One Lagrangian to rule them all, One Lagrangian to find them, One Lagrangian to bring them all and in the darkness bind them

(Work in progress with P. Álvarez and C. Quinzacara)

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Universidad San Sebastián, Concepción, Chile

Cosmoconce
November 7th, 2025


“One Ring to rule them all, One Ring to find them,
One Ring to bring them all and in the darkness bind them.”

One Lagrangian to rule them all, One Lagrangian to find them, One Lagrangian to bring them all and in the darkness bind them

How not to build a Theory of Everything

(Work in progress with P. Álvarez and C. Quinzacara)

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$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{2}\langle \mathbb{F} \wedge *\mathbb{F} \rangle$$

Unify everything in a single gauge principle? What about Yang-Mills gravity?

$$\mathbb{A} = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + \frac{1}{l}e^{a}\mathbb{P}_{a}.$$

$$
\begin{aligned}
\mathbb{F} &= \mathrm{d}\mathbb{A} + \frac{1}{2}\left[\mathbb{A}, \mathbb{A}\right], \\
&= \frac{1}{2}\left(R^{ab} + \frac{1}{l^2} e^a \wedge e^b\right)\mathbb{J}_{ab} + \frac{1}{l} T^a \mathbb{P}_a,
\end{aligned}
$$

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{2}\langle \mathbb{F} \wedge *\mathbb{F} \rangle$$

$$* : \Omega^{p}\left(M^{(d)}\right) \to \Omega^{d-p}\left(M^{(d)}\right)$$

$$* = *\,(e)$$

The gauge invariance is spoiled!

$$\delta e = -\mathrm{D}\lambda \implies \delta\,(*) \neq 0.$$

It almost works!

$$\mathcal{L}_{\text{YM}}\left(\omega,e\right)=\frac{1}{4l^{2}}\epsilon_{abcd}R^{ab}\wedge e^{c}\wedge e^{d}+\frac{1}{8l^{4}}\epsilon_{abcd}e^{a}\wedge e^{b}\wedge e^{c}\wedge e^{d}+\frac{1}{4}R^{ab}\wedge *R_{ab}-\frac{1}{2l^{2}}T^{a}\wedge *T_{a}.$$

It almost works!

$$\mathcal{L}_{\mathrm{YM}}(\omega, e) = \frac{1}{4l^2}\epsilon_{abcd}R^{ab}\wedge e^c \wedge e^d + \frac{1}{8l^4}\epsilon_{abcd}e^a \wedge e^b \wedge e^c \wedge e^d + \frac{1}{4}R^{ab}\wedge * R_{ab} - \frac{1}{2l^2}T^a \wedge * T_a.$$

$$\mathrm{d} * \mathrm{d} \neq 0.$$

What about fermions?

“Plurality is not to be posited without necessity.”

—William of Ockham (1287 – 1347)

Ockham's razor-X

$$\mathbb{A} = \underbrace{\frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + \frac{1}{l}e^{a}\mathbb{P}_{a}}_{\text{AdS}} + \cdots + \underbrace{A^{A}\mathbb{T}_{A}}_{\text{Internal}} + \underbrace{\frac{1}{2\sqrt{l}}\left(\bar{\mathbb{Q}}\Psi_{(3/2)} - \bar{\Psi}_{(3/2)}\mathbb{Q}\right)}_{\text{Super}}.$$

What if the only ingredient is a gauge connection?

$$\mathcal{L}_{\mathrm{YM}}\big|_{\psi} = -\frac{1}{2}\left\langle \mathbb{F} \wedge *\mathbb{F} \right\rangle ?$$

Add the constraint

$$\Psi_{(3/2)} = \not{e}\psi, \qquad \not{e} = e^{a}\Gamma_{a}.$$

It almost works!

$$\mathcal{L}_{\mathrm{YM}}|_{\psi} = \sqrt{|g|}\mathrm{d}x^4 \left(\frac{3!}{l^{11/2}} \left[\frac{1}{2} i \left(\bar{\psi}\Gamma^m \nabla_m \psi - \nabla_m \bar{\psi} \Gamma^m \psi \right) - \frac{3!}{l^{5/2}} \bar{\psi}\psi \right] + \right.$$

$$\left. + \frac{1}{l^3} \nabla_m \bar{\psi} \Gamma^{mn} \nabla_n \psi + \frac{3}{l^3} \nabla_m \bar{\psi} \nabla^m \psi \right) + \text{torsion terms.}$$

Solutions?

