Influence of Extreme Magnetic Fields on the Quark Anomalous Magnetic Moment

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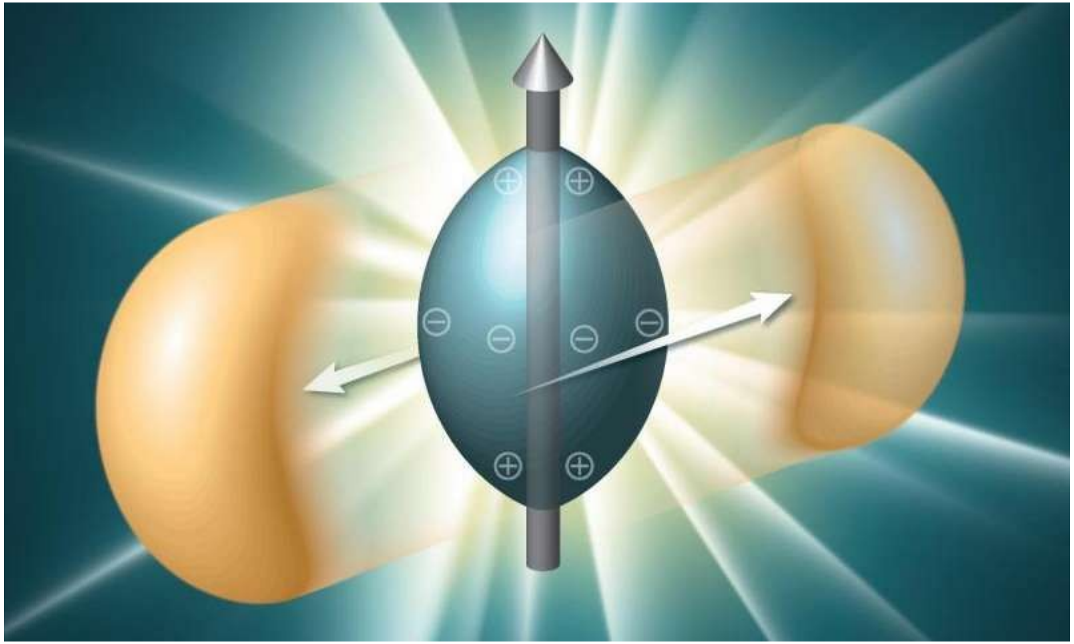
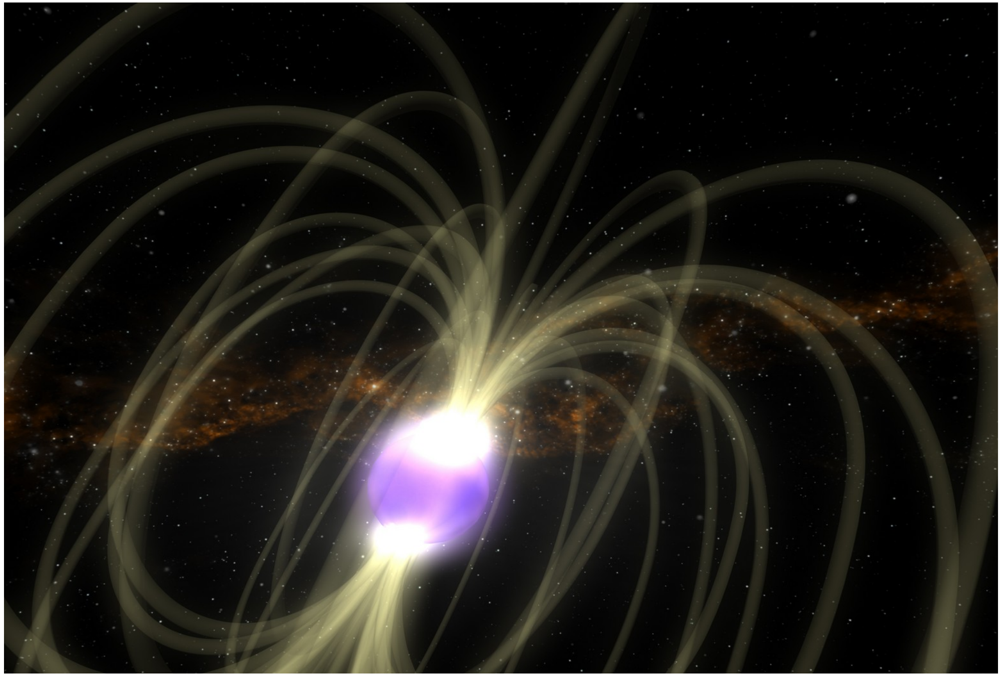
CAMPOS MAGNETICOS INTENSOS



