Reconstructing Warm Inflation

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-Introduction

- Inflation

-Perturbations

- Warm-inflation

- Reconstruction

- An example

-Some comments and conclusions

The standard Big-Bang model

-The Hubble expansion

- The abundance of light elements

- 3-K microwave background radiation

Some problems of the standard model (Big-Bang)

Horizon

Flatness

Cosmological perturbations

Problems of the standard model (Big-Bang)

$\Downarrow$

Inflation

The inflationary model :

-Typically: Einstein-Equation

Metric: FRW

$$ds^2 = dt^2 - a^2(t)\,d\Omega_k^2$$; a =scale factor

Matter: $T_{\mu\nu} = diag.(\rho, p, p, p)$

The Eqs. of motion :

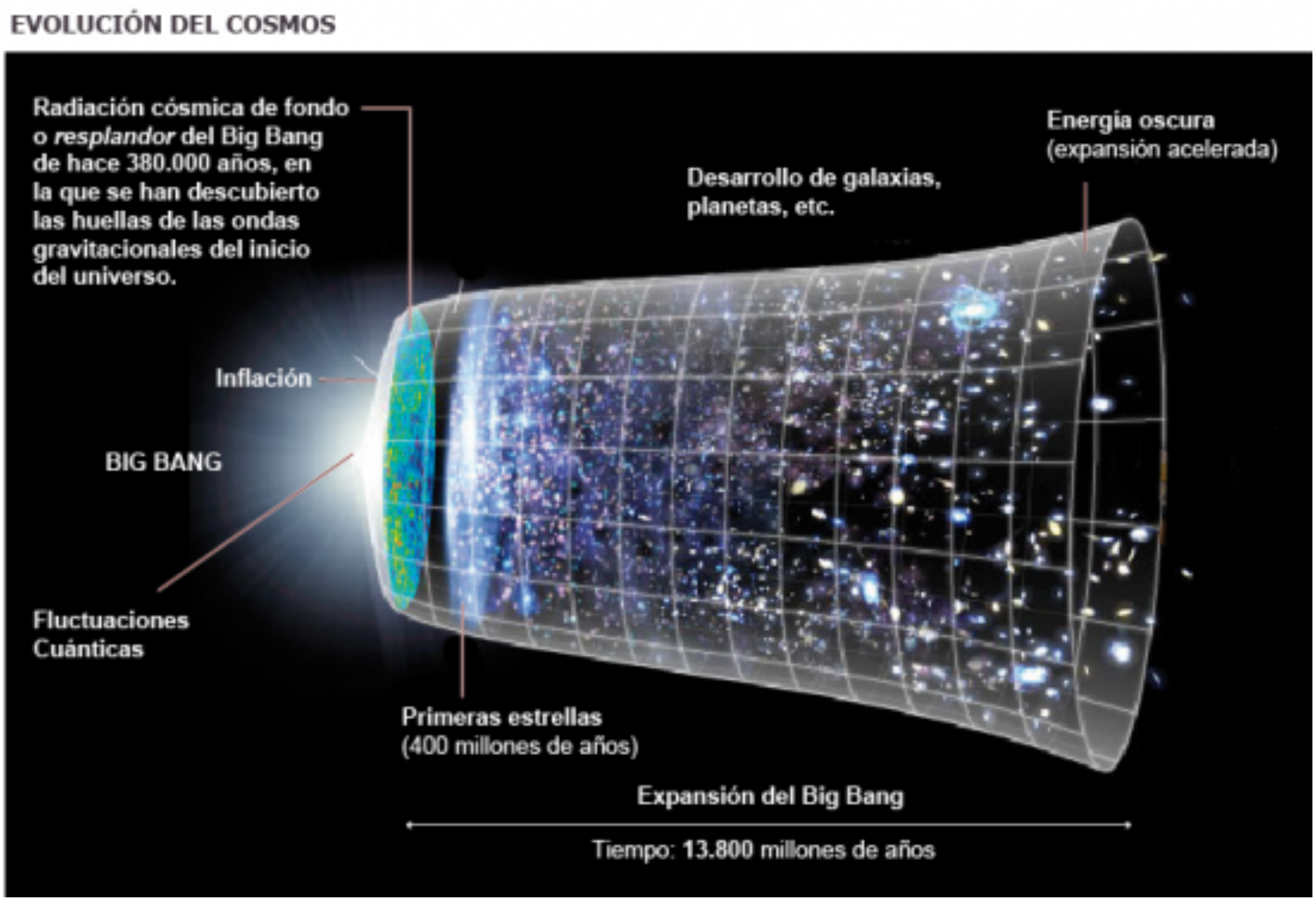
$$H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2},$$

$$\dot{\rho} + 3H(\rho + p) = 0,$$

- Hubble parameter

$$H = \frac{da(t)/dt}{a(t)} = \frac{\dot{a}}{a}$$

Inflation $\ddot{a} > 0$



model of inflation

Matter: scalar field ϕ

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Cold inflation

Scalar field

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V(\phi)}{\partial \phi},$$

Inflation : slow-roll Eqs.

