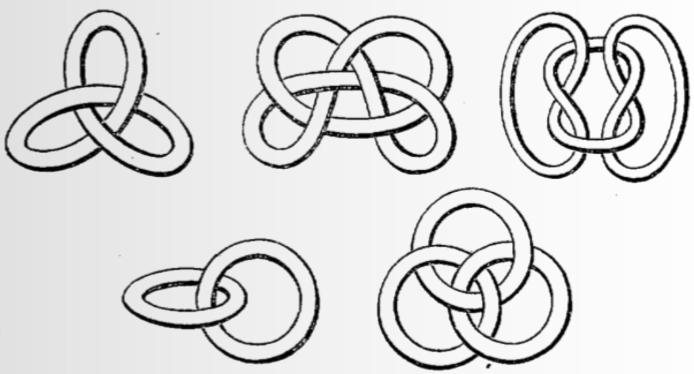
AdS$_4$/CFT$_3$



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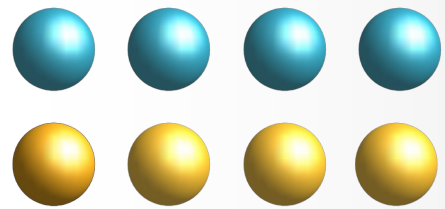

3 dimensions

supersymmetry

Aharony et al 2008

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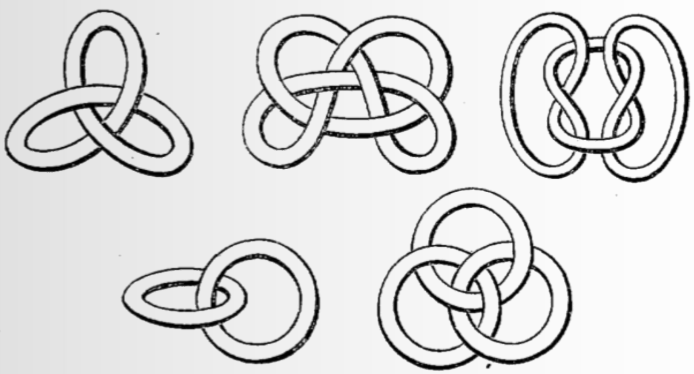
$\mathcal{N} = 6$ ABJM
SCFT in 3d
$k \gg N$

Type IIA ST on
$AdS_4 \times CP^3$
$k \ll N \ll k^5$

M-theory on
$AdS_4 \times S^7/\mathbb{Z}_k$
$N \gg k^5$

$\lambda = \frac{N}{k}$

AdS$_4$/CFT$_3$



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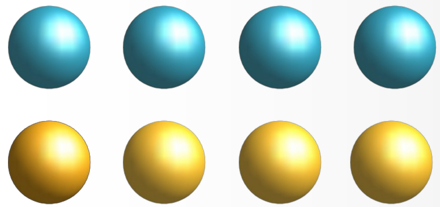

supersymmetry

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Exact tests via **supersymmetric Wilson loops**

$\mathcal{N} = 6$ ABJM
SCFT in 3d
$k \gg N$

localization

Type IIA ST on
$AdS_4 \times CP^3$
$k \ll N \ll k^5$

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exact results!!!

$\lambda = \frac{N}{k}$

Supersymmetric ABJM WL

The construction of supersymmetric Wilson loops in ABJM is a bit **convoluted**.

Supersymmetric ABJM WL

The construction of supersymmetric Wilson loops in ABJM is a bit **convoluted**. Let me omit it and only claim that

they can be *localized* on S^3 and their vev **computed exactly** in terms of a (*non-Gaussian) matrix model*:

Kapustin et al 2009

$$\langle \mathcal{W} \rangle = \int \prod_{a=1}^{N} d\lambda_a \, e^{i\pi k \lambda_a^2} \prod_{b=1}^{N} d\hat{\lambda}_b \, e^{-i\pi k \hat{\lambda}_b^2} \frac{\prod_{a<b}^{N} \sinh^2(\pi(\lambda_a - \lambda_b)) \prod_{a<b}^{N} \sinh^2(\pi(\hat{\lambda}_a - \hat{\lambda}_b))}{\prod_{a=1}^{N} \prod_{b=1}^{N} \cosh^2(\pi(\lambda_a - \hat{\lambda}_b))} \times \frac{1}{N} \sum_{a=1}^{N} e^{2\pi \lambda_a}$$

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expectation value at weak coupling:

$$\langle \mathcal{W} \rangle = 1 + i\,\pi \, \frac{N}{k} + \frac{1}{6}\left(1 + 2N^2\right) \pi^2 \, \frac{1}{k^2} + \frac{1}{6}\, i\, N \left(4 + N^2\right) \pi^3 \, \frac{1}{k^3} + \ldots ,$$

