Thin-shell Wormholes/Gravastars

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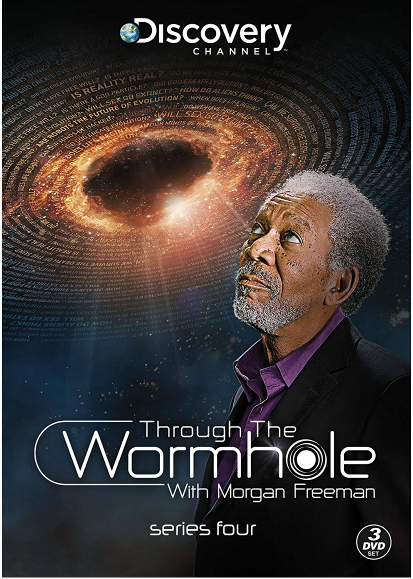


Figure 1: Can we make journeys to farther stars?


Time Travel Through
Wormhole
NZSA

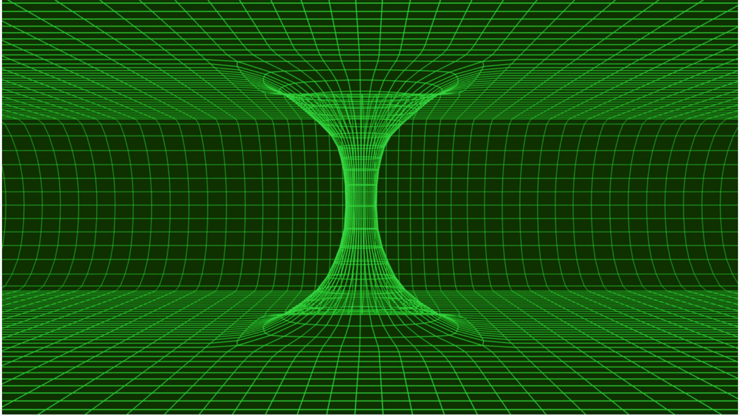


Figure 2: How can we open gate into space-time?

- How can we connect two regions of space-time?
- Can we make stable and traversable wormholes?

