Contracción generalizada de Inönü-Wigner como una S-expansión

Cosmoconce 2017

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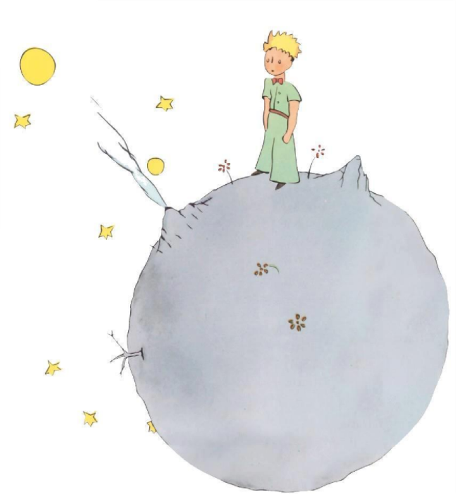
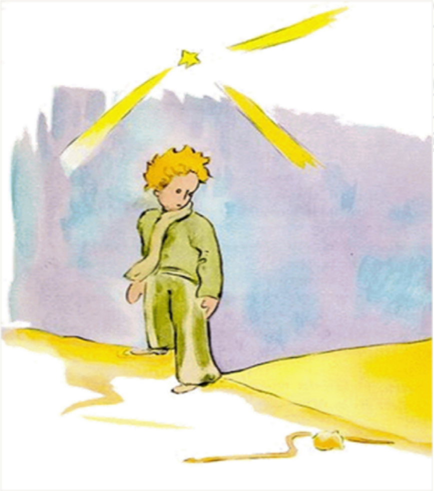
Principito estableciendo generadores de E_2 desde $SO(3)$[1]





[1] Le Petit Prince - Antoine de Saint-Exupéry

Estado Físico

$$U|\Phi\rangle = |\Phi'\rangle$$

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$$U = e^{\alpha^A T_A}$$

Grupos de Lie $\longrightarrow$ Algebras de Lie

Lie (super)algebra

Being $\mathfrak{g} = \{T_A\}$

$$[\,T_A, T_B\,\} = C_{AB}{}^{C}\,T_C$$

Contracción estándar de Inönü-Wigner

Sea $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{p}$

$$
\begin{aligned}
[\mathfrak{h}, \mathfrak{h}] &\subseteq \mathfrak{h} \\
[\mathfrak{h}, \mathfrak{p}] &\subseteq \mathfrak{p} \\
[\mathfrak{p}, \mathfrak{p}] &\subseteq \mathfrak{h} + \mathfrak{p}
\end{aligned}
$$

La contracción de IW: $\mathfrak{g} \longrightarrow \mathfrak{g}'$

$$
\begin{pmatrix} h' \\ p' \end{pmatrix} = \begin{pmatrix} I_{dim(\mathfrak{h})} & 0 \\ 0 & \epsilon I_{dim(\mathfrak{h})} \end{pmatrix} \begin{pmatrix} h \\ p \end{pmatrix}
$$

Contracción estándar de Inönü-Wigner

$\epsilon \longrightarrow 0$

$$
\begin{aligned}
[\mathfrak{h}', \mathfrak{h}'] &\subseteq \mathfrak{h}' \\
[\mathfrak{h}', \mathfrak{p}'] &\subseteq \mathfrak{p}' \\
[\mathfrak{p}', \mathfrak{p}'] &\subseteq 0
\end{aligned}
$$

Algebras de Lie $\longrightarrow$ Nuevas Algebras de Lie

Procedimiento de expansión con semigrupo abeliano

Consiste en combinar las constantes de estructura de una (super)algebra de Lie $\mathfrak{g}$ con una ley de producto interior de un (semi)grupo S definiendo así una nueva algebra de Lie

$$\mathfrak{g}_S \simeq S \times \mathfrak{g}.$$

Algebras de Lie $\longrightarrow$ Nuevas Algebras de Lie

Un semigroupo $S = \{\lambda_\alpha, \lambda_\beta, \ldots\}$ es una estructura algebraica provista de un producto cerrado y asociativo $\lambda_\alpha \lambda_\beta = K_{\alpha\beta}{}^{\rho} \lambda_\rho$.

