CosmoConce 05/11/2025

Unifying gravity and gauge bosons with supersymmetry in the adjoint representation

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Outline of the talk

1) Motivation

2) Main features of the idea

3) “Unifying models”

↳ current and future projects

Motivation

S-matrix

$$\langle\phi|\,\mathcal{S}\,|\psi\rangle = \lim_{t\to\infty} \langle\phi|\,\mathcal{T}[e^{-i\int_{-t}^{t}\hat{H}_I(t')dt'}]\,|\psi\rangle$$

Golfand & Likhtman;
Haag–Lopuszański–Sohnius
Theorem

Fermionic extension Poincare symmetry $\rtimes$ Internal symmetries

Motivation

Phenomenological susy
(linear representations)

- MSSM (global susy)

+ SUGRA MSSM (local susy)

-+ Chiral supermultiplet (matter and Higgs)
- Vector supermultiplet (gauge bosons)
+ Gravity supermultiplet (gravitino)

Motivation

Phenomenological susy
(linear representations)

- MSSM (global susy)
~ 100 free parameters

+ SUGRA MSSM (local susy)
~ 20 free parameters

Strong departure from unification ideas

tiplet (matter and Higgs)
iplet (gauge bosons)
tiplet (gravitino)

Motivation

Phenomenological susy
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~ 100 free parameters

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Strong departure from unification ideas

tiplet (matter and Higgs)
iplet (
tiplet

Where are all the superpartners?

Motivation

Phenomenological susy
(linear representations)

Is the implementation of susy
in the adjoint representation
more appropriate?

Strong departure
from
unification ideas

tiplet (matter and Higgs)
iplet (
tiplet

Where are all
the superpartners?

My collaborators in this topic

- J. Zanelli (CECs, USS), C. Corral (UTFS Vina del Mar), **A. Chavez** (phd student at Antofagasta U.)**, J. Ortiz** (Colombia)
- M. Valenzuela (CECs, USS), P. Pais (U. Austral), E. Rodriguez (U. Nac. Colombia), P. Salgado (Wroclaw Tech. U.), L. Delage
- Fernando Izaurieta, Adytia Sharma(postdoc cecs), etc...?

How? / Kinematics

"Geometric models"

Summary of the idea

- We study a general recipe to implement models for **gravity**, **gauge bosons** and **matter fields** using the **adjoint reresentation** of the **superconformal algebra**

$$\mathbb{A} = A^M \mathbb{G}_M + \overline{\mathbb{Q}} \not{e} \psi + \overline{\psi} \not{e} \mathbb{Q} \quad \in \quad su(2,2|N) \quad \text{or} \quad osp(N|4)$$

→ Fermion/boson matching of d.o.f. is **not mandatory**.

Sohnius '85
- adjoint representation
- nonlinear realizations

→ **Standard gauge kinetic terms** are included.

→ Models are highly predictive, **few free parameters** in the action.

→ Also included **non-minimal couplings**: scalar-tensor induced gravity, fermion quartic terms (NJL-mass), torsion couplings, etc…

How?

→ All fields in the gauge potential

- Bosons and fermions in the adjoint representation:

$$\mathbb{A} = A^M \mathbb{G}_M + \overline{\mathbb{Q}}\not{e}\psi + \bar{\psi}\not{e}\mathbb{Q}$$

$$A^M \mathbb{G}_M = W^S \mathbb{J}_S + A^I \mathbb{T}_I + A\mathbb{Z}$$

spacetime internal susy central

- Spinor matter fields require the introduction of a soldering form → gravity

$$\not{e}\psi = e^a{}_\mu dx^\mu \gamma_a \psi$$

$$g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$$

How?

→ All fields in the gauge potential

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Kinematics

- Matter in the adjoint representation: $\psi^\alpha \in A_\mu$
- Red. Reps. $\Psi^\alpha_\mu = 1 \otimes 1/2 = 3/2 \oplus 1/2$

(a) Gravitino (SUGRA) $\xi^\alpha_\mu : \gamma^\mu \xi^\alpha_\mu = 0$ $P_{(1/2)}\xi^\alpha_\mu = 0$

(b) USUSY $\psi^\alpha_\mu = \gamma_\mu \psi^\alpha$ $P_{(3/2)}\psi^\alpha_\mu = 0$