$$H^2 \approx \frac{\kappa^2}{3} V,$$

$$3H\,\dot{\phi} + V' = 0.$$

The slow-roll parameters

$$\varepsilon = -\frac{\dot{H}}{H^2} < 1 \quad \Rightarrow \quad \ddot{a} > 0$$

$$\eta \equiv -\frac{\ddot{H}}{H\dot{H}} < 1$$

The end of inflation $\varepsilon = 1$.

Cosmological Perturbations

Scalar Perturbation: Cold Inflation

$$\phi \rightarrow \phi_0(t) + \delta\phi(x,t)$$

$$ds^2 = (1-2\Phi)dt^2 + a^2[(1-2\psi)\delta_{ij} + \cdots]dx^i dx^j$$

$$P_s = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 \approx \left(\frac{V^3}{V_{,\phi}^2}\right)$$

$$P_s \approx k^{n-1+\frac{dn_R}{2dLnk}+\cdots}$$

Spectral index

Running-spectral index

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 = A_s\left(\frac{k}{k_*}\right)^{n_s-1+\frac{1}{2}\,dn_s/d\ln k\,\ln(k/k_*)+\frac{1}{6}\,\frac{d^2 n_s}{d\ln k^2}(\ln(k/k_*))^2+\cdots}$$

Tensor perturbations

$$P_T = \left(\frac{H}{2\pi}\right)^2 \approx V$$

The scalar-tensor ratio

$$r = \left(\frac{P_T}{P_s}\right)$$

Perturbations:

COBE-WMAP-PlANCK-satellites

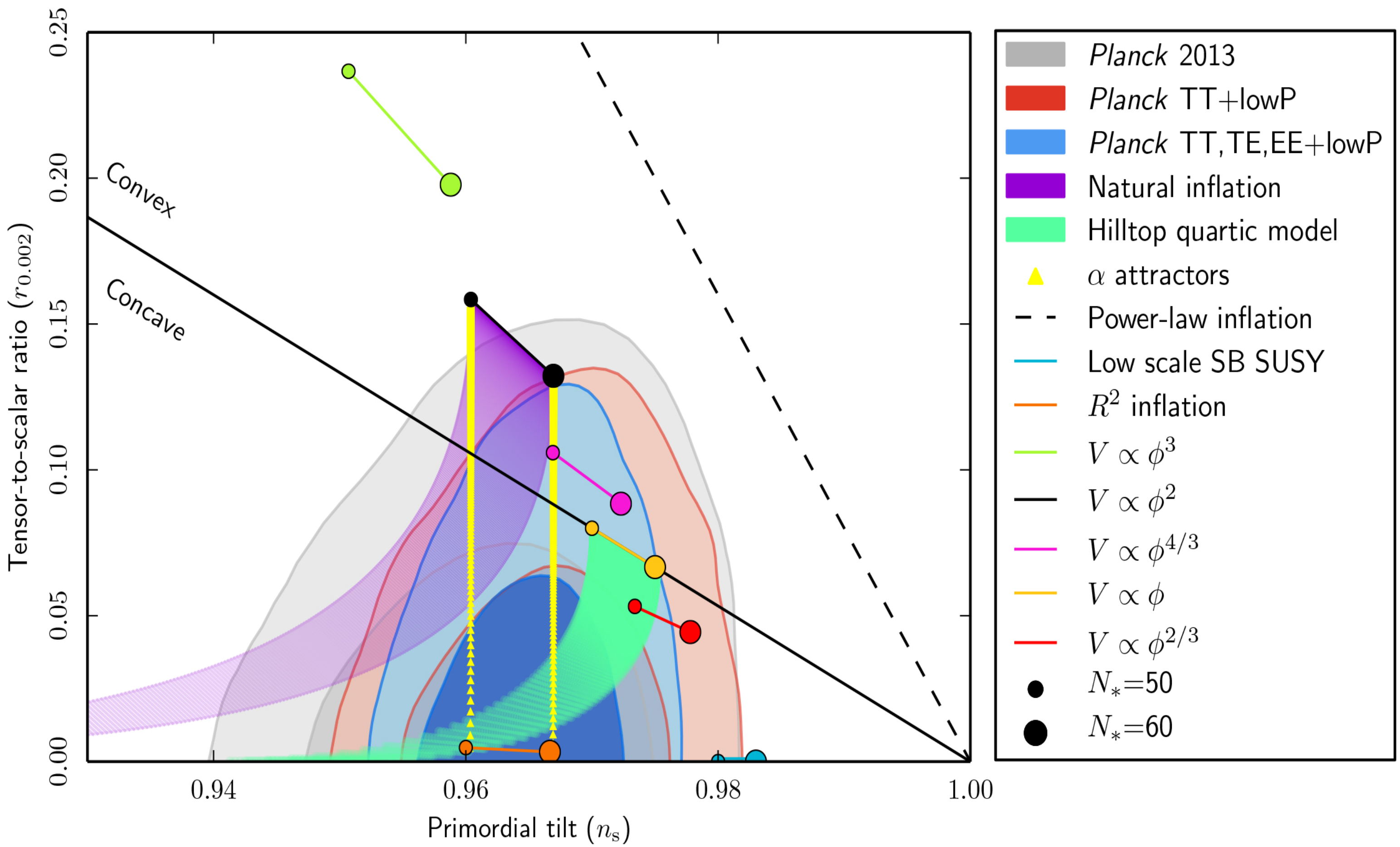
Observational constraints on Inflation

$$P_s^{1/2} \approx \Phi \approx H^2/\dot{\phi} \approx \Delta T/T \approx 10^{-5}$$

$$r < 0.1 (95\% \text{ CL})$$

$$n_R \approx 0.97$$

$r < 0.07 (95\% \text{ CL})$



Convex
Concave
Tensor-to-scalar ratio ($r_{0.002}$)
Primordial tilt (n_s)
0.00
0.05
0.10
0.15
0.20
0.25
0.94
0.96
0.98
1.00
Planck 2013
Planck TT+lowP
Planck TT,TE,EE+lowP
Natural inflation
Hilltop quartic model
α attractors
Power-law inflation
Low scale SB SUSY
R^2 inflation
$V \propto \phi^3$
$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
N_*=50
N_*=60

Reconstruction: Cold Inflation

Observable

attractor: $n_S(N)$

$$\underbrace{\hphantom{n_S(N)}}_{V(N)} \quad + \quad N=N(\phi) \Big\} V(\phi)$$

$$n_s - 1 = -6\epsilon + 2\eta.$$

$$r = 16\epsilon$$

$$N = \ln(a(t_{\rm end})/a(t)) = \int_t^{t_{\rm end}} H dt = \int_\phi^{\phi_{\rm end}} H \frac{d\phi}{\dot\phi} = \int_\phi^{\phi_{\rm end}} \frac{H}{-V'/3H} d\phi = \int_{\phi_{\rm end}}^{\phi} \frac{V}{V'} d\phi\,,$$