MacDowell-Mansouri gravity

$$* \longrightarrow \star$$

$$\star\left(e_a \wedge e_b\right) = *\left(e_a \wedge e_b\right) = \frac{1}{2}\epsilon_{abcd}e^c \wedge e^d,$$
$$\star R_{ab} = \frac{1}{2}\epsilon_{abcd}R^{cd},$$

$$R^{ab} \wedge *R_{ab} \to R^{ab} \wedge \star R_{ab} = \frac{1}{2}\epsilon_{abcd}R^{ab} \wedge R^{cd}.$$

Does it work too well?

Unconventional SUSY

$$\mathcal{L}_{\text{USUSY}} = \langle \mathbb{F} \wedge \circledast \mathbb{F} \rangle .$$

Does it work too well?

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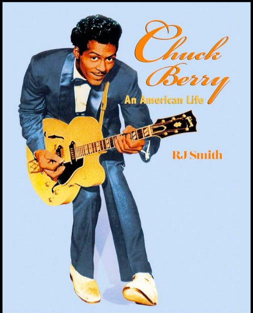
Classic $\longleftarrow$ Eccentric



Classic ⟷ Eccentric

What if the Yang-Mills term is only the first one from a series?

Ingredients:

1. $\mathbb{A} = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + \frac{1}{l}e^{a}\mathbb{P}_{a} + A^{A}\mathbb{T}_{A} + \frac{3}{\sqrt{2l}}\bar{\mathbb{Q}}\not{e}\psi,$
2. $* : \Omega^{p} \rightarrow \Omega^{d-p},$
3. $\not{e} = e^{a}\Gamma_{a}$ (e^{a} is already inside the Hodge, so it comes for free!)

$$L = \langle W_0 \wedge *W_0 \rangle + \langle W_1 \wedge *W_1 \rangle + \langle W_2 \wedge *W_2 \rangle + \cdots$$

$$W_n = W_n \left(\mathbb{F}, \not{\phi} \right)$$

Such series is not infinite!

$$\mathcal{L} = \lambda_0 \left\langle \mathbb{F} \wedge *\mathbb{F} \right\rangle + \lambda_1 \left\langle \not{\phi} \wedge \mathbb{F} \wedge * \left(\mathbb{F} \wedge \not{\phi} \right) \right\rangle + \lambda_2 \left\langle \not{\phi}^2 \wedge \mathbb{F} \wedge * \left(\mathbb{F} \wedge \not{\phi}^2 \right) \right\rangle +$$

$$+ \lambda_{[1]} \left\langle \left[\not{\phi}, \mathbb{F} \right] \wedge * \left[\mathbb{F}, \not{\phi} \right] \right\rangle + \lambda_{[2]} \left\langle \left[\not{\phi}^2, \mathbb{F} \right] \wedge * \left[\mathbb{F}, \not{\phi}^2 \right] \right\rangle .$$

Such series is not infinite!

$$\mathcal{L} = \lambda_0 \langle \mathbb{F} \wedge *\mathbb{F} \rangle + \lambda_1 \langle \not{\phi} \wedge \mathbb{F} \wedge *(\mathbb{F} \wedge \not{\phi}) \rangle + \lambda_2 \left\langle \not{\phi}^2 \wedge \mathbb{F} \wedge * \left(\mathbb{F} \wedge \not{\phi}^2 \right) \right\rangle + \\ + \lambda_{[1]} \langle [\not{\phi}, \mathbb{F}] \wedge * [\mathbb{F}, \not{\phi}] \rangle + \lambda_{[2]} \left\langle \left[\not{\phi}^2, \mathbb{F} \right] \wedge * \left[\mathbb{F}, \not{\phi}^2 \right] \right\rangle .$$