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Drukker et al 2008

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MB et al 2015

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$$\langle \mathcal{W} \rangle = 1 + i\,\pi\,\frac{N}{k} + \frac{1}{6}\left(1 + 2N^2\right)\pi^2\,\frac{1}{k^2} + \frac{1}{6}\,i\,N\left(4 + N^2\right)\pi^3\,\frac{1}{k^3} + \ldots\,,$$

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Localization based vevs exhibit **imaginary terms**

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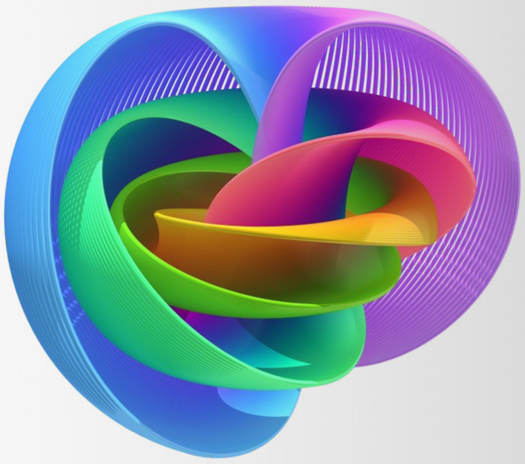

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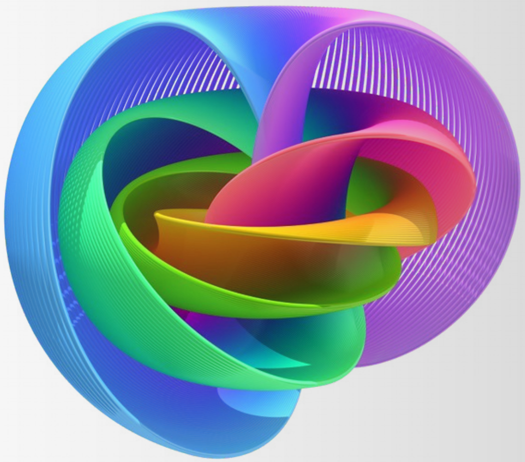

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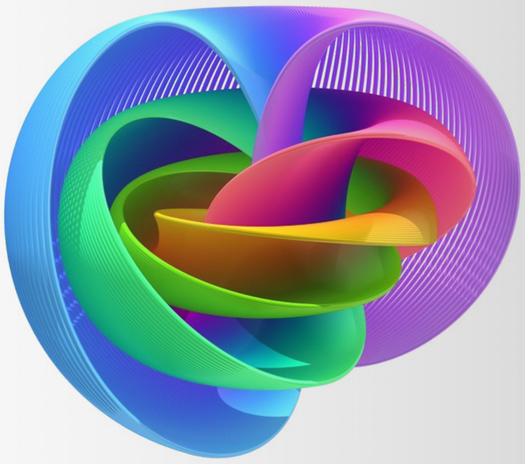

- Chern-Simons theory is topological and is sensitive to this linking ⇔ relation to **knot theory**

Witten 1988

Framing in pure CS

As an example, consider a pure CS Wilson loop on an arbitrary contour

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In perturbation theory its vev at 1 loop is given by

$$\int_\Gamma dx_1^\mu \int_\Gamma dx_2^\nu \, \epsilon_{\mu\nu\rho} \frac{(x_1 - x_2)^\rho}{|x_1 - x_2|^3}$$

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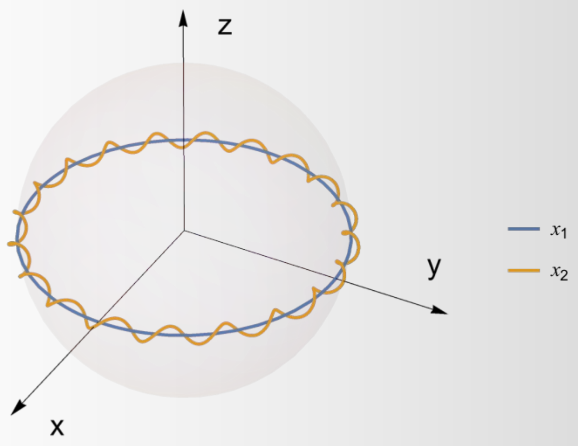
PT breaks CS topological invariance

define **framed contour** as a curve + *normal unit vector*



$$\Gamma_{\rm f} : x_2^\mu \to x_2^\mu + \delta n^\mu(\tau_2)\,, \qquad |n(\tau_2)| = 1$$