Unconventional SUSY: **fields in the adjoint rep**

$$\mathbb{A}_\mu \supset \overline{Q}^{i} e^a{}_\mu \gamma_a \psi_i$$

- We choose a basis of the conformal superalgebra with **complex Q's**, where the **R-symmetry is identified with the internal symmetry** (see Trigiante's lectures on supergravity)

Correct gauge transformations

$$\delta \mathbb{A} = DG \quad \Rightarrow \quad \begin{cases} \delta A_{SU(N)} = D\lambda_{SU(N)} \\ \delta\psi = [\lambda_{SU(N)}, \psi] \end{cases}$$

Unconventional SUSY: **fields in the adjoint rep**

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$$\delta\psi = [\lambda_{SU(N)}, \psi]$$

At our disposal are the algebra series

D=3	D=4
SU(2|N)	SU(2,2|N)
OSp(p,2)	OSp(N,4)
OSp(p,2) x OSp(q,2)	

How? Invariant terms USUSY

D = 3

$$S = \frac{1}{2}\int \langle \mathbb{A} d\mathbb{A} + \frac{2}{3}\mathbb{A}^3 \rangle \, .$$

Phys.Lett.B 738 (2014) 134-135; **1405.6657**
Phys.Lett.B 735 (2014) 314-321; **1306.1247**
JHEP 04 (2012) 058; **1109.3944**

D = 4

MacDowell, Mansouri type of action ‘77

$$S = \int \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

similarity can be exploited to study field equations and symmetries [PA, Chavez, Zanelli 2111.09845]

Geometric formulation MM

$$\epsilon_{abcd}(R^{ab} + e^a e^b)(R^{cd} + e^c e^d)$$

$\in$

$$S = \int \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

Geometric formulation MM

$$\epsilon_{abcd}(R^{ab} + e^a e^b)(R^{cd} + e^c e^d) \in S = \int \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

Lorentz invariant bilinears

$$\langle \mathbb{J}_{ab} \mathbb{J}_{cd} \rangle = -(\eta_{ac}\eta_{bd} - \eta_{bc}\eta_{ad})$$

Geometric formulation MM

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$$\langle \mathbb{J}_{ab} \mathbb{J}_{cd} \rangle = -(\eta_{ac}\eta_{bd} - \eta_{bc}\eta_{ad})$$

Lorentz connection

$$\mathbb{A} = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab}$$

$$\mathbb{F} = \frac{1}{2}R^{ab}\mathbb{J}_{ab}$$

Pontryagin

$$\int R^{ab} R_{ab}$$

Geometric formulation MM

$$\epsilon_{abcd}(R^{ab}+e^a e^b)(R^{cd}+e^c e^d) \in S = \int \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

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$$\mathbb{A} = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab}$$

$$\mathbb{F} = \frac{1}{2}R^{ab}\mathbb{J}_{ab}$$

AdS connection

$$\mathbb{A} = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + e^a\mathbb{J}_a$$

$$\mathbb{F} = \frac{1}{2}(R^{ab}+e^a e^b)\mathbb{J}_{ab} + T^a J_a$$

Pontryagin

$$\int R^{ab}R_{ab}$$

AdS Pontryagin

$$\int \mathcal{R}^{ab}\mathcal{R}_{ab}$$

Geometric formulation MM

$$\epsilon_{abcd}(R^{ab} + e^a e^b)(R^{cd} + e^c e^d) \in S = \int \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

MM formulation

$$\langle \mathbb{J}_{ab} S \mathbb{J}_{cd} \rangle \propto \epsilon_{abcd}$$

adS ~ sp(4)
(adS gravity)

Geometric formulation MM

$$\epsilon_{abcd}(R^{ab} + e^a e^b)(R^{cd} + e^c e^d) \in S = \int \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

MM formulation

$$\langle \mathbb{J}_{ab} S \mathbb{J}_{cd} \rangle \propto \epsilon_{abcd}$$

$$\langle \overline{\mathbb{Q}}_\alpha S \overline{\mathbb{Q}}_\beta \rangle \propto (\gamma_5)_{\alpha\beta}$$

