Does the Universe behave as a normal thermodynamic system?

Diego Pavón

Autonomous University of Barcelona and PUCV

In collaboration with Pedro C. Ferreira, Ninfa Radicella and José P. Mimoso

University of the Bio-Bio, Chile

Plan of the talk

1.- Connection gravitation-thermodynamics.

2.- The second law and the generalized second law.

3.- Cosmological apparent horizon.

4.- Is the Universe an ordinary thermodynamic system?

5.- Fluctuations of the fluxes on the apparent horizon.

6.- Conclusions.

Connection Gravitation-Thermodynamics 1

Equivalence principle at work:

Tolman's law: $T\sqrt{-g_{tt}} = \text{constant}$ (Tolman, 1934).

Heat flow through an accelerate body (Eckart, 1940).

Gravothermal catastrophe (Antonov, 1962) $\rightarrow$ Newtonian versus Einstein Gravity.

$\Rightarrow$ Entropy cannot become arbitrarily large.

Newtonian gravity is inconsistent with thermodynamics.

Connection Gravitation-Thermodynamics 2

Black holes $\rightarrow T_{bh} = 0$ versus $T_{bh} \propto 1/M$, $\quad S_{bh} \propto \mathcal{A}_{bh}$.

(Bekenstein, 1974, 1975; Hawking, 1975).

Cosmological event horizons possess $S \propto \mathcal{A}_{eh}$, $\quad T_{eh} \propto \kappa$
(Gibbons and Hawking, 1977).

Black holes and cosmological horizons possess entropy and temperature. Both behave thermodynamically in spite of their negative heat capacity.

The second law

Daily experience teaches us that macroscopic systems tend spontaneously to thermodynamical equilibrium. This constitutes the empirical basis of the second law of thermodynamics. The latter succinctly formalizes this by establishing that the entropy, S, of isolated systems never decreases, $S' \geq 0$, and it is concave $S'' < 0$, at least in the last leg of approaching equilibrium.

Thus far, all attempts to disprove this law by means of “counterexamples" have failed.

Beware of Mutilated versions of the law!!

In what follows, we examine whether the universe obeys this law.

The Generalised Second Law (GSL)

BH + its environment (Unruh & Wald, 1982; Bekenstein, 1976).

Event horizon + its environment (Davies, 1987; Pavón, 1990).

$$\dot{S}_{bh,\mathcal{A}} + \dot{S}_{surroundings} \geq 0$$

Cosmological apparent horizon

"Boundary surface of an antitrapped region with topology of a two-sphere"
(Bak & Rey, CQG (2000))

$$\mathrm{d}s^2 = h_{ab}\mathrm{d}x^a\,\mathrm{d}x^b + \tilde{r}^2(x)\,\mathrm{d}\Omega^2$$

$$\tilde{r} = a(t)r\,, \qquad h_{ab} = \mathrm{diag}\left[-1, \frac{a^2}{1-kr^2}\right] \qquad (k = +1, 0, -1)$$

$$h^{ab}\,\partial_a\tilde{r}\,\partial_b\tilde{r} = 0 \;\Rightarrow\; \tilde{r}_{\mathcal{A}} = \frac{1}{\sqrt{H^2 + ka^{-2}}}$$

$$\theta_{\rm in} = H - \frac{1}{\tilde{r}}\sqrt{1 - \frac{k\tilde{r}^2}{a^2}}\,, \qquad \theta_{\rm out} = H + \frac{1}{\tilde{r}}\sqrt{1 - \frac{k\tilde{r}^2}{a^2}}$$

Area and entropy

Antitrapped region

$$\tilde{r} > 1/\sqrt{H^2 + k\, a^{-2}}$$

Area & entropy

$$\mathcal{A} = 4\pi \tilde{r}_{\mathcal{A}}^2 = \frac{4\pi}{H^2 + \frac{k}{a^2}}$$

$$S_{\mathcal{A}} = k_B \frac{\mathcal{A}}{4\,\ell_{pl}^2}$$

At present the entropy of the horizon exceeds that of SMBHs, SBHs, relic neutrinos, and CMB photons, by 18, 25, 33, and 33 orders of magnitude, respectively (Egan et al., Ap. J. (2010)).

Area derivatives 1

$$\mathcal{A} = 4\pi/(H^2 + ka^{-2}) = \frac{3}{2\,G\,\rho}$$

$$\mathcal{A}' = -\frac{3}{2G}\frac{\rho'}{\rho^2} = 2G\mathcal{A}^2\left(\frac{\rho+p}{a}\right)$$

$$A' \geq 0 \;\Rightarrow\; \rho + p \geq 0$$

If the source of the gravitational field does not comply with the DEC, the area decreases regardless the universe is expanding, H > 0, or contracting, H < 0.