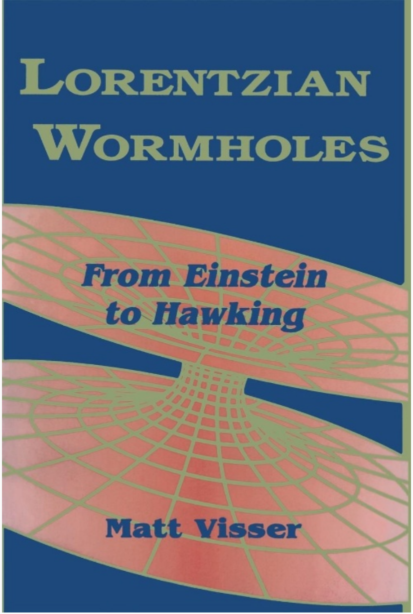


Figure 3: Published in 1995

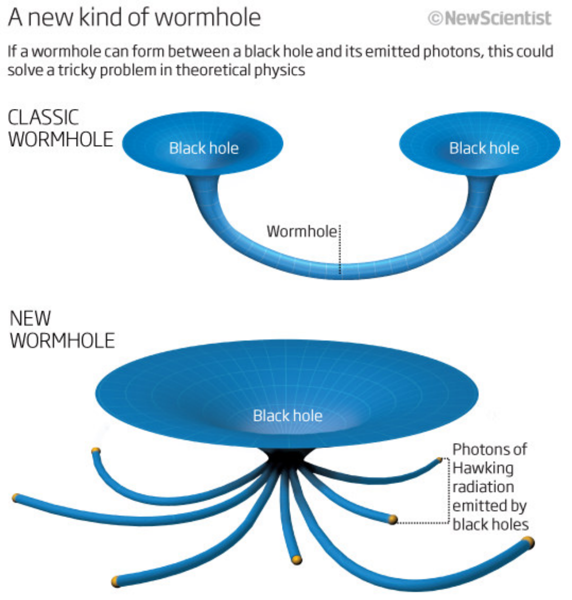


Figure 4: ER=EPR

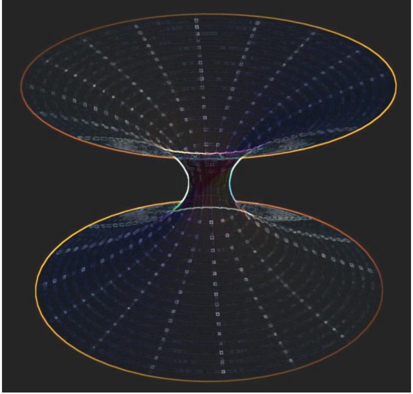


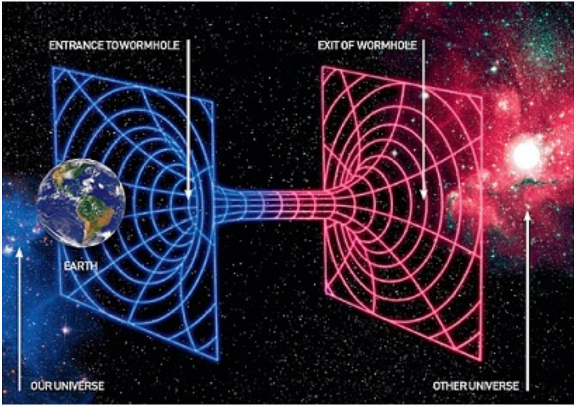
Figure 5: Wormhole

-We do not know how to open the throat without exotic matter.
-Thin-shell methods with Israel junction conditions can be used to minimize the exotic matter needed.

However, the stability must be saved.

Figure 6: How to realize Wormholes in real life

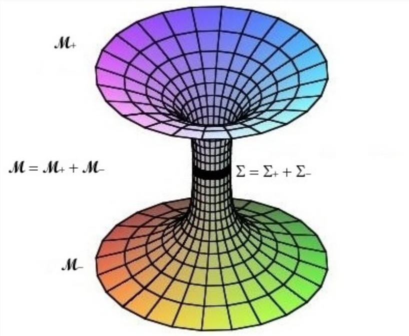
THIN-SHELL WORMHOLES

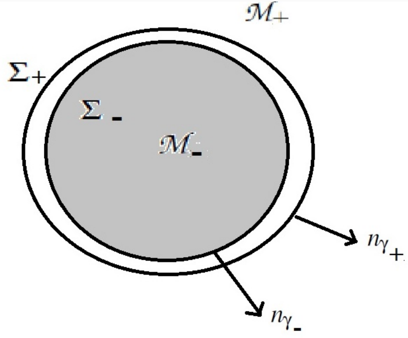


- Constructing WHs with non-exotic (normal matter) source is a difficult issue in General Relativity.
- First, Visser use the thin-shell method to construct WHs by minimizing the exotic matter on the throat of the WHs.

Formalism

- Using the cut-and-paste technique of Visser (**0809.0907, 1112.2057, 0809.0927, 9506083**),
- we take two copies of the space-time and remove from each manifold the four- dimensional regions which contain event horizons.
- single manifold $\mathcal{M}$ is obtained by gluing together two distinct spacetime manifolds, $\mathcal{M}_+$ and $\mathcal{M}_-$, at their boundaries $\Sigma = \Sigma_+ = \Sigma_-$.





- We consider two generic static spherically symmetric spacetimes given by the following line elements:

$$ds^2_{\pm} = -F(r)_{\pm} dt^2 + \frac{1}{G(r)_{\pm}} dr^2_{\pm} + r^2_{\pm} d\Omega^2_{\pm}. \tag{1}$$

with

$$d\Omega^2_{\pm} = d\theta^2_{\pm} + \sin^2 \theta_{\pm} d\phi^2_{\pm}. \tag{2}$$

- Remove from each spacetime the region described by

$$\Sigma_{\pm} \equiv \{r_{\pm} \leq a | \, a > r_h\} \ , \tag{3}$$

where a is a constant and r_h is the black hole event horizon.