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Ejemplo de estructura de (semi)grupo

$\cdot$	1	-1	i	$-i$
1	1	-1	i	$-i$
-1	-1	1	$-i$	i
i	i	$-i$	-1	1
$-i$	$-i$	i	1	-1

.

Algebras de Lie $\longrightarrow$ Nuevas Algebras de Lie

Un semigroupo $S = \{\lambda_\alpha, \lambda_\beta, \dots\}$ es una estructura algebraica provista de un producto cerrado y asociativo $\lambda_\alpha \lambda_\beta = K_{\alpha\beta}{}^{\rho} \lambda_\rho$.

Ejemplo de estructura de (semi)grupo

$$
\begin{array}{c|cccc}
\cdot & 1 & -1 & i & -i \\
\hline
1 & 1 & -1 & i & -i \\
-1 & -1 & 1 & -i & i \\
i & i & -i & -1 & 1 \\
-i & -i & i & 1 & -1
\end{array} .
$$

Los nuevos generadores son

$$
T_{(a,A)} \equiv \lambda_a \otimes T_A \in \mathfrak{g}_S
$$

Algebra Resonante

Si

$$\mathfrak{g} \simeq V_0 \oplus V_1 \oplus \ldots \oplus V_n$$

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Si

$$\mathfrak{g} \simeq V_0 \oplus V_1 \oplus \ldots \oplus V_n$$

y

$$S = S_0 \sqcup S_1 \sqcup \ldots \sqcup S_n$$

entonces

$$\mathfrak{g}_{SR} = \bigoplus W_p,$$

donde

$$W_p = S_p \oplus V_p$$

es una subalgebra de $\mathfrak{g}_S$ y es llamada *algebra resonante*.

Algebra Reducida

Además si

$$0_S \in S$$

entonces se tiene una *algebra reduced (o forzada)*

¿que ventaja existe en ocupar este procedimiento?

Theorem VII.1 Let S be an abelian semigroup, $\mathfrak{g}$ a Lie (super)algebra of basis $\{T_A\}$, and let $\langle T_{A_1}, T_{A_2}, \ldots, T_{A_n}\rangle$ be an invariant tensor of $\mathfrak{g}$. Then, the expression

$$\langle T_{(A_1,\alpha_1)}, T_{(A_2,\alpha_2)}, \ldots, T_{(A_n,\alpha_n)}\rangle = \alpha_\gamma K_{a_1\cdots a_n}{}^{\gamma}\langle T_{A_1}, T_{A_2}, \ldots, T_{A_n}\rangle$$

where γ are arbitrary constants and $K_{a_1\ldots a_n}{}^{\gamma}$ is the n-selector for S, corresponds to an invariant tensor for the S-expanded algebra $\mathfrak{g}_S = S \times \mathfrak{g}$ F. Izaurieta, E. Rodriguez, P. Salgado, 2006.

Chern-Simons

La forma Pontryagin-Chern $\mathcal{P}_{2n+2} = \langle F^{n+1} \rangle$ satisface la condición

$$d\mathcal{P}_{2n+2} = 0,$$

donde $F = dA + A^2$. A partir de lema de Poincaré

$$\mathcal{P}_{2n+2} = d\mathcal{C}_{2n+1}.$$

$\mathcal{C}_{2n+1}$ es llamada forma Chern-Simons la cual es quasi invariante bajo transformaciones de gauge.

S-expansión en teorías de la gravitación Chern-Simons

¡Poderosa herramienta para construir nuevas teorías en (super)gravedad!

$$S_{CS} = \int_{\partial M} \mathcal{L}_{2n+1}$$

donde

$$d\mathcal{L}_{2n+1} = \int_M \langle F^{2n} \rangle = \int_M F^{a_1} \cdots F^{a_{n2}} \langle T_{a_1} \cdots T_{a_{2n}} \rangle$$

$$\begin{aligned} F =& dA + AA, \\ A =& \phi^a T_a + \theta^\alpha T_\alpha . \end{aligned}$$

Contracción generalizada de Inönü-Wigner

$$\begin{pmatrix} \mathfrak{h}' \\ \mathfrak{p}' \\ \mathfrak{k}' \end{pmatrix} = \begin{pmatrix} I_{dim(\mathfrak{h})} & 0 & 0 \\ 0 & \epsilon I_{dim(\mathfrak{p})} & 0 \\ 0 & 0 & \epsilon^2 I_{dim(\mathfrak{k})} \end{pmatrix} \begin{pmatrix} \mathfrak{h} \\ \mathfrak{p} \\ \mathfrak{k} \end{pmatrix}$$