Area derivatives 2

$$\mathcal{A}'' = 2G \left(\frac{\mathcal{A}}{a}\right)^2 \left\{ \frac{6}{\rho}(\rho + p)^2 \; - \; 4(\rho + p) \; + \; ap' \right\}$$

Both, in radiation dominated and dust dominated universes,

$$\mathcal{A}'' \propto \left(\frac{\mathcal{A}}{a}\right)^2 > 0$$

hence, a state of equilibrium (maximum entropy) is not attainable regardless of whether the universe is expanding or contracting.

NEED of AN ENERGY COMPONENT with $-1 \leq \frac{p}{\rho} \leq -\frac{2}{3}$
i.e., DARK ENERGY

Area derivatives 3

For a universe dominated by pressureless matter and dark energy

$$w = \frac{p_m + p_{de}}{\rho_m + \rho_{de}}$$

$$\Rightarrow \quad \mathcal{A}'' = 2G\left(\frac{\mathcal{A}}{a}\right)^2 (1+w)(6+3w-4)\,\rho$$

Accordingly, if

$$1+w \geq 0 \quad \Rightarrow \quad \mathcal{A}'' \geq 0 \;\text{ for }\; w \geq -2/3 \;\;\&\;\; \mathcal{A}'' \leq 0 \;\text{ for }\; -1 < w \leq -2/3$$

and if

$$1 + w < 0 \quad \Rightarrow \quad \mathcal{A}'' > 0$$

Is the universe an ordinary thermodynamic system? 1

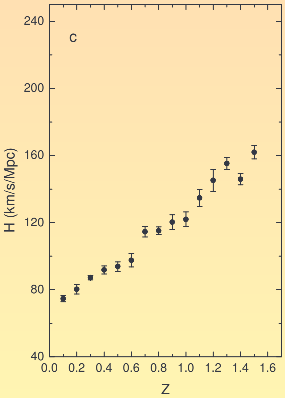

Figure: A Montecarlo realization of 1000 simulated values of the Hubble parameter with a precision of 1% (Carvalho & Alcaniz, 2011).

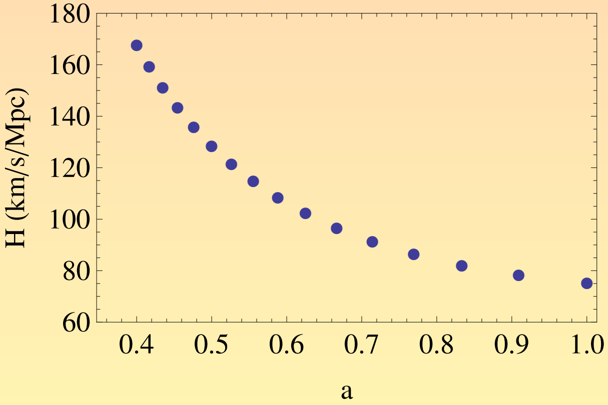

Figure: Projected history of the Hubble factor based on Montecarlo simulations on recent H(z) data (Carvalho & Alcaniz, 2011). Clearly, $H' < 0$ and $H'' > 0$.

Is the universe an ordinary thermodynamic system? 3

Assume , for simplicity, $k = 0$; then, the restrictions

$$H'(a) < 0\,, \quad \text{and} \quad H''(a) > 0$$

are fully consistent with severe observational constraints from the matter power spectrum and the CMB (after the decoupling era) on previous periods of acceleration-deceleration (Linder, PRD 2010 & 2011). Then,

$$\mathcal{A}' \propto -\frac{2H'}{H^3} > 0.$$

But what about the sign of

$$\mathcal{A}'' \propto \frac{2}{H^2}\left[3\left(\frac{H'}{H}\right)^2 - \frac{H''}{H}\right] ?$$

Is the universe an ordinary thermodynamic system? 4

Note that

$$\mathcal{A}'' < 0 \quad \Leftrightarrow \quad 3\left(\frac{H'}{H}\right)^2 < \frac{H''}{H}$$

Assuming the restrictions $H' < 0$ and $H'' > 0$ to hold also when $a > 1$, and $q = -[(1 + (aH'/H)] < 0$ at late times, it follows that

$$\mathcal{A}'' < 0$$

The set of points in last Figure can be roughly approximated by each of the two functions

$$H = H_* \exp(\lambda/a) \qquad \text{and} \qquad H = H_* (1 + \lambda\, a^{-n})$$

which fulfill the said restrictions, and accordingly $\mathcal{A}'' < 0$ at late times.

