Nonperturbative non Gaussianity from multifield inflation

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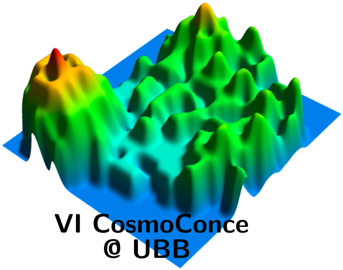
dfi, *fcfm*, UChile



VI CosmoConce
@ UBB

Based on

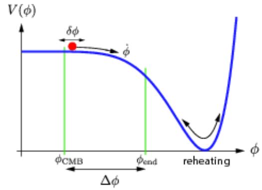
1712.XXXXX

in collaboration with:

Xingang Chen, Gonzalo Palma, Wálter Riquelme and Bruno Scheihing

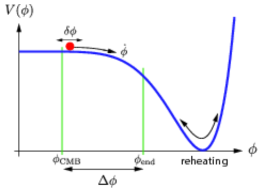
Outline

Scalar Perturbations in single field inflation



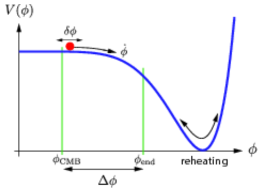
flat gauge: $g_{ij} = e^{2\rho}(\delta_{ij} + \gamma_{ij})$ and $\phi = \phi_0(t) + \delta\phi(x,t)$

Scalar Perturbations in single field inflation



unitary gauge: $g_{ij} = e^{2\rho+\zeta}(\delta_{ij} + \gamma_{ij})$ and $\phi = \phi_0(t)$

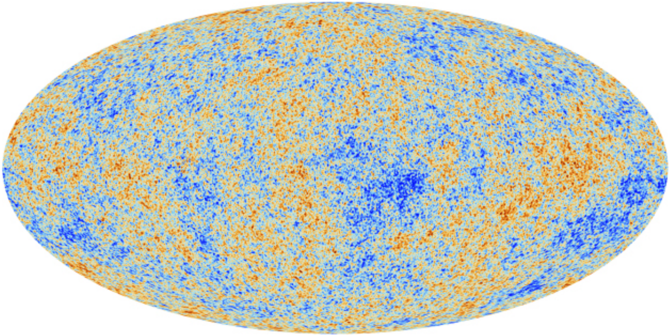
Scalar Perturbations in single field inflation



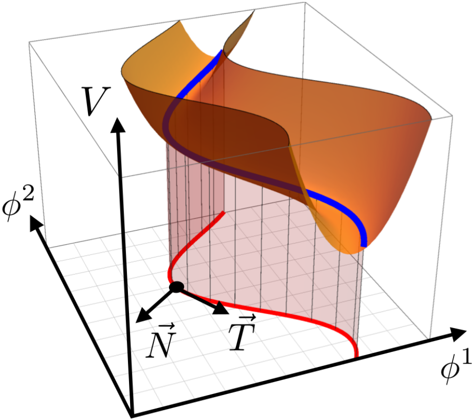
unitary gauge: $\boxed{g_{ij} = e^{2\rho+\zeta}(\delta_{ij} + \gamma_{ij}) \text{ and } \phi = \phi_0(t)}$

$$H\delta\phi \sim \zeta \sim H\delta T$$

ζ causes tiny $\mathcal{O}(10^{-5})$ temperature anisotropies in the CMB!
These small anisotropies also provide the seeds for large scale structure formation.



Scalar Perturbations in multifield inflation



Measuring the distribution of these small anisotropies in the CMB has been our main window into the very early Universe

⋆ Second moment (Power spectrum) **Measured!**

$$\langle \zeta\zeta \rangle \sim \frac{H^2}{M_{\rm Pl}^2 \epsilon}$$

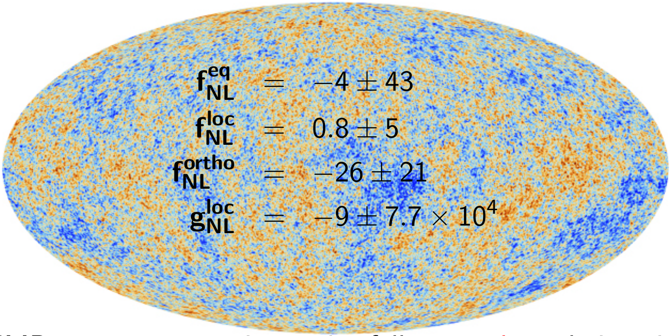
⋆ Third moment (Bispectrum) **Constrained!**

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3) \rangle \sim f_{NL}(\epsilon) S(k_1, k_2, k_3)$$

⋆ Fourth moment (Trispectrum) **Constrained!**

$$\langle \zeta(k_1)\zeta(k_2)\zeta(k_3)\zeta(k_4) \rangle \sim g_{NL}(\epsilon) S(k_1, k_2, k_3, k_4)$$