the famous relation

Large N

$$n_s - 1 = -\frac{2}{N},$$

$$V(\phi) = \frac{1}{\beta}\tanh^2\left(\gamma(\phi - C)/2\right),$$

$$V(\phi) \simeq \frac{1}{\beta}\left(1 - 4e^{-\gamma(\phi - C)}\right).$$

$$r = \frac{8}{N+\gamma^2 N^2}$$

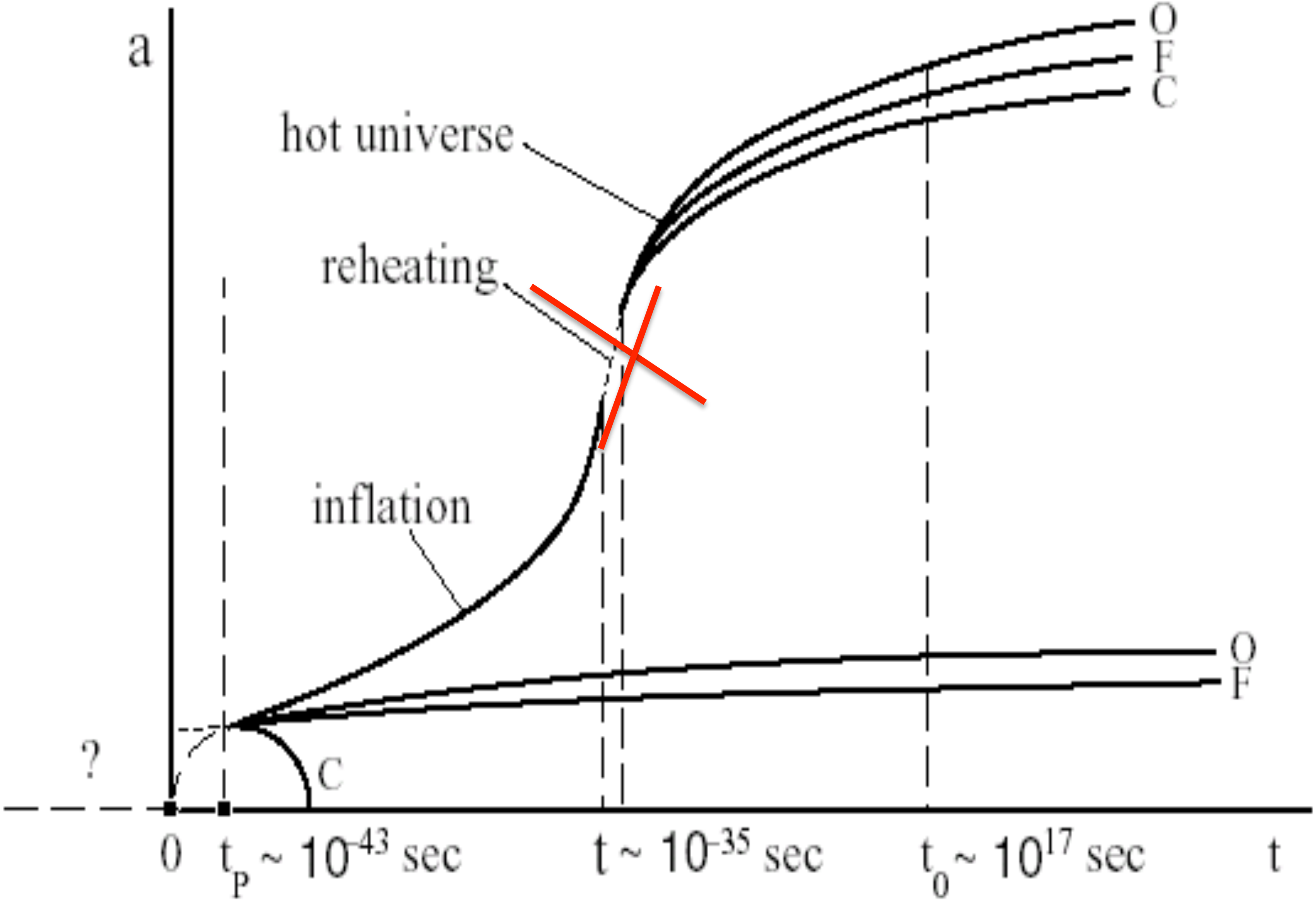
Warm- inflation

$$\rho = \rho_v + \rho_r,$$

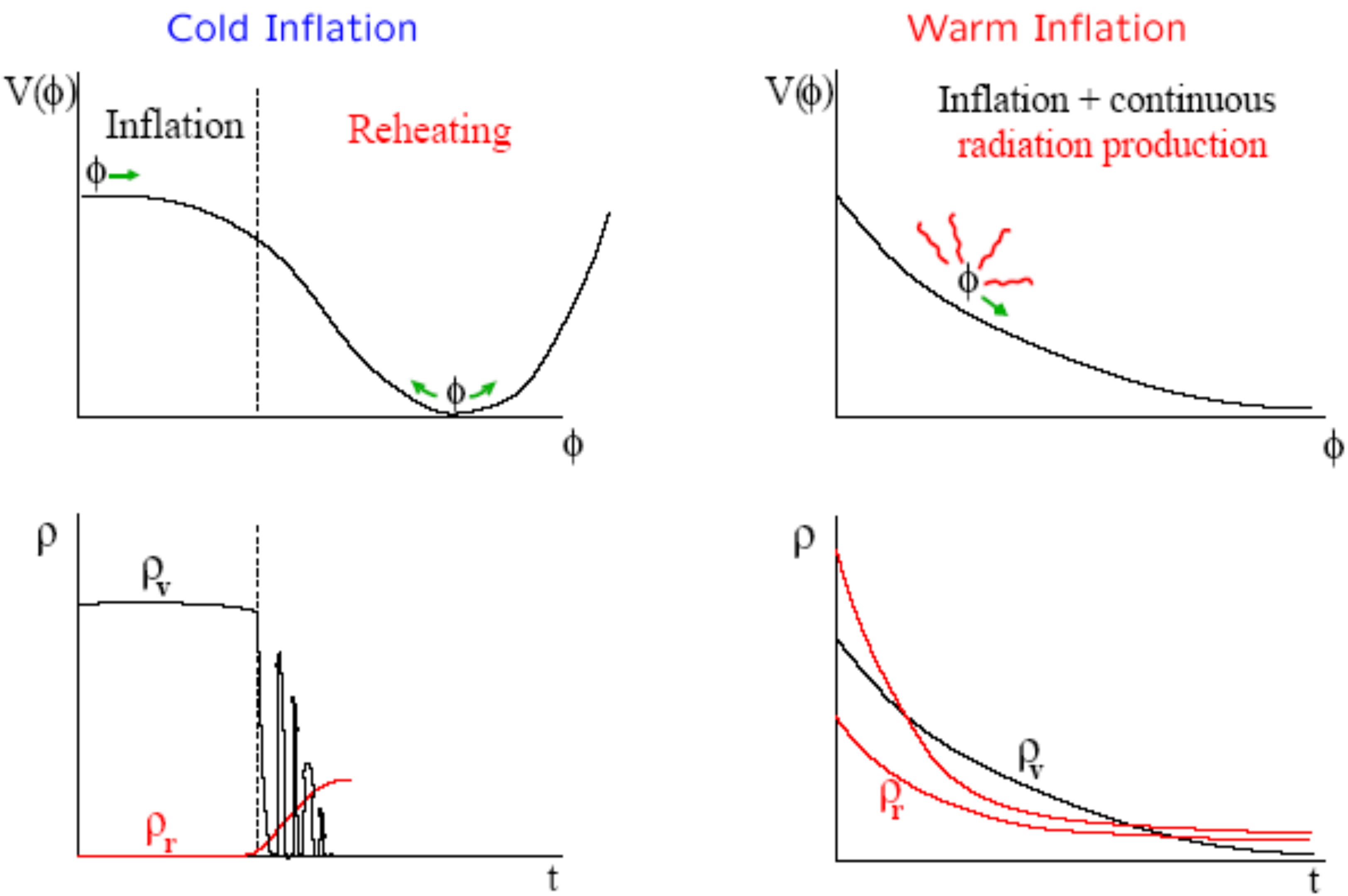
where

ρ_v = inflaton ϕ,

ρ_r = radiation field.


a
hot universe
reheating
inflation
O
F
C
O
F
?
C
0
$t_P \sim 10^{-43}$ sec
$t \sim 10^{-35}$ sec
$t_0 \sim 10^{17}$ sec
t

Comparison of the cold and warm inflationary pictures



Standard Field

$$\dot{\rho} + 3H(\rho + p) = 0, \qquad \begin{cases} \dot{\rho_\phi} + 3\,H\,(\rho_\phi + P_\phi) = -\Gamma\ \dot{\phi}^2, \\ \dot{\rho}_r + 4H\rho_r = \Gamma\dot{\phi}^2, \end{cases}$$

Slow-roll Eqs.

$$H^2 \simeq \frac{1}{3}\rho_\phi \simeq \frac{1}{3}V,$$

$$R = \frac{\Gamma}{3H}.$$