Also valid in osp(1,4) (simple sugra)

$$\mathbb{A} = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + e^a \mathbb{J}_a + \overline{\mathbb{Q}}_\alpha \chi^\alpha$$

$$\mathbb{F} = \frac{1}{2}F^{ab}\mathbb{J}_{ab} + T^a J_a + \overline{\mathbb{Q}}_\alpha \mathcal{D}\chi^\alpha$$

$$\mathcal{D}\chi = D\chi + e^a \gamma_a \chi$$

$$D\chi(C\gamma_5)D\chi = 0$$

$$\mathcal{L}_{\mathrm{RS}} \propto \chi e^a \gamma_a D\chi$$

Unconventional SUSY

$$S = \int \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

$$\left(\begin{array}{c|c} (\text{Spacetime})_{4\times 4} & (\text{Q's})_{4\times n} \\ \hline (\text{Q's})^{\dagger}_{4\times n} & (\text{Internal})_{n\times n} \end{array} \right)$$

Gauge bosons

Unconventional SUSY

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Gauge bosons

We also include matter fields s=1/2

$$\mathbb{A} = \frac{1}{2}\omega^{ab}\mathbb{J}_{ab} + e^{a}\mathbb{J}_{a} + \frac{1}{2}F^{ij}\mathbb{J}_{ij} + \mathbb{Q}_{\alpha}\not{e}\psi^{\alpha}$$

$$\chi^{\alpha} \quad \rightarrow \quad (\not{e}\psi)^{\alpha}$$

Unconventional SUSY

$$S = \int \langle \mathbb{F} \circledast \mathbb{F} \rangle$$

$$\left(\begin{array}{c|c} (\text{Spacetime})_{4\times 4} & (\text{Q}'\text{s})_{4\times n} \\ \hline (\text{Q}'\text{s})^{\dagger}_{4\times n} & (\text{Internal})_{n\times n} \end{array}\right)$$

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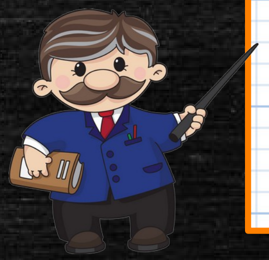
$$\chi^{\alpha} \quad \rightarrow \quad (\not{\phi}\psi)^{\alpha}$$



we refined the definition of the generalized dual operator in d=4

d=4 case
Dual operator

D=4

$$SU(2,2|N)$$

Properties of the dual operator

Action

$$I = -\int_{\mathcal{M}} \langle \mathbb{F} \wedge \circledast \mathbb{F} \rangle \,,$$

"Generalized Townsend-MacDowell-Mansouri"

Conformal algebra grading

$$S\,G - \varepsilon G\,S = 0\,, \qquad \text{where} \quad \begin{cases} \varepsilon = +1\,, \text{if } G \in \{\mathbb{J}_{ab}, \mathbb{D}, \mathbb{Z}, \mathbb{T}_I\}\,, \\ \varepsilon = -1\,, \text{if } G \in \{\mathbb{J}_a, \mathbb{K}_a\}\,. \end{cases}$$

Invariant traces

$$\langle \mathbb{E}_1 \circledast \mathbb{E}_2 \rangle = \langle \mathbb{E}_2 \circledast \mathbb{E}_1 \rangle \,, \qquad \langle \mathbb{O}_1 \circledast \mathbb{O}_2 \rangle = -\langle \mathbb{O}_2 \circledast \mathbb{O}_1 \rangle \,,$$

$$\langle \mathbb{E} \circledast \mathbb{O} \rangle = 0 = \langle \mathbb{O} \circledast \mathbb{E} \rangle \,,$$

PA, Valenzuela, Zanelli '20
PA, Delage, Valenzuela, Zanelli,'21
PA, Chavez, Zanelli '22

Auxiliar fields →

$$\langle \mathbb{F}^- \wedge \circledast \mathbb{F}^- \rangle = 0\,,$$

D=4

Auxiliar fields

$$\langle \mathbb{F}^- \wedge \circledast \mathbb{F}^- \rangle = 0\,,$$

$\circledast$-Odd

$$\mathbb{F}^- = \mathcal{F}^a \mathbb{J}_a + \mathcal{G}^a \mathbb{K}_a\,.$$

"Compensating fields"

f^a and g^a do not acquire independent kinetic terms

Algebraic equations

D=4

Auxiliar fields

$$\langle \mathbb{F}^- \wedge \circledast \mathbb{F}^- \rangle = 0\,,$$

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"Compensating fields"

f^a and g^a do not acquire independent kinetic terms

Algebraic equations

Field eqs. have cubic and linear terms → **Strong deviation from conformal sugra**