- The removal of the regions results in two geodesically incomplete manifolds, with boundaries given by the following timelike hypersurfaces

$$\partial\Omega_{1,2} \equiv \{r_{1,2} = a | \, a > r_b\} \ . \tag{4}$$

- Identifying these two timelike hypersurfaces, $\partial\Omega_1 = \partial\Omega_2$,
- The intrinsic metric to Σ is obtained as

$$ds^2_{\Sigma} = -d\tau^2 + a(\tau)^2 \left(d\theta^2 + \sin^2 \theta \, d\phi^2\right). \tag{5}$$

- Use the Darmois-Israel formalism to determine the surface stresses at the junction boundary.
- The intrinsic surface stress-energy tensor, S_{ij}, is given by the Lanczos equations: $S^i{}_j = -\frac{1}{8\pi}\left(\kappa^i{}_j - \delta^i{}_j \kappa^k{}_k\right)$
- The discontinuity in the second fundamental form or extrinsic curvatures is given by $\kappa_{ij} = K^+_{ij} - K^-_{ij}$.
- Setting coordinates $\xi^i = (\tau, \theta)$, the extrinsic curvature formula connecting the two sides of the shell is simply given by

$$K^{\pm}_{ij} = -n^{\pm}_{\gamma}\left(\frac{\partial^2 x^{\gamma}}{\partial\xi^i\partial\xi^j} + \Gamma^{\gamma}_{\alpha\beta}\frac{\partial x^{\alpha}}{\partial\xi^i}\frac{\partial x^{\beta}}{\partial\xi^j}\right), \tag{6}$$

where the unit 4-normal to $\partial\Omega$ are

$$n^{\pm}_{\gamma} = \pm\left|g^{\alpha\beta}\frac{\partial H}{\partial x^{\alpha}}\frac{\partial H}{\partial x^{\beta}}\right|^{-1/2}\frac{\partial H}{\partial x^{\gamma}}, \tag{7}$$

with $n^{\mu}\, n_{\mu} = +1$ and $H(r) = r - a(\tau)$.

$$K^{\tau}{}_{\tau}{}^{(\pm)} = \pm \frac{\dot{a}^2 F G' - \dot{a}^2 G F' - 2FG\ddot{a} - G^2 F'}{2F\sqrt{FG(\dot{a}^2 + G)}}, \tag{8}$$

$$K^{\theta}{}_{\theta}{}^{(\pm)} = K^{\phi}{}_{\phi}{}^{(\pm)} = \mp \frac{\sqrt{FG\,(\dot{a}^2 + G)}}{aF}. \tag{9}$$

- Use surface stress-energy tensors to find the surface energy density, σ, and the surface pressure, p, as

$$S^{i}{}_{j} = \mathrm{diag}(-\sigma, p, p)\,,$$

which taking into account the Lanczos equations, reduce to

$$\sigma = -\frac{1}{4\pi}\kappa^{\theta}{}_{\theta}\,, \tag{10}$$

$$p = \frac{1}{8\pi}(\kappa^{\tau}{}_{\tau} + \kappa^{\theta}{}_{\theta})\,. \tag{11}$$

- The energy and pressure densities are obtained as

$$\sigma = -\frac{1}{2}\frac{\sqrt{FG\,(\dot{a}^2 + G)}}{\pi F a}, \tag{12}$$

and the surface pressure p

$$p = \frac{1}{8}\frac{a(aGF'\dot{a}^2 - aFG'\dot{a}^2 + 2\,aGF\ddot{a} + aF'G^2 + 2\,GF\dot{a}^2 + 2\,FG^2)}{\sqrt{FG\dot{a}^2 + G}F\pi}. \tag{13}$$

The above equations imply the conservation of the surface stress-energy tensor

$$\dot{\sigma} = -2\,(\sigma + p)\,\frac{\dot{a}}{a} \tag{14}$$

or

$$\frac{d\,(\sigma A)}{d\tau} + p\,\frac{dA}{d\tau} = 0\,, \tag{15}$$

where $A = 4\pi a^2$ is the area of the wormhole throat. The first term represents the variation of the internal energy of the throat, and the second term is the work done by the throat's internal forces.