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H(1+R)}.$$

$R \ll 1$ (or equivalently $\Gamma \ll 3H$) weak dissip. regime

$$\rho_\gamma = C_\gamma T^4 \simeq \frac{\Gamma\dot{\phi}^2}{4H} = \frac{R}{4(1+R)^2}\frac{V_{,\phi}^2}{V},$$

$R \gg 1$ ($\Gamma \gg 3H$) strong dissip. regime

$$T = \left[\frac{R}{4C_\gamma\,(1+R)^2}\,\frac{V_{,\phi}^2}{V} \right]^{1/4}. \qquad N = \int_t^{t_e} H\,dt' = \int_\phi^{\phi_e} H\,\frac{d\phi'}{\dot\phi} \simeq \int_{\phi_e}^{\phi} \frac{V(1+R)}{V_{,\phi}} d\phi'.$$

$$\mathcal{P}_\mathcal{S} = \frac{\sqrt{\pi}}{2}\,\frac{H^3\,T}{\dot\phi^2}\,\sqrt{(1+R)}. \qquad n_S - 1 = -\frac{(9R+17)}{4(1+R)^2}\,\epsilon - \frac{(9R+1)}{4(1+R)^2}\,\beta + \frac{3}{2}\frac{1}{(1+R)}\,\eta,$$

$$\epsilon = \frac{1}{2}\left(\frac{V_{,\phi}}{V}\right)^2, \quad \eta = \frac{V_{,\phi\phi}}{V}, \ \text{and} \ \beta = \frac{V_{,\phi}\,\Gamma_{,\phi}}{V\Gamma}.$$