Colisiones periféricas



magnetar

Brief introduction to AMM (QED)

$$\left[-i\not{D}-m\right]\left[i\not{D}-m\right]\psi = \left[D^2+m^2-g\frac{e_q}{4}F_{\mu\nu}\sigma^{\mu\nu}\right]\psi = 0$$

$g = 2$

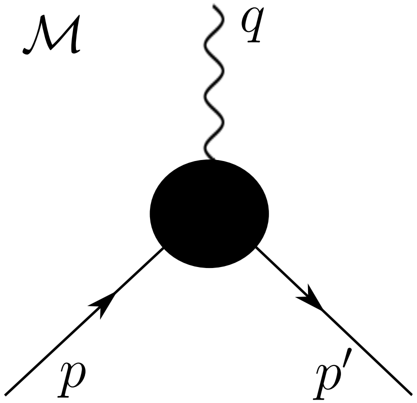
(g-factor)

$$\sigma_{\mu\nu}F^{\mu\nu} = -2\begin{pmatrix}(\boldsymbol{B}+i\boldsymbol{E})\cdot\boldsymbol{\sigma} & 0 \\ 0 & (\boldsymbol{B}-i\boldsymbol{E})\cdot\boldsymbol{\sigma}\end{pmatrix}$$

Electric field here is Lorentz-invariant completion of the magnetic moment

Radiative corrections $\Rightarrow$ $g = 2(1+a)$ AMM

Vertex



on-shell

$$\bar{u}(p')(\not{p}' - m) = 0, \qquad (\not{p} - m)u(p) = 0$$

Gordon identity

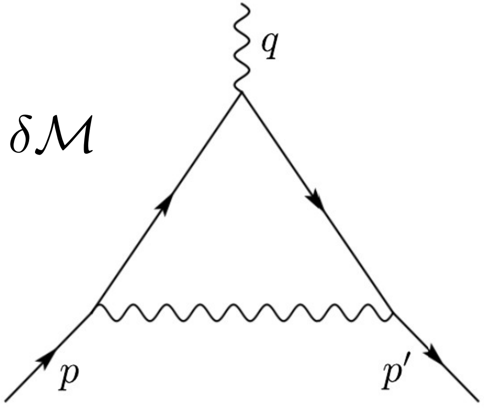
$$\bar{u}(p')(p'^\mu + p^\mu)u(p) = \bar{u}(p')\left[2m\gamma^\mu - i\sigma^{\mu\nu}\underbrace{(p'_\nu - p_\nu)}_{q_\nu}\right]u(p)$$

EM coupling AMM

$$\mathcal{M}^\mu = e_q\overline{u}(p')\left[F_1\left(\frac{q^2}{m^2}\right)\gamma^\mu + F_2\left(\frac{q^2}{m^2}\right)\frac{i\sigma^{\mu\nu}}{2m}q_\nu\right]u(p)$$

$$g = 2\left[1 + F_2\left(\frac{q^2}{m^2}\right)\right] \qquad a = F_2(0)$$

Vertex correction



$$i\delta\mathcal{M}^\mu = (ie_q)^3 \int_k D_{\mu\nu}(k)\,\overline{u}(p')\gamma^\nu S(k+p')\gamma^\mu S(k+p)\gamma^\rho\,u(p)$$

$$\Rightarrow \quad F_2(q^2) = \frac{\alpha}{\pi} m^2 \int_{xyz} \frac{z(1-z)}{(1-z)^2 m^2 - xyq^2}$$