Contracción generalizada de Inönü-Wigner

$$\begin{pmatrix}\mathfrak{h}'\\ \mathfrak{p}'\\ \mathfrak{k}'\end{pmatrix}=\begin{pmatrix}I_{dim(\mathfrak{h})} & 0 & 0\\ 0 & \epsilon I_{dim(\mathfrak{p})} & 0\\ 0 & 0 & \epsilon^2 I_{dim(\mathfrak{k})}\end{pmatrix}\begin{pmatrix}\mathfrak{h}\\ \mathfrak{p}\\ \mathfrak{k}\end{pmatrix}$$

Al tener

$$[\epsilon\mathfrak{p},\epsilon\mathfrak{p}]\subseteq\epsilon^2\mathfrak{p}+\epsilon^2\mathfrak{k},$$

redefinimos $\mathfrak{h}'=\mathfrak{h}$, $\mathfrak{p}'=\epsilon\mathfrak{p}$, $\mathfrak{k}'=\epsilon^2\mathfrak{k}$, con $\epsilon\longrightarrow 0$ y se obtiene

$$[\mathfrak{p}',\mathfrak{p}']\subseteq\mathfrak{k}'.$$

S-expansion infinita con sustracción de ideal

Sea

$$\mathfrak{g}_S^{\infty} = S^{\infty} \otimes \mathfrak{g}$$

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$$\mathfrak{g}_S^{\infty} = S^{\infty} \otimes \mathfrak{g} = \mathfrak{g}_S \oplus \mathcal{I}$$

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El requerimiento para que $\mathcal{I}$ sea un ideal es que

$$[\mathfrak{g}_S, \mathcal{I}] \subset \mathcal{I},$$
$$[\mathcal{I}, \mathcal{I}] \subset \mathcal{I},$$

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Sea

$$\mathfrak{g}_S^\infty = S^\infty \otimes \mathfrak{g} = \mathfrak{g}_S \oplus \mathcal{I}$$

El requerimiento para que $\mathcal{I}$ sea un ideal es que

$$\begin{aligned}[\mathfrak{g}_S, \mathcal{I}] &\subset \mathcal{I}, \\ [\mathcal{I}, \mathcal{I}] &\subset \mathcal{I},\end{aligned}$$

$$[\mathfrak{g}_S, \mathfrak{g}_S] \subset \mathfrak{g}_S + \mathcal{I} \subset \mathfrak{g}_S^\infty .$$

Resonancia

$$
\begin{aligned}
\mathfrak{g}_S^{\infty} &= \{\lambda_n\}_{n=0}^{\infty} \times \mathfrak{g} = \\
&= \{\lambda_n\}_0^{\infty}\, \mathfrak{g}^{(0)} \oplus \{\lambda_n\}_0^{\infty}\, \mathfrak{g}^{(1)} \oplus \cdots \oplus \{\lambda_n\}_0^{\infty}\, \mathfrak{g}^{(n)}.
\end{aligned}
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$$

$$
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$$

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$$
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\end{aligned}
$$

$$
S^{(\infty)} = \bigcup_{p \in I} S_p .
$$

$$
\mathfrak{g}_R^{(\infty)} = \bigoplus_{p \in I} W_p \tag{1}
$$

Theorem 3. *Let $\mathfrak{g}$ be a Lie (super)algebra and let $\mathfrak{g}_S^\infty = S^{(\infty)} \times \mathfrak{g}$ be the infinite S-expanded (super)algebra obtained using the infinite abelian semigroup $S^{(\infty)} = \{\lambda_\alpha,\ \alpha = 0, \ldots, \infty\}$.*

Let $\mathfrak{g}_R^\infty$ be an infinite resonant subalgebra of $\mathfrak{g}_S^\infty$ and let $\mathcal{I}$ be an infinite ideal subalgebra of $\mathfrak{g}_R^\infty$. Then, the (super)algebra

$$\check{\mathfrak{g}}_R = \mathfrak{g}_R^\infty \ominus \mathcal{I} \tag{3.36}$$

is a reduced (super)algebra.