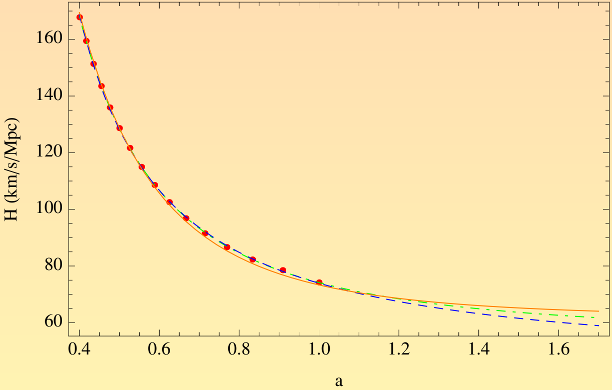

Figure: Hubble expansion rate vs. scale factor. The dashed, dot-dashed, and solid lines represent the best fits to the set of points of the functions of above and the flat ΛCDM, respectively.

Best fit values (at 95% CL) for:

$$H = H_* \exp(\lambda/a)$$

$$H_* = 42.6 \pm 0.4\,\mathrm{Km/s/Mpc}\,, \qquad \lambda = 0.550 \pm 0.005$$

$$H = H_* \left(1 + \lambda\, a^{-n}\right)$$

$$H_* = 54.78 \pm 0.06\,\mathrm{Km/s/Mpc}\,, \quad \lambda = 0.3535 \pm 0.0013\,,\, n = 1.928 \pm 0.002$$

$$H = H_0 \sqrt{\Omega_{m0}\, a^{-3} + (1 - \Omega_{m0})}$$

$$H_0 = 73.3 \pm 1.2\,\mathrm{Km/s/Mpc}\,, \quad \Omega_{m0} = 0.29 \pm 0.2$$

For completeness ...

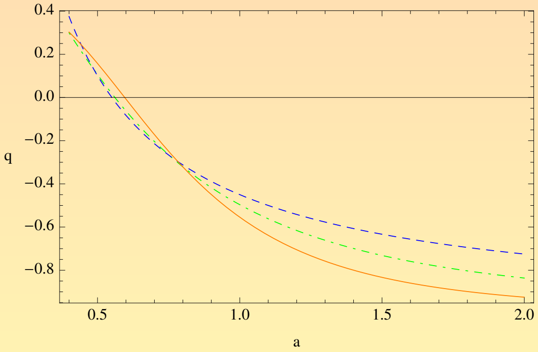

Figure: Deceleration parameter vs. scale factor. The dashed, dot-dashed, and solid lines represent the best fits to the set of points of the functions of above and the flat ΛCDM, respectively.

(Pavon and Radicella, 2013).

Note that not all Hubble functions that fulfill the observational restrictions $H' < 0$ and $H'' > 0$ comply with the inequality

$$3\left(\frac{H'}{H}\right)^2 < \frac{H''}{H}$$

This is, for instance, the case of the expansion laws

$$H = H_* \left[\exp\left(\lambda\, a^{-1}\right) - 1\right] \quad \text{and} \quad H = H_* \, \exp\left(-\lambda\, a\right)$$

(with $\lambda > 0$). The entropy of a universe that obeyed any of these two laws would increase without bound in the long run, similarly to the entropy of Antonov's sphere in Newtonian gravity. Note, however, that the said functions do not correspond to realistic universes. In the first case the universe never accelerates; in the second one the universe accelerates at early times (when $a < 1/\lambda$) to decelerate for ever afterwards.

What if $k \neq 0$?

If $\mathcal{A}' \geq 0 \quad \Rightarrow \quad HH' \leq \frac{k}{a^2} \quad \Rightarrow H\frac{\mathrm{d}H}{\mathrm{d}z} \geq -k(1+z)$

The latter two inequalities are trivially fulfilled for $k = 1$ and $k = 0$ so long as H is positive and does not decrease with redshift (which is reasonably supported by observation).

For $k = -1 \quad \Rightarrow \quad H\frac{\mathrm{d}H}{\mathrm{d}z} \geq 1 + z$

Integrating,

$$H_2^2 - H_1^2 \geq 2(z_2 - z_1) + (z_2^2 - z_1^2) \qquad (z_2 > z_1)$$

Is the universe an ordinary thermodynamic system? 10

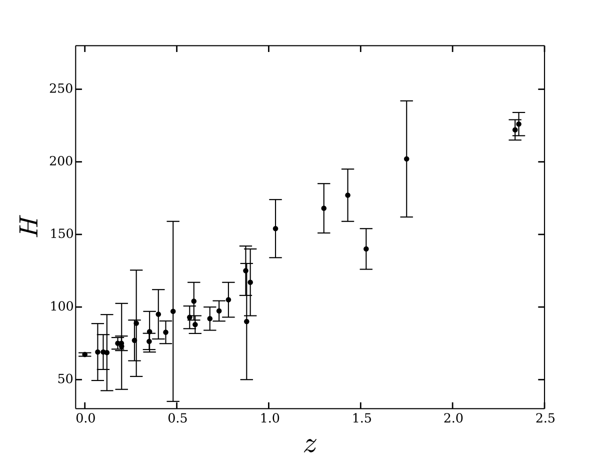

Figure: 29 data points with their 1σ error bars.