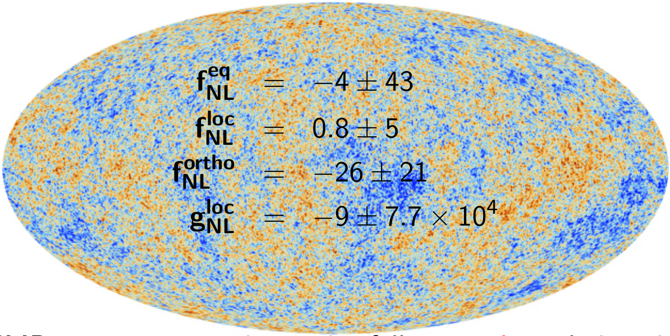
PLANCK 2015 @ 68% CL



$$
\begin{aligned}
f_{NL}^{eq} &= -4 \pm 43 \\
f_{NL}^{loc} &= 0.8 \pm 5 \\
f_{NL}^{ortho} &= -26 \pm 21 \\
g_{NL}^{loc} &= -9 \pm 7.7 \times 10^4
\end{aligned}
$$

⋆ CMB temperature anisotropies follow nearly scale invariant, almost Gaussian statistics (Consistent with ΛCDM)

Planck 2015 @ 68% CL



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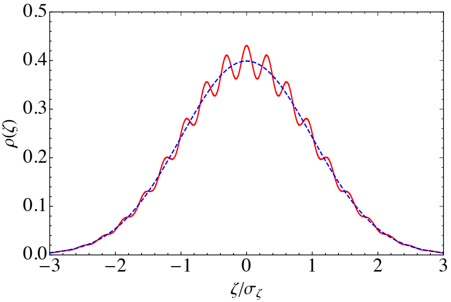
- ⋆ CMB temperature anisotropies follow nearly scale invariant, almost Gaussian statistics (Consistent with ΛCDM)
- ⋆ End of the story?

The PLANCK analysis is optimised for the search of nonzero moments of the PDF. What about this PDF for example?



It is clearly non Gaussian but has $f_{nl} = 0$
Can we get something like that?

Palma et al. '16

$$S = \int dx^4 a^3 \left[\epsilon\dot{\zeta}^2 - 2\epsilon\alpha\psi\dot{\zeta} - \epsilon a^{-2}(\nabla\zeta)^2 + \frac{1}{2}\left(\dot{\psi}^2 - a^{-2}(\nabla\psi)^2 - m^2\psi^2\right)\right]$$

Rewrite:

$$S = \int dx^4 a^3 \left[\epsilon\left(\dot{\zeta} - \alpha\psi\right)^2 - \epsilon a^{-2}(\nabla\zeta)^2 + \frac{1}{2}\left(\dot{\psi}^2 - a^{-2}(\nabla\psi)^2 - \mu^2\psi^2\right)\right]$$

where $\mu = \sqrt{m^2 + 2\epsilon\alpha^2}$ is the entropic mass.

- $\mu \gg H$ integrate out $\rightarrow c_s < 1$
- $\mu \sim H$ quasi-single field inflation
- we will be interested in the case where the field cannot be integrated out: $\mu \sim 0$

Ultralight ψ, $\mu = 0$

$$S = \int dx^4 a^3 \left[\epsilon\left(\dot{\zeta} - \alpha\psi\right)^2 - \epsilon a^{-2}(\nabla\zeta)^2 + \frac{1}{2}\left(\dot{\psi}^2 - a^{-2}(\nabla\psi)^2\right)\right]$$

Symmetries:

$$\zeta \to \zeta + C$$

$$\psi \to \psi + C_1 \quad \& \quad \dot{\zeta} \to \dot{\zeta} + \alpha C_1$$

This leads to

$$\zeta \simeq \psi_0 \frac{\alpha}{H}\Delta N + \zeta_0 \Rightarrow \boxed{P_\zeta = \frac{\alpha^2}{H^2}\Delta N^2 P_\psi}$$

Chen, Palma, Riquelme, Scheihing, SS '17

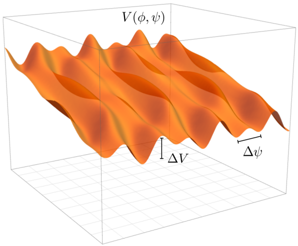
What if we add a potential instead of an entropic mass term?

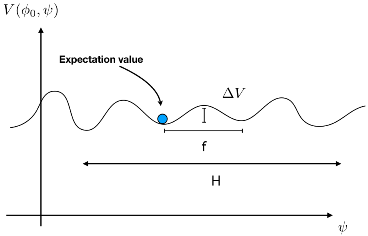
$$S = \int dx^4 a^3 \left[\epsilon \left(\dot{\zeta} - \alpha\psi \right)^2 - \epsilon a^{-2} (\nabla\zeta)^2 + \frac{1}{2} \left(\dot{\psi}^2 - a^{-2} (\nabla\psi)^2 - V(\psi) \right) \right]$$

Can we preserve the symmetries?

$$\boxed{V(\psi) = \Lambda^4 (1 - \cos(\psi/f))}$$

So by the same arguments we expect to transfer to ψ statistics to ζ. But now we have nonzero even n-point functions!