D=4

$$SU(2,2|N)$$

$$I = -\int_{\mathcal{M}} \langle \mathbb{F} \wedge \circledast \mathbb{F} \rangle \,,$$

Variation

$$\delta(-\langle \mathbb{F} \circledast \mathbb{F} \rangle) = -2d\langle \delta\mathbb{A} \circledast (\mathbb{F} - \mathbb{F}^-)\rangle - 2\langle \delta\mathbb{A} D_{\mathbb{A}} \circledast (\mathbb{F} - \mathbb{F}^-)\rangle \,.$$

Field eqs

$$D_{\mathbb{A}} \circledast (\mathbb{F} - \mathbb{F}^-) = 0 \,, \quad \Rightarrow \quad [\mathbb{F}, \circledast(\mathbb{F} - \mathbb{F}^-)] = 0 \,.$$

$$\Rightarrow \quad [\mathbb{F}, \circledast(\mathbb{F} - \mathbb{F}^-)] = [\mathbb{F}^+, \circledast\mathbb{F}^+] + [\mathbb{F}^-, \circledast\mathbb{F}^+] + [\mathbb{X}, \circledast\mathbb{X}] + [(\mathbb{F} - \mathbb{X}), \circledast\mathbb{X}] + [\mathbb{X}, \circledast\mathbb{F}^+] \,,$$

Integrability condition

$$[\mathbb{F}^+, \circledast\mathbb{F}^+] \equiv 0$$

- “Easy” tivial terms,
- “healthy” gauge symmetries
- YM only has these

$$[\mathbb{X}, \circledast\mathbb{X}] \sim B^- \,.$$

Conditions on Traslation components

$$[(\mathbb{F} - \mathbb{X}), \circledast\mathbb{X}] + [\mathbb{X}, \circledast\mathbb{F}^+] \,,$$

Conditions on susy components

D=4

$$SU(2,2|N)$$

$$I=-\int_{\mathcal{M}}\langle \mathbb{F}\wedge \circledast\mathbb{F}\rangle\,,$$

Gauge variation

$$\delta(-\langle\mathbb{F}\circledast\mathbb{F}\rangle)=-2d\langle D_{\mathbb{A}}G\circledast(\mathbb{F}-\mathbb{F}^{-})+GD_{\mathbb{A}}\circledast(\mathbb{F}-\mathbb{F}^{-})\rangle+2\langle G[\mathbb{F},\circledast(\mathbb{F}-\mathbb{F}^{-})]\rangle\,.$$

generator

$$G=G^{+}+G^{-}+\overline{\mathbb{Q}}\epsilon-\bar{\epsilon}\mathbb{Q}\,,$$

→ **same condition as before (Integrability condition)**

Aros at al 2000 in the context of AdS gravity

D=4

$$SU(2,2|N)$$

$$I = -\int_{\mathcal{M}} \langle \mathbb{F} \wedge \circledast \mathbb{F} \rangle \,,$$

Generalization
of self-duality
PA Corral Zanelli

generalized notion
of ***self-duality***

$$\circledast\left(\mathbb{F} - \mathbb{F}^{-}\right) = \pm(\mathbb{F} - \mathbb{F}^{-})\,.$$

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Generalization of self-duality PA Corral Zanelli

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$$\circledast\,(\mathbb{F} - \mathbb{F}^{-}) = \pm(\mathbb{F} - \mathbb{F}^{-})\,.$$

$$\circledast\left(\frac{1}{2}\mathcal{F}^{ab}\mathbb{J}_{ab} + \mathcal{F}^{a}\mathbb{J}_{a} + \mathcal{G}^{a}\mathbb{K}_{a}\right) = S\left(\frac{1}{2}\mathcal{F}^{ab}\mathbb{J}_{ab} + \mathcal{F}^{a}\mathbb{J}_{a} + \mathcal{G}^{a}\mathbb{K}_{a}\right)$$
$$= \frac{1}{2}\mathcal{F}^{ab}S\mathbb{J}_{ab} + \mathcal{F}^{a}S\mathbb{J}_{a} + \mathcal{G}^{a}S\mathbb{K}_{a}\,.$$