Unless you choose the coeficients in a particular way!

$$\mathcal{L}_{\mathrm{LL}}^{(4)} = \frac{1}{3!}\Bigg[-\langle \mathbb{F} \wedge *\mathbb{F}\rangle + \langle \not{\phi} \wedge \mathbb{F} \wedge *(\mathbb{F} \wedge \not{\phi})\rangle + \frac{1}{4}\left\langle \not{\phi}^2 \wedge \mathbb{F} \wedge *\left(\mathbb{F} \wedge \not{\phi}^2\right)\right\rangle + \\ + \frac{2}{3}\left(\langle [\not{\phi}, \mathbb{F}] \wedge *[\mathbb{F}, \not{\phi}]\rangle + \frac{1}{4}\left\langle \left[\not{\phi}^2, \mathbb{F}\right] \wedge *\left[\mathbb{F}, \not{\phi}^2\right]\right\rangle\right)\Bigg]$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{LL}}^{(4)} = {} & \frac{1}{4!}\epsilon_{abcd}\left(R^{ab} + \frac{1}{l^2}e^a \wedge e^b\right) \wedge \left(R^{cd} + \frac{1}{l^2}e^c \wedge e^d\right) + \\
& - \frac{i}{l^3}\left(\nabla\bar{\psi}^I \wedge \not{e} + \frac{1}{2l^{5/2}}\bar{\psi}^I \not{e}^2\right) \wedge \left(\not{e}\Gamma_5 \wedge \nabla\psi_I + \frac{1}{2l^{5/2}}\not{e}^2\Gamma_5\psi_I\right) + \\
& + \frac{1}{2}\left\langle \mathbb{F}_{\mathrm{int}} \wedge * \mathbb{F}_{\mathrm{int}} \right\rangle + \text{torsion} * \text{-terms.}
\end{aligned}
$$

$$
\mathrm{d}^2 = 0.
$$


A path to the One Ring or to Mount Doom?

What about exploring something so conservative it becomes excentric?

Higgs?

$$\mathbb{A} = \underbrace{\frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + \frac{1}{l}e^{a}\mathbb{P}_{a}}_{\text{AdS}} + \cdots + \underbrace{A^{A}\mathbb{T}_{A}}_{\text{Internal}} + \underbrace{\frac{1}{2\sqrt{l}}\left(\bar{\mathbb{Q}}\Psi_{(3/2)} - \bar{\Psi}_{(3/2)}\mathbb{Q}\right)}_{\text{Super}}.$$

Let us split the internal algebra in a subalgebra (+) and an ideal (−),

$$\mathbb{A}_{\text{int}} = \mathbb{A}_{(+)} + \mathbb{A}_{(-)} + \bar{\mathbb{A}}_{(-)},$$

$$\begin{aligned} [+,+] &= +, \\ [+,-] &= -, \\ [-^{*},-] &= +, \end{aligned}$$

satisfying

$$\begin{aligned} \langle +,+ \rangle &\neq 0, \\ \langle -^{*},- \rangle &\neq 0. \end{aligned}$$

Let us constraint $\mathbb{A}_{(-)}$ in such a way that it can be written as

$$\mathbb{A}_{(-)} = \mathbb{X}_j \Phi^j$$

with

$$\mathbb{X}_j = \mathbb{T}_i^{(-)} e^a \left[X_a\right]^i{}_j,$$

where the X_a are constant matrices satisfying,

$$\left[\bar{X}_a\right]^m{}_i \left[X_b\right]^n{}_j \left\langle \bar{\mathbb{T}}_m^{(-)} \mathbb{T}_n^{(-)} \right\rangle = k\eta_{ab}\delta_{ij}$$

and Φ^j is a bosonic 0-form.