Linearized stability analysis Then they reduce to simple form in a static configuration ($a = a_0$)

$$\sigma_0 = -\frac{4}{a_0}\sqrt{f(a_0)} \tag{16}$$

and

$$p_0 = 2\left(\frac{\sqrt{f(a_0)}}{a_0} + \frac{f'(a_0)/2}{\sqrt{f(a_0)}}\right). \tag{17}$$

Once $\sigma \geq 0$ and $\sigma + p \geq 0$ hold, then WEC is satisfied.

- It is obvious hat negative energy density violates the WEC, and consequently we are in need of the exotic matter for constructing thin-shell WH.
- For static solution (at a_0) to the equation of motion $\frac{1}{2}\dot{a}^2 + V(a) = 0$, and so also a solution of $\ddot{a} = -V'(a)$.

$$V(a) = \frac{4F\pi^2 a^2\sigma^2 - G^2}{G}. \tag{18}$$

- Generally a Taylor expansion of $V(a)$ around a_0 to second order yields with $\dot{a}_0 = \ddot{a}_0 = 0$, $V(a_0) = V'(a_0) = 0$,

$$V(a) = \frac{1}{2} V''(a_0)(a - a_0)^2 + O[(a - a_0)^3] \,. \tag{19}$$

- If $V''(a_0) < 0$ is verified, then the potential $V(a_0)$ has a local maximum at a_0, where a small perturbation in the wormhole throat's radius will provoke an irreversible contraction or expansion of the throat.
- Thus, the solution is stable if and only if $V(a_0)$ has a local minimum at a_0 and $V''(a_0) > 0$
- Stability of such a WH is investigated by applying a linear perturbation with the following EoS

$$p = \psi(\sigma) \tag{20}$$

APPLICATIONS: HAYWARD THIN-SHELL WH IN 3+1-D

A.Ovgun M.Halilsoy and S.H.Mazharimousavi ***1312.6665***

- *The metric of the Hayward BH is given by*

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2. \tag{21}$$

with the metric function

$$f(r) = \left(1 - \frac{2mr^2}{r^3 + 2ml^2}\right) \tag{22}$$

SOME MODELS OF EXOTIC MATTER SUPPORTING THE TSW

Linear gas

For a LG, EoS is choosen as

$$\psi = \eta_0 (\sigma - \sigma_0) + p_0 \tag{23}$$

in which η_0 is a constant.

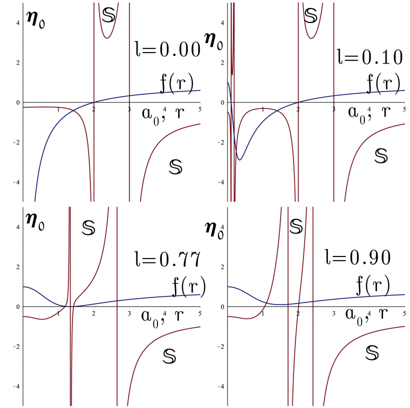


Figure 7: Stability of Thin-Shell WH supported by LG.

Stability of TSW supported by LG in terms of a_0 and η_0. The value of $m = 1$. The effect of Hayward's constant is to increase the stability of the TSW.

Chaplygin gas

$$\psi = \eta_0 \left(\frac{1}{\sigma} - \frac{1}{\sigma_0} \right) + p_0 \tag{24}$$

where η_0 is a constant.