$$r = \frac{\mathcal{P}_\mathcal{T}}{\mathcal{P}_\mathcal{S}} = \frac{2\epsilon}{(1+R)^3}\frac{H}{T}.$$

$$\dot\Phi + H\Phi = \frac{4\pi}{m_p^2}\left[-\frac{4\rho_\gamma\ a\ v}{3k} + \frac{V\dot\phi}{\sqrt{(1-\dot\phi^2)}}\,\delta\phi \right],$$

$$\frac{\ddot{(\delta\phi)}}{1-\dot\phi^2} + \left[3H + \frac{\Gamma}{V}\right]\dot{(\delta\phi)} + \left[\frac{k^2}{a^2(1-\dot\phi^2)} + (\ln(V))_{,\phi\phi} + \dot\phi\left(\frac{\Gamma}{V}\right)_{,\phi}\right]\,\delta\phi$$
$$= \left[\frac{1}{1-\dot\phi^2} + 3\right]\dot\phi\,\dot\Phi + \left[\dot\phi\frac{\Gamma}{V} - 2(\ln(V))_{,\phi}\right]\,\Phi,$$

$$\dot{(\delta\rho_\gamma)} + 4H\delta\rho_\gamma + \frac{4}{3}ka\rho_\gamma v - 4\rho_\gamma\dot\Phi - \dot\phi^2\Gamma_{,\phi}\delta\phi - \Gamma\dot\phi[2\dot{(\delta\phi)} - 3\dot\phi\Phi] = 0,$$

and

$$\dot v + 4Hv + \frac{k}{a}\left[\Phi + \frac{\delta\rho_\gamma}{4\rho_\gamma} + \frac{3\Gamma\dot\phi}{4\rho_\gamma}\delta\phi\right] = 0,$$

Reconstruction

Observable

attracts: $n_S(N)$ and $r(N)$

$$\underbrace{\hspace{3cm}}$$

$V(N)$ and $\Gamma(N)$

$$+ \quad N = N(\phi) \quad \Big\} \begin{matrix} V(\phi) \\ \Gamma(\phi) \end{matrix}$$

$$\epsilon = \frac{V_{,N}}{2V}\,(1+R), \qquad \beta = \frac{\Gamma_{,N}}{\Gamma}\,(1+R),$$

$$\eta = \frac{1}{2VV_{,N}}\,\left[(1+R)[V_{,N}^2 + V\,V_{,NN}] + V\,V_{,N}\,R_{,N}\right],$$

$$n_S - 1 = -\frac{(9R+17)}{8(1+R)}\,\frac{V_{,N}}{V} - \frac{(9R+1)}{4(1+R)}\,\frac{\Gamma_{,N}}{\Gamma}$$

$$+\frac{3}{4}\frac{1}{(1+R)}\,\frac{1}{VV_{,N}}\,\left[(1+R)[V_{,N}^2 + V\,V_{,NN}] + V\,V_{,N}\,R_{,N}\right].$$

$$r = \frac{\mathcal{P}_{\mathcal{T}}}{\mathcal{P}_{\mathcal{S}}} = \frac{V_{,N}}{(1+R)^2}\frac{1}{\sqrt{3V}\,T},$$

$$\int \left[\frac{V_{,N}}{V(1+R)}\right]^{1/2} dN = \int d\phi.$$

THE WEAK DISSIPATIVE REGIME. $\Gamma \ll 3H$ (or equivalently $R \ll 1$).

$$V(N) = V = \frac{r}{\tilde{C}_\gamma^{1/4}} \exp[\int (1 - n_S) dN],$$

$$\Gamma(N) = \Gamma = \frac{\tilde{C}_\gamma}{r^4} \left[\frac{V_{,N}}{V^{1/2}}\right]^3 = \left[\frac{\tilde{C}_\gamma^{1/4}}{r}\right]^{5/2} \left[\frac{r_{,N}}{r} + (1 - n_S)\right]^3 \exp\left[\frac{3}{2} \int (1 - n_s) dN\right],$$

$$\int \left[\frac{V_{,N}}{V}\right]^{1/2} dN = \int d\phi.$$

THE STRONG DISSIPATIVE REGIME. $\Gamma \gg 3H$.

$$V(N) = \left(\frac{-1}{5 \times 18^2\, C_\gamma} \int \left[r^4 \exp[\frac{16}{3} \int (1 - n_S) dN]\right] dN\right)^{1/5},$$

$$\Gamma(N) = 2 \times 3^{7/4}\, C_\gamma^{1/2} \left(\frac{V^{5/2}}{r^2} \exp[2 \int (n_S - 1) dN]\right).$$

$$\int \left[\frac{V_{,N}}{V\, R}\right]^{1/2} dN = \int \left[\frac{\sqrt{3}\, V_{,N}}{\sqrt{V}\, \Gamma}\right]^{1/2} dN = \int d\phi.$$