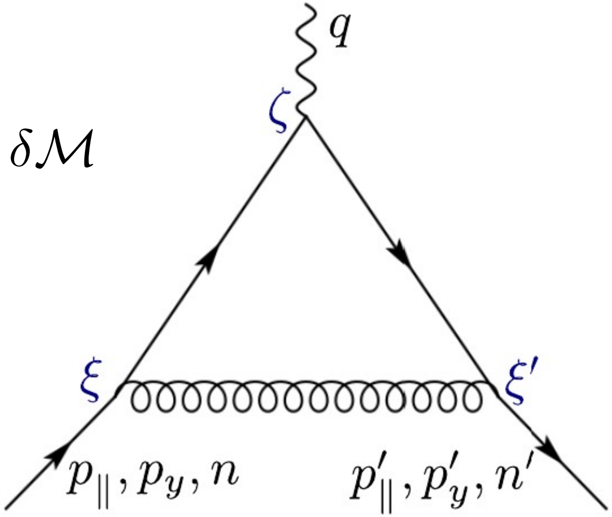
$$a = F_2(0) = \frac{\alpha}{2\pi}$$

$$\int_{xyz} = \int_0^1 dx\; dy\; dz\; \delta(x+y+z-1)$$

For quarks with a gluon interchange

$$a_q^{\mathrm{vac}} \equiv F_2^{\mathrm{vac}}(0) = \frac{\alpha_s}{2\pi}\frac{N_c^2-1}{2N_c}$$

AMM under extreme magnetic field



$$\boldsymbol{B}_{\mathrm{ext}} = B_{\mathrm{ext}}\hat{\boldsymbol{z}}$$ Landau Gauge

$$\mathcal{T} = (2\pi)^2\delta^{(2)}(q_\parallel - p'_\parallel + p_\parallel)\,\varepsilon_\mu\,\delta\mathcal{M}^\mu_{nn'}$$

Matrix elements

$$\langle 0|\mathcal{A}_\mu(\zeta)|q\rangle = \varepsilon_\mu(q)\,e^{-iq\cdot\zeta}$$

$$\langle 0|\psi(\xi)|p_\parallel, p_y, n\rangle = f_n(p_y, \xi_\perp)u_n(p_\parallel)\,e^{-ip\cdot\xi_\parallel}$$

on-shell $\quad \ell_q = |e_q B_{\mathrm{ext}}|^{1/2}$

$$p_0 = \sqrt{p_z^2 + 2n\ell_q^{-2} + m^2},$$
$$p'_0 = \sqrt{p_z'^2 + 2n'\ell_q^{-2} + m^2},$$
$$q_0 = |\boldsymbol{q}|,$$

Lowest Landau Level → $\quad n = n' = 0$

Structure of form factors in LLL

on-shell

$$\bar{u}_0(p'_{\parallel})(\not{p}'_{\parallel} - m) = 0, \qquad (\not{p}_{\parallel} - m)u_0(p_{\parallel}) = 0$$

Gordon identity

$$\bar{u}_0(p')(p'^{\mu} + p^{\mu})u_0(p) = \bar{u}_0(p'_{\parallel})\left[2m\gamma^{\mu}_{\parallel} - i\sigma^{\mu\nu}_{\parallel} q^{\parallel}_{\nu}\right] u_0(p_{\parallel})$$