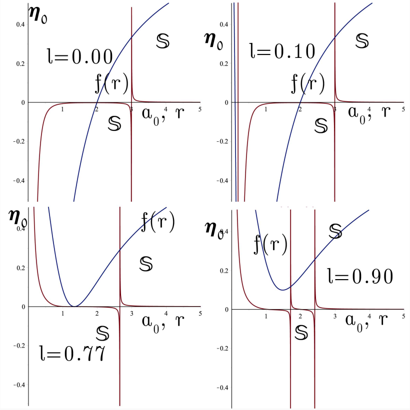


Figure 8: Stability of Thin-Shell WH supported by CG.

Modified Generalized Chaplygin gas

$$\psi(\sigma) = \xi_0 (\sigma - \sigma_0) - \eta_0 \left(\frac{1}{\sigma^\nu} - \frac{1}{\sigma_0^\nu} \right) + p_0 \qquad (25)$$

in which ξ_0, η_0 and ν are free parameters.

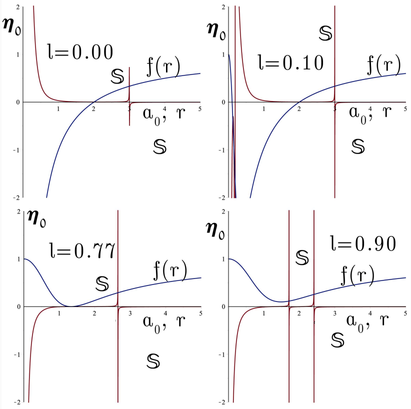


Figure 9: Stability of Thin-Shell WH supported by MGCG.

Logarithmic gas

$$\psi(\sigma) = \eta_0 \ln\left|\frac{\sigma}{\sigma_0}\right| + p_0 \tag{26}$$

in which η_0 is a constant.

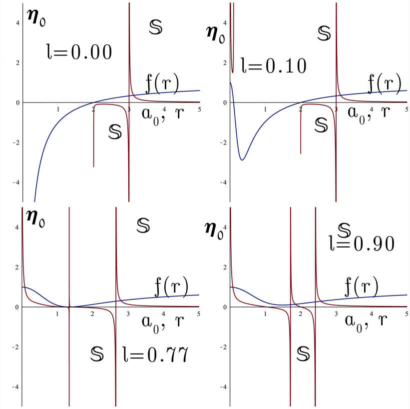


Figure 10: Stability of Thin-Shell WH supported by LogG.

Conclusions

- On the thin-shell we use the different type of EoS with the form $p = \psi(\sigma)$ and plot possible stable regions.
- We show the stable and unstable regions on the plots.
- Stability simply depends on the condition of $V''(a_0) > 0$.
- We show that the parameter ℓ, which is known as Hayward parameter has a important role.
- Moreover, for higher ℓ value the stable regions are increased.
- Hence, energy density of the WH is found negative so that we need exotic matter.

APPLICATION 2: ROTATING THIN-SHELL WORMHOLE

A. Ovgun, ***1604.08477***

The 5-d rotating Myers-Perry (5DRMP) black hole solution:

$$ds^2 = -F(r)^2 dt^2 + G(r)^2 dr^2 + r^2 \widehat{g}_{ab} dx^a dx^b + H(r)^2 \left[d\psi + B_a dx^a - K(r) dt \right]^2 , \tag{27}$$

in which

$$G(r)^2 = \left(1 + \frac{r^2}{\ell^2} - \frac{2M\Xi}{r^2} + \frac{2Ma^2}{r^4} \right)^{-1} , \tag{28}$$

$$H(r)^2 = r^2 \left(1 + \frac{2Ma^2}{r^4} \right) , \qquad K(r) = \frac{2Ma}{r^2 H(r)^2} , \tag{29}$$

$$F(r) = \frac{r}{G(r)H(r)} , \qquad \Xi = 1 - \frac{a^2}{\ell^2} , \tag{30}$$

where $B = B_a dx^a$ *and*

$$\widehat{g}_{ab} dx^a dx^b = \frac{1}{4} \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) , \quad B = \frac{1}{2} \cos\theta \, d\phi . \tag{31}$$