AN EXAMPLE.

$$n_S - 1 = -\frac{2}{N}, \qquad\qquad r = \frac{1}{N(1+\xi N)},$$

A. The weak regime.

$$V(N) = \frac{1}{\alpha\, \tilde{C}_\gamma^{1/4}} \left[\frac{1}{\xi + 1/N}\right] .$$

$$\Gamma(N) = \Gamma_0\, N^{5/2}\, (1+\xi N)^{-1/2}, \quad \text{where} \quad \Gamma_0 = \frac{\tilde{C}_\gamma^{\;5/8}}{\alpha^{3/2}}.$$

$$\int \sqrt{\frac{1}{N(1+\xi N)}}\, dN = \int d\phi,$$

In the case in which $\xi > 0$.

$$V(\phi) = V_0 \,\tanh^2\left[\frac{\sqrt{\xi}}{2}(\phi - \phi_0)\right], \quad \text{where} \quad V_0 = \frac{1}{\alpha\, \xi\, \tilde{C}_\gamma^{1/4}}.$$

$$\Gamma(\phi) = \frac{\Gamma_0}{\xi^{5/2}} \,\tanh\left[\frac{\sqrt{\xi}}{2}(\phi - \phi_0)\right] \sinh^4\left[\frac{\sqrt{\xi}}{2}(\phi - \phi_0)\right].$$

$$V(\phi) = -V_0 \, \tan^2\left[\frac{\sqrt{-\xi}}{2}(\phi-\phi_0)\right].$$

the constant $\xi < 0$

$$\Gamma(\phi) = \frac{\Gamma_0}{(-\xi)^{5/2}} \, \tan\left[\frac{\sqrt{-\xi}}{2}(\phi-\phi_0)\right] \, \sin^4\left[\frac{\sqrt{-\xi}}{2}(\phi-\phi_0)\right].$$

$$V(\phi) = \frac{1}{4\alpha\, \tilde{C}_\gamma^{1/4}}\,(\phi-\phi_0)^2,$$

the constant $\xi = 0$

$$\Gamma(\phi) = \frac{\Gamma_0}{32}\,(\phi-\phi_0)^5,$$

lower bound for the integration constant $\alpha \gg \frac{\tilde{C}_\gamma^{3/4}}{\sqrt{3}}\, N^2$

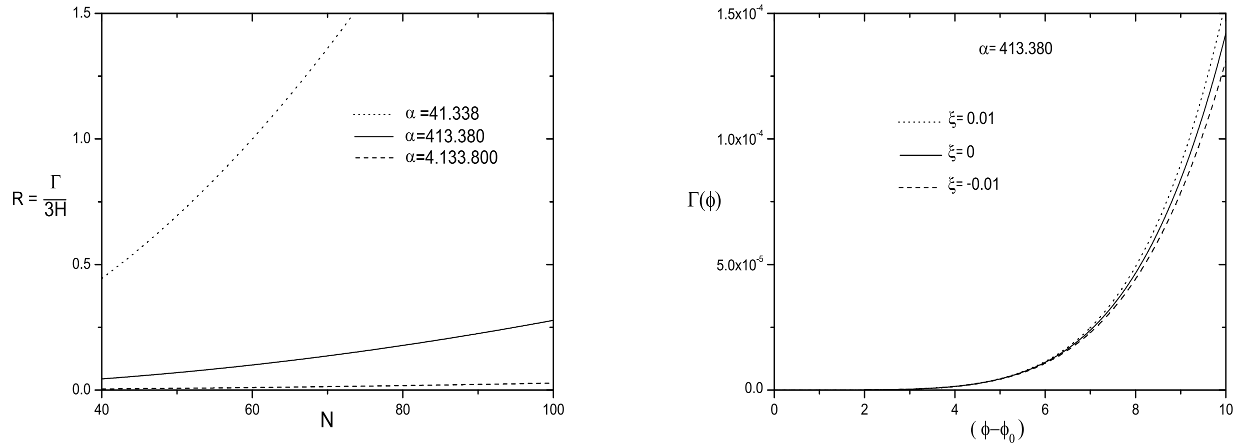


FIG. 1: The dependence of the ratio $R = \frac{\Gamma}{3H}$ versus the number of e-folds N (left panel) and the dependence of the dissipation coefficient Γ versus the scalar field (right panel) during the weak dissipative regime. From Eq.(41) we plot $R = R(N)$ in which the dotted solid and dashed lines correspond to three different values of α (left panel). From Eqs.(45), (48) and (50) we plot $\Gamma = \Gamma(\phi)$ for three different values of $\xi \gtreqless 0$, in which we have fixed $\alpha = 413.380$ (right panel). Also, in these plots we have used $C_\gamma = 70$.