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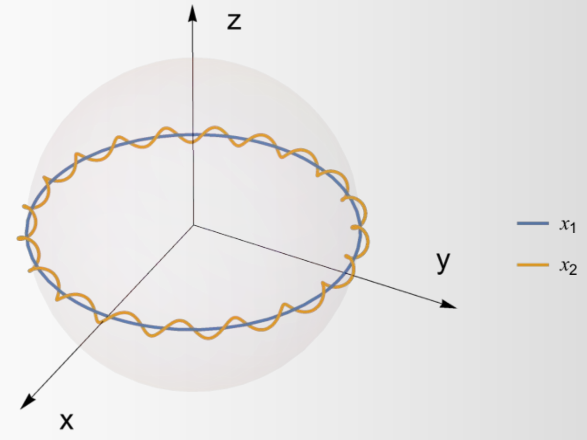

net effect of framing on WL vev in pure Chern-Simons is a **phase**

$$\langle W_{\rm CS}\rangle_{\rm f} = e^{\frac{i\pi N}{k}{\rm f}}\langle W_{\rm CS}\rangle_{\rm f=0}$$

Framing in ABJM at weak coupling

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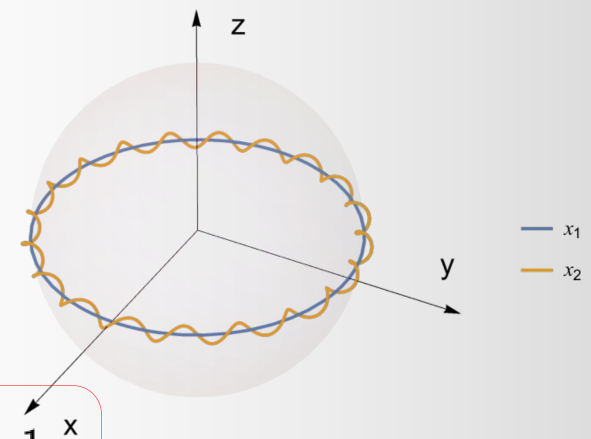

ABJM is not topological theory, due to matter, but **imaginary terms** still appear via the same mechanism

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MB et al 2015

Framing at strong coupling?

What about the **holographic** description of WL?

The strong coupling vevs expansions from matrix model:

$$\langle \mathcal{W} \rangle = \frac{N}{4\pi\lambda}\, e^{\pi\sqrt{2\lambda}}\, e^{\pi \mathrm{i} B} + \ldots$$

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Drukker et al 2008
Rey et al 2008
Chen et al 2008

Correa et al 2014
Faraggi et al 2018
Giombi et al 2020

The **real exponential** has been calculated from strings

$$ds^2 = L^2 \left(ds^2_{AdS_4} + 4 ds^2_{\mathbb{CP}^3} \right)$$

$$ds^2_{AdS_4} = -\cosh^2\rho \, dt^2 + d\rho^2 + \sinh^2\rho \left(d\vartheta^2 + \sin^2\vartheta \, d\psi^2 \right)$$

$$ds^2_{\mathbb{CP}^3} = \frac{1}{4}\Big[d\alpha^2 + \cos^2\frac{\alpha}{2}\left(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2 \right) + \sin^2\frac{\alpha}{2}\left(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2 \right) + \sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}\left(d\chi + \cos\theta_1 d\varphi_1 - \cos\theta_2 d\varphi_2 \right)^2 \Big],$$

$$t = 0, \quad \rho = \rho(\sigma), \quad \vartheta = \pi/2, \quad \psi = \tau, \quad \theta_1 = \theta(\sigma), \quad \varphi_1 = \tau, \quad \alpha = 0$$

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But **not the phase**.

How do we get imaginary corrections in holography?

What is the strong coupling interpretation of framing?

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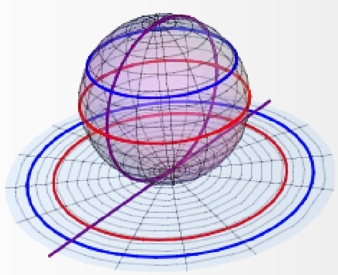
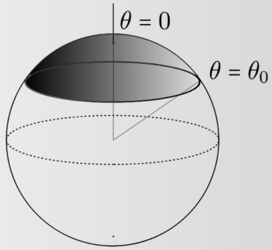
Latitude deformation: coupling to *scalars* means internal path sweeps a CP^1-cycle at polar angle θ and strings couple to B

Correa et al 2014





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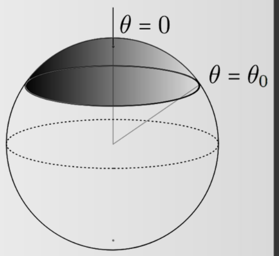

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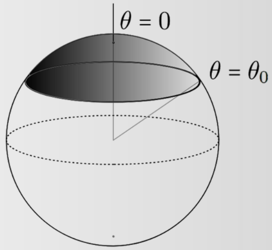

Same logic as Aharonov-Bohm: shrinking contour to a point doesn't kill the phase if enclosed flux threads a nontrivial cycle

Conclusions

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- Wilson loops are natural observables in gauge theories
- We explored Wilson loops within AdS_4 / CFT_3 correspondence
- We addressed the **holographic interpretation of framing**, that is figuring out how imaginary contributions to the string action may arise in holography.
- The solution is via a topological term in the string action, coupling the dual WL solution to the background B-field of ABJM

Gracias!