We move to comoving frame to eliminate cross terms in the induced metrics by introducing

$$d\psi \longrightarrow d\psi' + K_{\pm}(\mathcal{R}(t))dt\,. \tag{32}$$

The line element in the interior and the exterior sides become

$$ds^2_{\pm} \;\; = \;\; -F_{\pm}(r)^2 dt^2 + G_{\pm}(r)^2 dr^2 + r^2 d\Omega + H_{\pm}(r)^2 \{d\psi' + B_a dx^a + [K_{\pm}(\mathcal{R}(t))$$

For simplicity in the comoving frame, we drop the prime on ψ'.

$$\rho \;\; = \;\; -\frac{\beta(\mathcal{R}^2 H)'}{4\pi\,\mathcal{R}^3}\,,\; \varphi = -\frac{\mathcal{J}(\mathcal{R}H)'}{2\pi^2\mathcal{R}^4 H}\,, \tag{34}$$

$$P \;\; = \;\; \frac{H}{4\pi\mathcal{R}^3}\left[\mathcal{R}^2\beta\right]',\; \Delta P = \frac{\beta}{4\pi}\left[\frac{H}{\mathcal{R}}\right]', \tag{35}$$

where primes stand for $d/d\mathcal{R}$ *and*

$$\beta \equiv F(\mathcal{R})\sqrt{1 + G(\mathcal{R})^2\dot{\mathcal{R}}^2}\,. \tag{36}$$

Without rotation or in the case of a corotating frame, the momentum φ *and the anisotropic pressure term* ΔP *are equal to zero.*

Note that $a = \omega = 0$ corresponds to a non-rotating case.

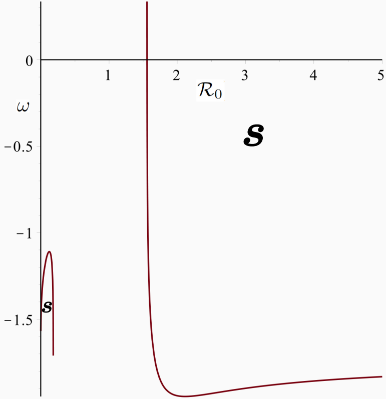


Figure 11: *Stability of wormhole supported by linear gas in terms of ω and R_0 for $a = 0.1$.*

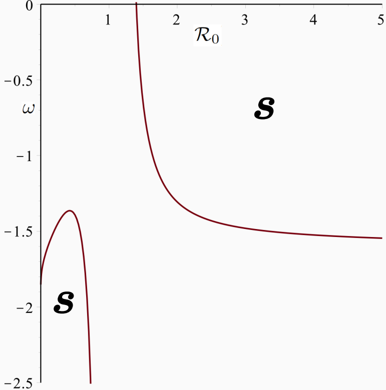


Figure 12: *Stability of wormhole supported by linear gas in terms of* ω *and* R_0 *for* $a = 0.4$.

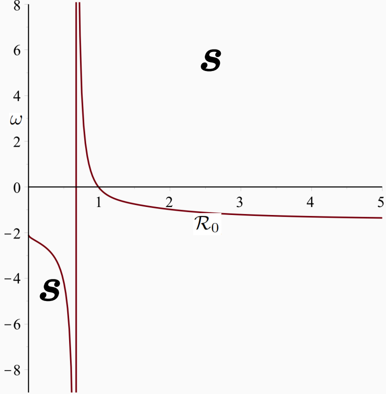


Figure 13: *Stability of wormhole supported by linear gas in terms of* ω *and* R_0 *for* $a = 1$.