B. The strong regime.

$$V(N) = \left[\frac{3\tilde{\alpha}^4}{2618} \left[\frac{770 + 3740\xi N + 7140\xi^2 N^2 + 6545\xi^3 N^3 + 2618\xi^4 N^4}{N^{17/3}} \right] + C_1 \right]^{-1/5},$$

$$\Gamma(N) = 2 \times 3^{7/4} C_\gamma^{1/2} \alpha \left[\frac{(1+\xi N)^2 \, V^{5/2}}{N^2} \right],$$

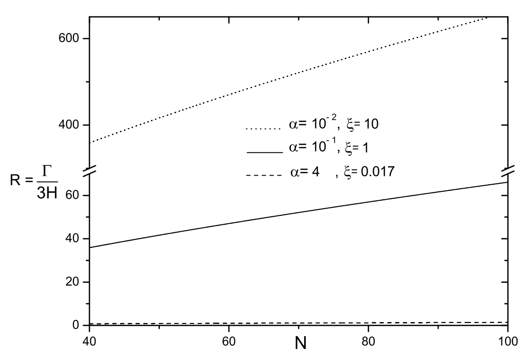
the asymptotic form of the potential and the dissipation coefficient in the limit $N \gg 1/\xi$ or $N \ll 1/\xi$

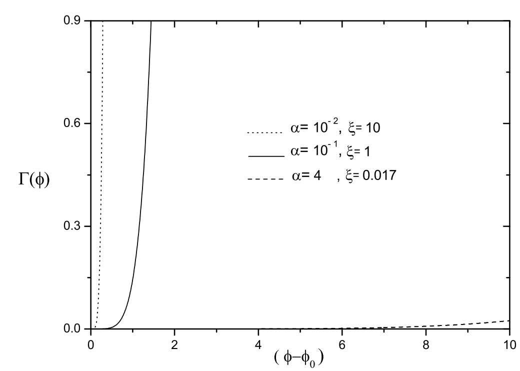
We start with the limit $\xi N \gg 1$

$$r(N) = \frac{1}{N(1+\xi N)} \approx \frac{1}{\xi \, N^2},$$

$$V(\phi) \approx V_0 \, (\phi - \phi_0)^2, \quad \text{where} \quad V_0 = \frac{\alpha_1}{4 \, 3^{3/2} \tilde{\beta}^{12/5}}.$$

$$\Gamma(\phi) \approx \Gamma_0 \, (\phi - \phi_0)^5, \quad \text{where} \quad , \; \Gamma_0 = \frac{\alpha_1^{7/2}}{2^5 \, 3^{15/4} \, \tilde{\beta}^6},$$





now we consider the limit in which $\xi N \ll 1$ where $r(N) \approx 1/N$.

$$V(N) \approx \left[\frac{15}{17}\tilde{\alpha}^4\right]^{-1/5} N^{17/15}, \qquad \Gamma(N) \approx 3^{-1/4}\left[\frac{17}{15}\right]^{1/2} \frac{N^{5/6}}{\alpha}.$$

$$R(N) \approx \frac{1}{3^{7/20}}\left[\frac{\sqrt{2}\,17}{15}\right]^{2/5}\left(\frac{\sqrt{C_\gamma}}{\alpha^3}\right)^{1/5} N^{4/15}.$$

$$V(\phi) \approx \tilde{V}_0\,(\phi - \tilde{\phi}_0)^{34/11}, \text{ where } \tilde{V}_0 = \left(\frac{17}{15\,\tilde{\alpha}^4},\right)^{1/5} \tilde{N}_0^{\,17/15},$$

$$\Gamma(\phi) \approx \tilde{\Gamma}_0\,(\phi - \tilde{\phi}_0)^{25/11}, \text{ where } \tilde{\Gamma}_0 = \frac{1}{3^{1/4}\,\alpha}\left(\frac{17}{15}\right)^{1/2} \tilde{N}_0^{\,5/6}.$$

Some comments and conclusions

- Warm-Inf. : Avoid the reheating period- Thermal Fluctuations
- W.-I. $\Rightarrow$ Perturbations$\Rightarrow$Background

(reconstruction)

- W.-I. $\Rightarrow$ $n_S(N)$ and $r(N)$
- W.I. we have used slow roll app.
- W.I. analytical solutions ???

References

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