Is the universe an ordinary thermodynamic system? 11

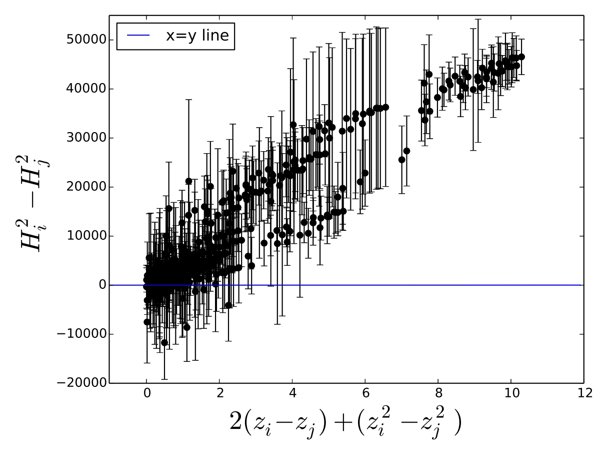

Figure: Within statistical errors, $H_i^2 - H_j^2 \geq 2(z_i - z_j) + (z_i^2 - z_j^2)$ for all pairs (H_{z_i}, H_{z_j}) with $i > j$.

Is the universe an ordinary thermodynamic system? 12

Landau-Lifshitz method for thermodynamical fluctuations

$$\dot{y}_i = \Sigma_j \Gamma_{ij} Y_j \, + \, \delta \dot{y}_i$$

$$\dot{S} = \Sigma_i (\pm Y_i \dot{y}_i)\,, \quad \text{Exemple}: \ \dot{S} = \vec{q} \cdot (-\nabla(1/T))$$

where

$$Y_i = (\partial \dot{S} / \partial \dot{y}_i)$$

$$< \delta \dot{y}_i \delta \dot{y}_j >= (\Gamma_{ij} \, + \, \Gamma_{ji}) \delta_{ij} \, \delta(t_i \, - \, t_j)$$

Obviously,

$$< \delta \dot{y}_i >= 0$$

Is the universe an ordinary thermodynamic system? 13

Energy flux through the apparent horizon

$$-\dot{E} = \mathcal{A}_H(\rho + p)H\,\tilde{r}_H = -\frac{\mathcal{A}_H}{4\pi G}\left(\dot{H} - \frac{k}{a^2}\right)\frac{H}{\sqrt{H^2 + ka^{-2}}} + \delta\dot{E}$$

$$\dot{S}_H = \frac{1}{4\ell_P^2}\,\dot{\mathcal{A}}_H = 2\pi\,G\,\frac{\mathcal{A}_H^2}{\ell_P^2}\,H\,(\rho + p)$$

$$< (\delta(-\dot{E}))^2 >= 2\Gamma\, \delta(t) = \frac{3\,\ell_P^2}{8\,\pi^2 G^2}\, H\, \frac{\rho + p}{\rho}\, \delta(t)$$

The fact that

$$< (\delta(-\dot{E}))^2 >$$

increases with

$$H$$

tells us the strength of the fluctuations grow with the horizon temperature and decrease with area of the horizon. It parallels the behavior of the fluctuations in systems absent of gravity.

Conclusions 1

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- Cosmological models worth of consideration must fulfill the laws of thermodynamics.
- Neither radiation dominated nor CDM dominated universes tend to thermodynamic equilibrium in the long run, by contrast, the ΛCDM model does.
- Phantom models do not.
- The entropy of the Universe seems to approach some maximum value (of the order of H^{-2} when $a \to \infty$), but in order to reach a firmer conclusion, greater accuracy in H(z) measurements should be achieved.

Conclusions 2

- General Relativity seems compatible with thermodynamics also at large scales.

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- General Relativity seems compatible with thermodynamics also at large scales.
- The existence of dark energy could have been anticipated prior to its discovery.
- The fluctuations of the flux of energy through the apparent horizon show clear similarities with the typical fluctuations in which gravity does not play any main role. This strengthens our belief that the universe behaves as an ordinary thermodynamic system.

Pressureless matter yielding to dark energy