Explicit form of the operator acting on spacetime generators

Even

$$(\mathbb{J}_{ab})^{A}{}_{B} = \frac{1}{4}[\gamma_a, \gamma_b]^{A}{}_{B} = (\Sigma_{ab})^{A}{}_{B}$$

$$(\mathbb{D})^{A}{}_{B} = \frac{1}{2}(\gamma_5)^{A}{}_{B}$$

$$S = i(\gamma_5)^{A}{}_{B}$$

Odd

$$(\mathbb{J}_{a})^{A}{}_{B} = \frac{s}{2}(\gamma_a)^{\alpha}{}_{\beta}\delta^{A}{}_{\alpha}\delta^{\beta}{}_{B} = \frac{s}{2}(\gamma_a)^{A}{}_{B}$$

$$(\mathbb{K}_{a})^{A}{}_{B} = \frac{1}{2}(\tilde{\gamma}_a)^{A}{}_{B}$$

$$\tilde{\gamma}_a = \frac{i}{3!}\epsilon_{abcd}\gamma^{bcd} = -\gamma_5\gamma_a$$

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Generalization of self-duality
PA Corral Zanelli

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$$\circledast\,(\mathbb{F} - \mathbb{F}^{-}) = \pm(\mathbb{F} - \mathbb{F}^{-})\,.$$

Concrete definition

$$\circledast\left(\frac{1}{2}\mathcal{F}^{ab}\mathbb{J}_{ab} + \mathcal{F}^{a}\mathbb{J}_{a} + \mathcal{G}^{a}\mathbb{K}_{a}\right) = S\left(\frac{1}{2}\mathcal{F}^{ab}\mathbb{J}_{ab} + \mathcal{F}^{a}\mathbb{J}_{a} + \mathcal{G}^{a}\mathbb{K}_{a}\right)$$
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Explicit form of the operator acting on spacetime generators

$$\frac{1}{2}\varepsilon_s \epsilon^{ab}{}_{cd}\mathcal{F}^{cd} = \pm\frac{1}{2}\mathcal{F}^{ab}\,,$$
$$\varepsilon_1 * \mathcal{H} = \pm\mathcal{H}\,,$$
$$\varepsilon_2 * \mathcal{F}^{I} = \pm\mathcal{F}^{I}\,,$$
$$\varepsilon_3 * \mathcal{F} = \pm\mathcal{F}\,,$$
$$(-i\varepsilon_\psi\gamma_5)\mathcal{X} = \pm\mathcal{X}\,,$$
$$\overline{\mathcal{X}}(-i\varepsilon_\psi\gamma_5) = \pm\overline{\mathcal{X}}\,.$$

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Generalization of self-duality PA Corral Zanelli

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$$(-i\varepsilon_\psi\gamma_5)\mathcal{X} = \pm\mathcal{X} ,$$

$$\overline{\mathcal{X}}(-i\varepsilon_\psi\gamma_5) = \pm\overline{\mathcal{X}} .$$

Action on fermionic components

Even

$$(\mathbb{J}_{ab})^{A}{}_{B} = \frac{1}{4}[\gamma_a, \gamma_b]^{A}{}_{B} = (\Sigma_{ab})^{A}{}_{B}$$

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D=4

$$SU(2,2|N)$$

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Generalization of self-duality PA Corral Zanelli

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$$\circledast \left(\frac{1}{2} \mathcal{F}^{ab} \mathbb{J}_{ab} + \mathcal{F}^{a} \mathbb{J}_{a} + \mathcal{G}^{a} \mathbb{K}_{a} \right) = S \left(\frac{1}{2} \mathcal{F}^{ab} \mathbb{J}_{ab} + \mathcal{F}^{a} \mathbb{J}_{a} + \mathcal{G}^{a} \mathbb{K}_{a} \right)$$
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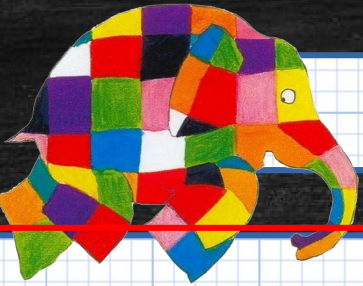
$$(-i \varepsilon_\psi \gamma_5) \mathcal{X} = \pm \mathcal{X} ,$$
$$\overline{\mathcal{X}} (-i \varepsilon_\psi \gamma_5) = \pm \overline{\mathcal{X}} .$$

Self-dual configurations:
gravitational instantons with torsion

torsional generalization of
- Taub-NUT/Bolt-AdS
- Eguchi-Hanson metric

PA, Corral, Zanelli 2310.02769

Models and Milestones



D=4

- Implementation of USUSY in d=4

  dirac term is reproducted by USUSY in MM type of action
  Phys.Lett.B 735 (2014) 314-321