$$\frac{1}{2}\left\langle \mathbb{F}_{\text{int}} \wedge *\mathbb{F}_{\text{int}} \right\rangle = \frac{1}{2}\left\langle \mathbb{F}_{(+)} \wedge *\mathbb{F}_{(+)} \right\rangle + 3k\mathrm{D}_{(+)}\bar{\Phi}^{i} \wedge *\mathrm{D}_{(+)}\Phi_{i} + \\ + \left\langle \mathbb{F}_{(+)} \wedge * \left[\bar{\mathbb{X}}_{i}, \mathbb{X}_{j}\right] \right\rangle \bar{\Phi}^{i}\Phi^{j} + \frac{1}{8}\left\langle \left[\bar{\mathbb{X}}_{i}, \mathbb{X}_{j}\right] \wedge * \left[\bar{\mathbb{X}}_{k}, \mathbb{X}_{l}\right] \right\rangle \bar{\Phi}^{i}\Phi^{j}\bar{\Phi}^{k}\Phi^{l}.$$

$$\mathbb{A} = \underbrace{\frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + \frac{1}{l}e^{a}\mathbb{P}_{a}}_{\text{AdS}} + \cdots + \underbrace{A^{A}\mathbb{T}_{A}}_{\text{Internal}} + \underbrace{\frac{1}{2\sqrt{l}}\left(\bar{\mathbb{Q}}\Psi_{(3/2)} - \bar{\Psi}_{(3/2)}\mathbb{Q}\right)}_{\text{Super}}.$$

Do we need only a single ingredient for a unified theory of everything in $d = 4$?

$$\mathcal{L}^{(4)}_{\text{LL}} \ldots ?$$

Lucía Izaurieta

Lizzie Álvarez

Contents

A ?

- Is there an underlying math structure? Symmetry? Why torsion is always in the way?

- Cosmology? Neutrino mass? Higgs?


Encuentro
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¡Gracias!

$$
\begin{aligned}
\mathcal{L}_{\mathrm{LL}}^{(4)} = \frac{1}{3!}\Bigg(&\frac{1}{3!}\left[\epsilon_{abcd}F^{ab}\wedge F^{cd}+\frac{1}{2}\epsilon_{abcd}F^{ab}\wedge\tilde{F}^{cd}\right]+\\
&+\frac{1}{12}e^{m}\wedge \mathrm{D}T_{m}\wedge *\left(e^{p}\wedge \mathrm{D}T_{p}\right)-\frac{7}{3}\left(\mathrm{D}T^{b}\wedge e^{a}\right)\wedge *\left(\mathrm{D}T_{a}\wedge e_{b}\right)+\\
&+\frac{2}{l^{2}}\left[2T^{b}\wedge *T_{b}-T^{(3)}\wedge *T^{(3)}-T^{b}\wedge e^{a}\wedge *\left(T_{a}\wedge e_{b}\right)\right]\Bigg)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{(4)}_{\mathrm{LL}} = \; & \sqrt{|\mathrm{g}|}\mathrm{d}x^4 \left(\frac{1}{3!l^2}\mathcal{R} + \frac{1}{l^4} + \frac{1}{4!}\left(\mathcal{R}^2 - 4\mathcal{R}^a{}_b\mathcal{R}^b{}_a + R^{ab}{}_{cd}R^{cd}{}_{ab}\right) + \right.\\
& + \frac{1}{3}\mathcal{R}^{ab}\left(\mathcal{R}_{ab} - \mathcal{R}_{ba}\right) - \frac{1}{3!}R^{abcd}R_{abcd} + \frac{1}{3}R^{abcd}R_{acbd} + \\
& \left. + \frac{1}{3l^2}T_{abc}T^{bac} + \frac{1}{3l^2}\mathcal{T}_a\mathcal{T}^a \right)
\end{aligned}
$$