Separated in polarization projections $\delta\mathcal{M}^{\mu}_{\mathrm{LLL}} = \delta\mathcal{M}^{\mu}_{+} + \delta\mathcal{M}^{\mu}_{-}$

$$\delta\mathcal{M}^{\mu}_{\pm} = e_q \bar{u}_0(p'_{\parallel})\left[\delta F^{\pm}_1(q^2_{\parallel})\,\gamma^{\mu}_{\parallel} + F^{\pm}_2(q^2_{\parallel})\,\sigma^{\mu\nu}_{\parallel}\frac{iq^{\parallel}_{\nu}}{2m} + F^{\pm}_3(q^2_{\parallel})\,q^{\mu}_{\parallel}\right]\mathcal{P}_{\pm}\,u_0(p_{\parallel})$$

contributtes $\partial \cdot \mathcal{A}_{\parallel}$

$$\mathcal{P}_{\pm} = \frac{1}{2}\left[1 \pm i\ \mathrm{sign}(eB)\gamma_1\gamma_2\right]$$

Structure of form factors in LLL

$$(1+a)\sigma_{\mu\nu}F^{\mu\nu} \to \sum_{s=\pm}(\sigma_{\mu\nu}F^{\mu\nu} + a^s\sigma^{\parallel}_{\mu\nu}F^{\mu\nu}_{\parallel})\mathcal{P}_s$$

$$\sum_{s=\pm} a^s\sigma^{\parallel}_{\mu\nu}F^{\mu\nu}_{\parallel}\mathcal{P}_s = -2iE_z\,\mathrm{diag}(a^+, -a^-, -a^+, a^-)$$

$$= -(a^+ + a^-)\begin{pmatrix} iE_z\sigma_z & 0 \\ 0 & -iE_z\sigma_z \end{pmatrix} + (a^+ - a^-)E_z i\gamma_5$$

AMM at LLL

We can set $q_\parallel^2 = 0$ if $p'_z = p_z$ or $p' \gg m$ and $p \gg m$

$$\boxed{a_q^\pm \equiv F_2^\pm(0) = a_q^{\rm vac} \int_0^1 dz \int_0^\infty d\eta \, \frac{e^{-(m\ell_q)^2\eta} f^\pm(z)(1-z)}{\left[(1-z)^2 + (m_g/m)^2 z + 2\eta z\right]^2}}$$

$$f^+(z) = \tfrac{2}{3}(1+6z), \qquad f^-(z) = z(4-z) - \tfrac{2}{3}$$

IR regularization

Infrared divergence when $m_g \to 0$

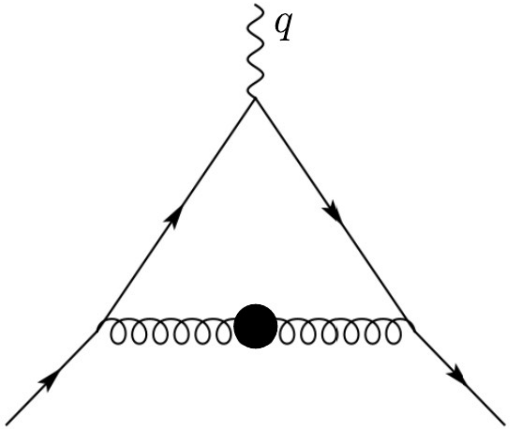
1. fixed effective masses $m_g = 0.3 \text{ GeV}$
   $m_q = 0.35 \text{ GeV}$

2. $m_q(\mu)$ and magnetically-dressed gluon self-energy in LLL

$$\Pi_g^{\mu\nu}(k) = \left(g_{\parallel}^{\mu\nu} - \frac{k_{\parallel}^{\mu} k_{\parallel}^{\nu}}{k_{\parallel}^2} \right) \Pi_g(k_{\parallel}^2, k_{\perp}^2)$$

$$\Pi_g(k_{\parallel}^2, k_{\perp}^2) = \frac{\alpha_s}{\pi} N_c \sum_q \frac{e^{-\frac{1}{2}\ell_q^2 k_{\perp}^2}}{\pi^2 \ell_q^2} g(m_q^2/k_{\parallel}^2)$$