- Chiral gauge theory from USUSY, on-shell symmetries

  dual operator, chiral gauge theory (left handed fermions from left-right symmetric theory)
  JHEP 07 (2020) 07, 205

- SU(2,2|N) model

  paving the road to GUT models 1/2
  PA, Chavez,
  J.Math.Phys. 63 (2022) 042304

- Embedding of rank 2 supercharges

  paving the road to GUT models 2/2
  PA, Chavez,
  J.Math.Phys. 63 (2022) 042304

- SU(2|2,10) GUT model SU(5) inspired

  SU(5) inspired GUT
  PA, Chavez, Zanelli
  J.Math.Phys. 63 (2022) 042304

- Black holes solutions

  PA Corral, Zanelli,
  2211.15585, JHEP 01 (2023) 009

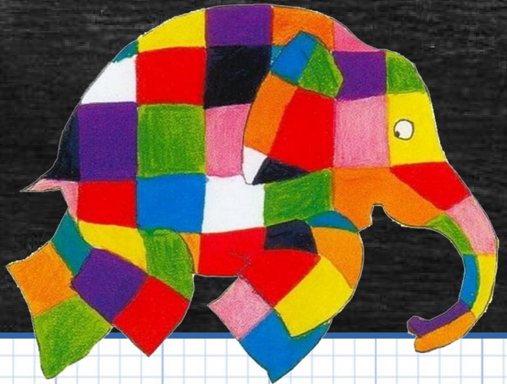
- Self-dual solutions

  Notion of self-duality
  PA, Corral, Zanelli,
  JHEP 01 (2024) 065

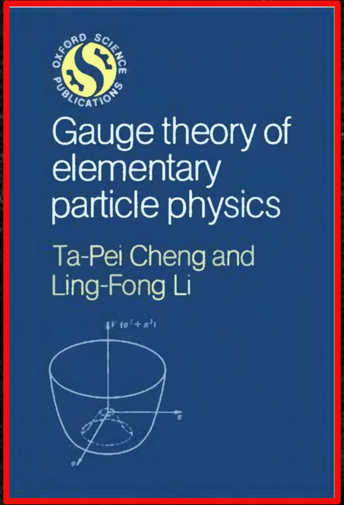
- Cosmological "induced gravity model"

  work in progress...

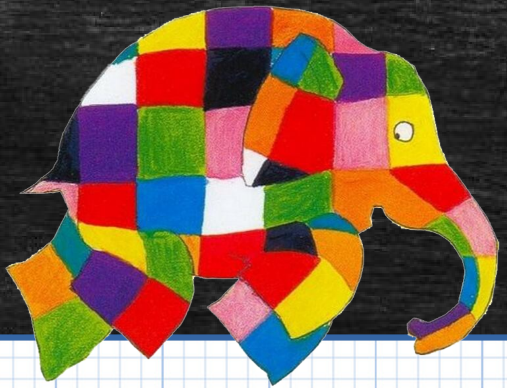
D=4



Conformal superalgebra GUT



D=4



Conformal superalgebra GUT

Punch line:

→ We embeded the SU(5) Georgi-Glashow in SU(2,2|10) USUSY

→ the model incudes gravity fields in the same gauge connection

things to improve:

PA, Chavez, Zanelli
hep-th/2211.12473

→ Resulting model is anomalous

→ Anomalies can be computed by traces of generators in the algebra but USUSY taught us that we should also learn how to compute them super-traces!

→ SO(10) model has more chances of being anomaly free

Current/Future developments

- generalization of the dual operator
- masses of psi, dark sectors? DM? cosmology with DE and DM (improvement to “induced gravity”)
- implementation of the coset idea to construct gauge invariant models in general
- SO(10) spin representation, anomalies?
- better understanding of the integrability conditions?

Current/Future developments

- generalization of the dual operator
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Hay becas y dinero, solo faltan valientes...

THANK YOU!

GUT/solutions

- PA, Chavez, Zanelli, J Mat Phys 63 (4) p. 042304; **2110.06828**
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- Phys. Lett. B 735 (2014) 314-321
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- JHEP 04 (2012) 058; **1109.3944**
- Class. Quant. Grav. 39 245007, **2208.07897**