Thin-shell Gravastars by M. Visser et al. 0310107, 1112.5253, 1512.07659

Exterior Of Gravastars: Noncommutative geometry inspired Charged BHs *A.Ovgun, A.Banerjee and K.Jusufi* ***1704.00603***

- *The metric of a noncommutative charged black hole is described by the metric given in S. Ansoldi et al. (0612035):*

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2, \tag{37}$$

with $f(r) = \left(1 - \frac{2M_\theta}{r} + \frac{Q_\theta^2}{r^2}\right)$.

STRUCTURE EQUATIONS OF CHARGED GRAVASTARS

- *The metrics of interior is the nonsingular de Sitter spacetimes:*

$$ds^2 = -(1 - \frac{r_-^2}{\alpha^2})dt_-^2 + (1 - \frac{r_-^2}{\alpha^2})^{-1}dr_-^2 + r_-^2 d\Omega_-^2 \tag{38}$$

and exterior of noncommutative geometry inspired charged spacetimes:

$$ds^2 = -f(r)_+ dt_+^2 + f(r)_+^{-1}dr_+^2 + r_+^2 d\Omega_+^2. \tag{39}$$

- *Then using the relation of* $S^i{}_j = \mathrm{diag}(-\sigma, P, P)$*, one can find the surface energy density,* σ*, and the surface pressure,* P*, as follows:*

$$\sigma = -\frac{1}{4\pi a}\left[\sqrt{1-\frac{2M_{\theta+}}{a}+\frac{Q_{\theta+}^2}{a^2}+\dot{a}^2}-\sqrt{\left(1-\frac{a^2}{\alpha^2}\right)+\dot{a}^2}\right], \tag{40}$$

$$\mathcal{P} = \frac{1}{8\pi a}\left[\frac{1+\dot{a}^2+a\ddot{a}-\frac{M_{\theta+}}{a}}{\sqrt{1-\frac{2M_{\theta+}}{a}+\frac{Q_{\theta+}^2}{a^2}+\dot{a}^2}}-\frac{\left(1+a\ddot{a}+\dot{a}^2-\frac{2a^2}{\alpha^2}\right)}{\sqrt{\left(1-\frac{a^2}{\alpha^2}\right)+\dot{a}^2}}\right]. \tag{41}$$

- *To calculate the surface mass of the thin-shell, one can use this equation* $M_s(a) = 4\pi a^2\sigma$*. To find stable solution, we consider a static case* $[a_0 \in (r_-, r_+)]$*.*

Then the surface charge and pressure at static case reduce to

$$\sigma(a_0) = -\frac{1}{4\pi a_0}\left[\sqrt{1-\frac{2M_{\theta+}}{a_0}+\frac{Q_{\theta+}^2}{a_0^2}}-\sqrt{\left(1-\frac{a_0^2}{\alpha^2}\right)}\right], \quad (42)$$

$$P(a_0) = \frac{1}{8\pi a_0}\left[\frac{1-\frac{M_{\theta+}}{a_0}}{\sqrt{1-\frac{M_{\theta+}}{a_0}+\frac{Q_{\theta+}^2}{a_0^2}}}-\frac{\left(1-\frac{2a_0^2}{\alpha^2}\right)}{\sqrt{\left(1-\frac{a_0^2}{\alpha^2}\right)}}\right]. \quad (43)$$

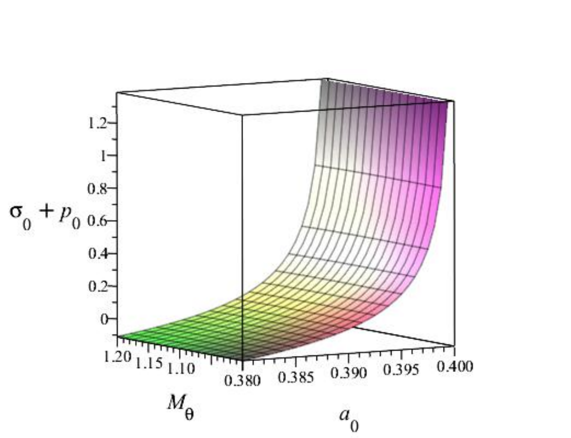


Figure 14: *We plot $\sigma_0 + p_0$ as a function of M_θ and a_0. We choose $Q_\theta = 1$ and $\alpha = 0.4$. Note that in this region the NEC is satisfied.*