$V(\phi,\psi)$
ΔV
$\Delta\psi$


$V(\phi_0,\psi)$
Expectation value
ΔV
f
H
ψ

Palma, Riquelme '17

Decoupled dynamics $\alpha = 0$

$$S = \frac{1}{2}\int dx^4 a^3 \left[\dot{\psi}^2 - a^{-2}(\nabla\psi)^2 - \Lambda^4 \left(1 - \cos(\psi/f)\right)\right]$$

If we stay perturbative in Λ^4 the PDF for the axion field can be derived nonperturbatively in f (resummation of the cosine series).

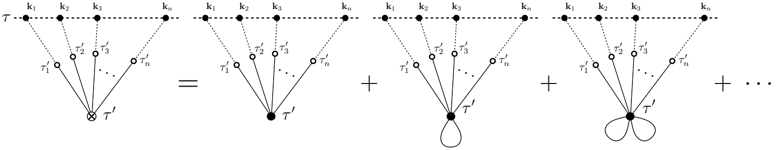
$$\rho(\psi) = \frac{e^{-\frac{\psi^2}{2\sigma_L^2}}}{\sqrt{2\pi}\sigma_L}\left[1 - A^2\left(\frac{\sigma_L^2 - \psi^2 - \sigma_L^4/f^2}{2\sigma_L^4}\right)\cos(\psi/f)\right]$$

where $\sigma_L^2 = \int_{k_L} \psi\psi^*$, so that $\langle\psi^n\rangle = \int d\psi \psi^n \rho(\psi)$.

We may now turn on the α coupling and see if we can perform the resummation and derive a PDF for the curvature perturbation. We write

$$1 - \cos(\psi/f) = -\sum_{m=1}^{\infty} \frac{(-1)^m}{(2m)!}(\psi/f)^{2m}$$

and compute the even n-point function using the in-in formalism.



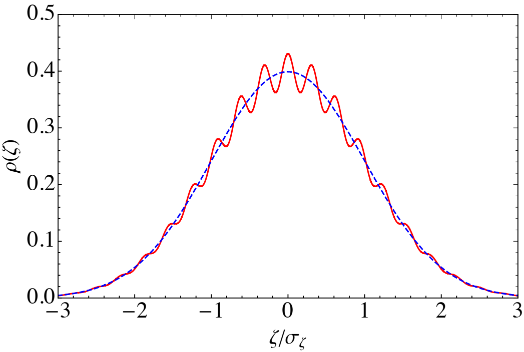
Loops can be resummed to $\exp(-\sigma^2/2f^2)$ as before.

Having the n-th moment for arbitrary n (the n-point correlator) one can compute the whole PDF for the curvature perturbation ζ as

$$\rho(\zeta) = \frac{e^{-\zeta^2/2\sigma_\zeta^2}}{\sqrt{2\pi}\sigma_\zeta}\left[1 - B^2\left(\frac{\sigma_\zeta^2 - \zeta^2 - \sigma_\zeta^4/f_\zeta^2}{2\sigma_\zeta^4}\right)\cos(\zeta/f_\zeta)\right]$$

where $\sigma_\zeta^2 = \alpha^2 \Delta N^2 \int_{k_L} \zeta\zeta^*$ and $B^2 = \frac{1}{6}\frac{\Lambda^4}{\alpha^2} e^{-\sigma_S^2/2f^2}$.

This PDF has a Gaussian part plus a non Gaussian correction. The Gaussian part comes out because we have only considered the cuadratic Lagrangian. This can be corrected in a perturbative manner by cubic + ... contributions.



An example of the PDF for the choice of parameters $f_\zeta/\sigma_\zeta = 5 \times 10^{-2}$ and $B^2/\sigma_\zeta^2 = 10^{-6}$ (red solid curve). For comparison, we have plotted a Gaussian distribution of variance σ_ζ (blue dashed curve).

We have considered an axion potential. A general periodic potential can be expanded as

$$V(\psi) = \Lambda^4 \sum_n [A_n \sin(\psi/f_n) + B_n \cos(\psi/f_n)]$$

The sine part can be resummed into an exponential too!
We could have a more general example of an imprint of the landscape on the CMB.
What about a completely generic nonperiodic potential? (recall that we are not talking about the inflaton potential...)

Buchert, France and Steiner '17

Recently Buchert, France and Steiner searched for a non Gaussian PDF in the Planck 2015 data claiming a clear signature at 2σ.
Discrepancy functions:

$$\Delta_P(\tau) = \frac{P(\tau) - P^G(\tau)}{P^G(0)}$$

Non Gaussianity coefficients:

$$\Delta_P(\tau) = e^{-\tau^2/2\sigma_0} \sum_n \frac{a_P(n)}{n!} \mathrm{He}_n\left(\frac{\tau}{\sigma_0}\right)$$

The a_P's characterise skewness, kurtosis etc.

We have shown in a concrete example involving stringy axions how the PDF of the curvature fluctuation can inherit a rich non Gaussian structure. This kind of non Gaussianity has not been fully constrained!

To do:

- ⋆ What about a nonperiodic landscape?
- ⋆ Test such a PDF with the PLANCK data (a la Buchert et al.)
- ⋆ How would this affect structure formation?
  - ⋆ Simulate LSS with this initial condition?

¡Gracias!