$$g(m^2/k_{\parallel}^2) \approx g(0) = 1$$



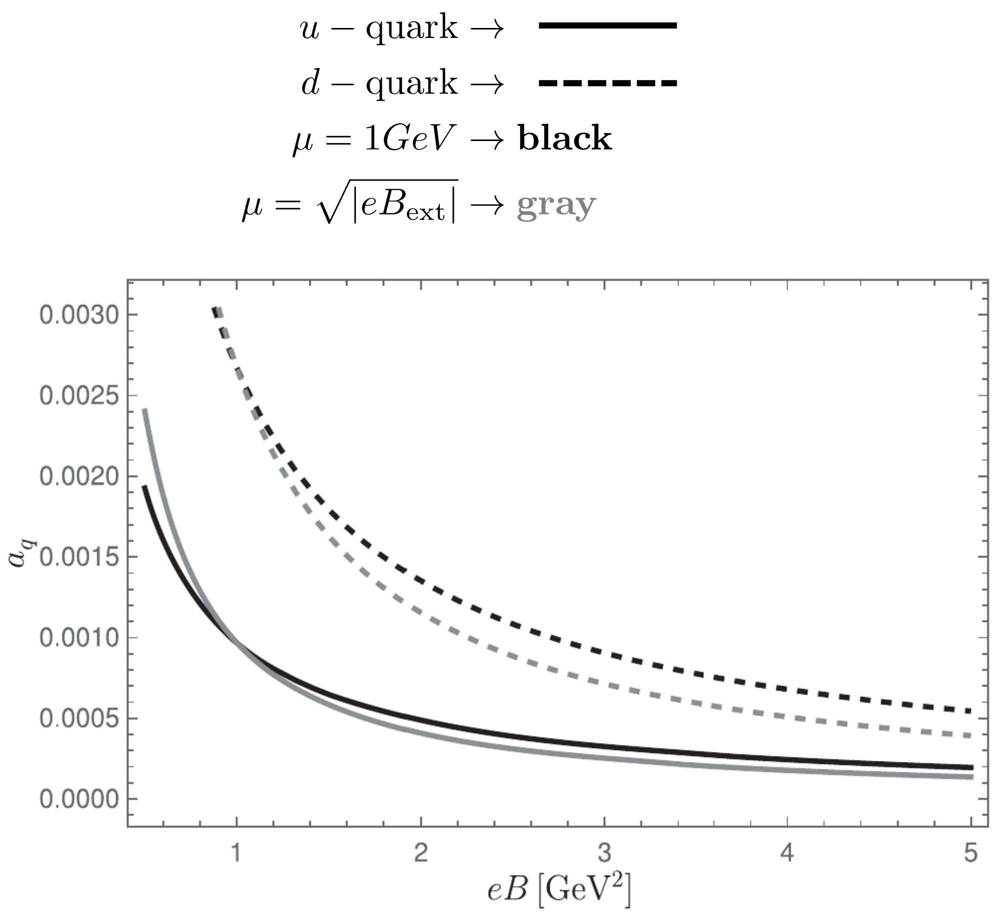
Scales $\alpha_s(\mu)$ $m_q(\mu)$

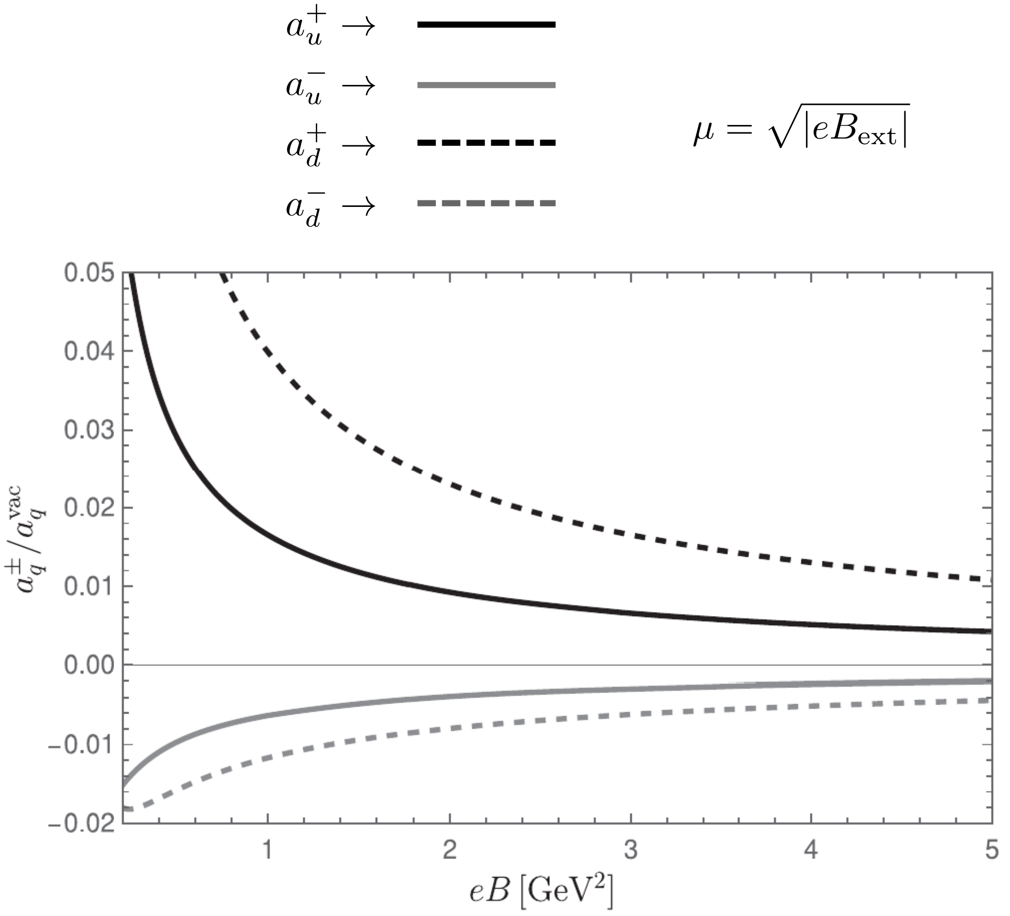
$a.$ $\mu = 1 \text{ GeV}$

$b.$ $\mu = \sqrt{|eB_{\text{ext}}|}$

Numerical results

$$m_g^2 = \Pi_g, \quad m_q(\mu)$$





Numerical results

$$m_g = 0.3\ \text{GeV}, \quad m_q = 0.35\ \text{GeV},$$

$u - \text{quark} \rightarrow$