Stability of the Charged Thin-shell Gravastars in Noncommutative Geometry

- *Therefore, in this work the range of η will be relaxed and we use graphical reputation to determine the stability regions*

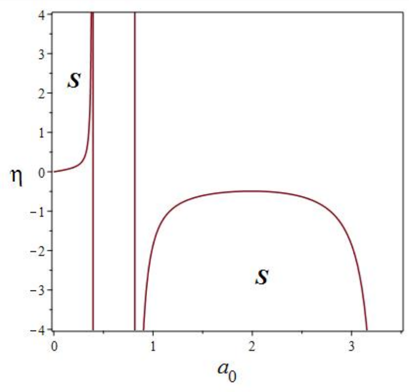


Figure 15: *Stability regions of the charged gravastar in terms of $\eta = P'/\sigma'$ as a function of a_0. We choose $M_\theta = 2$, $Q_\theta = 1.5$, $\alpha = 0.4$.*

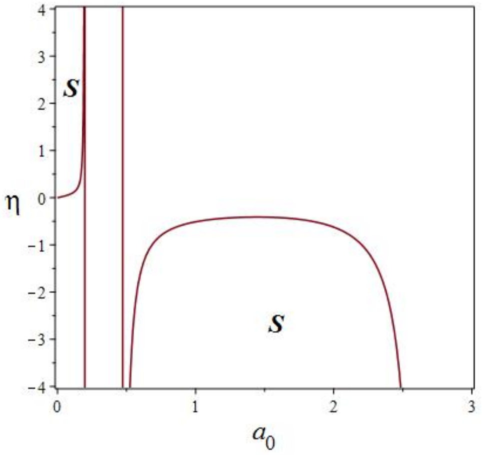


Figure 16: *Stability regions of the charged gravastar in terms of* $\eta = P'/\sigma'$ *as a function of* a_0. *We choose* $M_\theta = 1.5$, $Q_\theta = 1$, $\alpha = 0.2$.

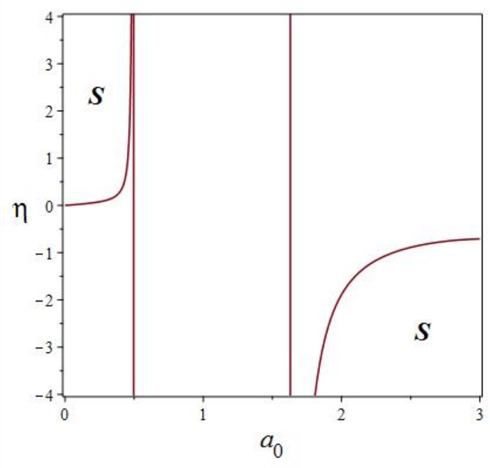


Figure 17: *Stability regions of the charged gravastar in terms of* $\eta = P'/\sigma'$ *as a function of* a_0*. We choose* $M_\theta = 3$, $Q_\theta = 2.5$, $\alpha = 0.5$.

Conclusions

- *We have studied the stability of a particular class of thin-shell gravastar solutions, in the context of charged noncommutative geometry.*
- *We have considered the de Sitter geometry in the interior of the gravastar by matching an exterior charged noncommutative solution at a junction interface situated outside the event horizon.*
- *We have showed that gravastar's shell satisfies the null energy conditions.*
- *We further explored the gravastar solution by the dynamical stability of the transition layer, which is sufficient close to the event horizon.*
- *We have found that for specific choices of mass M_θ, charge Q_θ and the values of α, the stable configurations of the surface layer do exists which is sufficiently close to where the event horizon is expected to form.*

¡Muchas gracias!