$d - \text{quark} \rightarrow$

$\mu = 1GeV \rightarrow$ **black**

$\mu = \sqrt{|eB_{\text{ext}}|} \rightarrow$ **gray**

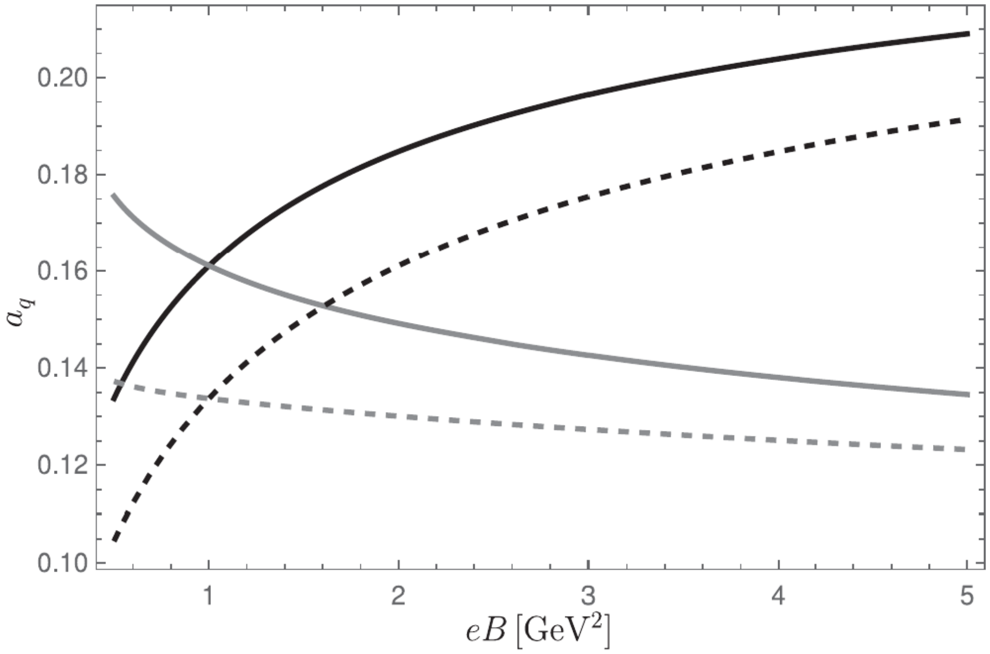
$a_u^+ \rightarrow$

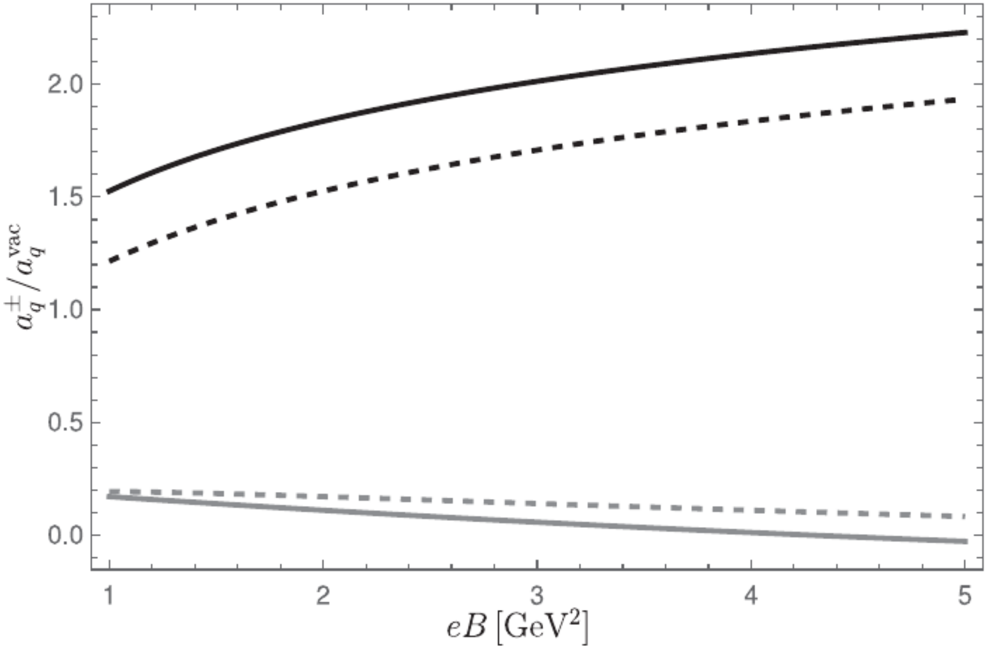
$a_u^- \rightarrow$

$a_d^+ \rightarrow$

$a_d^- \rightarrow$

$\mu = \sqrt{|eB_{\text{ext}}|}$





Conclusions and outlook

- AMM at LLL contributes to the electric part
  The other contributions are completely suppressed $\sim iE_z\sigma_z$

- IR divergences controled with gluon mass

- Results with gluon SE more closer to a physical description

- Results with effective fixed masses are of the order of the ones obtained recently in arXiv: 2506.20246

→ Full LL description

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GRACIAS!