Ideal

Consideremos el homomorfismo

$$\varphi : G \longrightarrow G/H,$$

donde G es un grupo de Lie y H un grupo normal. Por definición $H = ker\varphi$ y G/H es un grupo de Lie.

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Sea A el algebra de G y $\hat{A}$ el algebra de G/H, entonces φ induce un homomorfismo

$$\hat{\varphi} : \; A \longrightarrow \hat{A}$$

tal que $a \in \mathcal{A} \longrightarrow \varphi(e^{a}) = e^{\hat{\varphi}(a)}$

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Sea A el algebra de G y $\hat{A}$ el algebra de G/H, entonces φ induce un homomorfismo

$$\hat{\varphi} : \; A \longrightarrow \hat{A}$$

tal que $a \in \mathcal{A} \longrightarrow \varphi(e^{a}) = e^{\hat{\varphi}(a)}$ Si $\forall a \in \mathcal{I}, \; e^{a} \in H$

$$\Longrightarrow \varphi(e^{a}) = e' \in G/H. \Longleftrightarrow \hat{\varphi}(a) = 0.$$

Consequently $\hat{\varphi}(\mathcal{I}) = 0$.

Bajo el punto de vista de representaciones ρ: $G \longrightarrow Aut(V)$, siendo V un espacio vectorial, esto induce una representación G/H

$$\rho(H) \cdot \Psi = \Psi.$$

Es más, $\forall\, [\rho] \in G/H$,

$$\rho([g]) = \rho(g),$$

ya que,

$$\begin{aligned}\rho(g \cdot h)\Psi =& \rho(g)\rho(h)\Psi \\ =& \rho(g)\Psi, \quad \forall\Psi,\end{aligned}$$

podemos decir entonces que $\rho(G/H) = \rho(G)$ es la propiedad que necesitamos para exigir que

$$\rho(\mathcal{I}) = 0,$$

Entonces ρ provee de una representación de $\mathcal{A} \ominus \mathcal{I}$:

$$\rho(\mathcal{A}) = \rho(\mathcal{A} \ominus \mathcal{I}).$$

Entonces, si escribimos

$$\mathcal{A} \ominus \mathcal{I} = \mathcal{A}_0$$

donde $\mathcal{A}_0$ es espacio coseto, podemos decir que $\rho(\mathcal{A}_0)$ es homomorfico a $\rho(\mathcal{A})$, y

$$\rho(\mathcal{A})\Psi = \rho(\mathcal{A}_0)\Psi, \quad \forall \Psi.$$

S-expansion infinita con sustracción de ideal

Infinite S-expansion with Ideal reduction

$$\mathfrak{g}_S = \mathfrak{g}_S^{\infty} \ominus \mathcal{I} \tag{2}$$

$$S^{\infty} = \{z^n = \exp(n\phi), \ n \in \mathbb{N}\}_{n=0}^{\infty}$$

$$z^n z^m = z^{n+m} = \delta_l^{n+m} z^l$$

Theorem

Let $\langle T_{A_1} \dots T_{A_N} \rangle$ be an invariant tensor of an algebra $\mathfrak{g}$, being T_{A_i} the N generators ($i = 1, 2, \dots, N$) of $\mathfrak{g}$, and let the algebra $\mathfrak{g}_S^{\infty)} = S_E^{(\infty)} \times \mathfrak{g}$ be the one constructed by infinite S-expansion involving the abelian semigroup $S_E^{(\infty)}$. Denote the generators of $\mathfrak{g}_S^{\infty}$ as $\lambda_{a_i} T_{A_i} \equiv T_{A_i}^{a_i}$, $a = 0, 1, 2, \dots \infty$, and define a particular ideal $\mathcal{I}$ of $\mathfrak{g}_S^{\infty}$. The invariant tensor of the algebra $\mathfrak{g}_S = \mathfrak{g}_S^{\infty} \ominus \mathcal{I}$ can be then written in the form

$$\langle T_{A_1}^{a_1} \dots T_{A_N}^{a_N} \rangle = \sum_{m=0}^{n_{max}} \alpha^m \delta_m^{a_1+a_2+\dots+a_N} \langle T_{A_1} \dots T_{A_N} \rangle, \tag{3}$$

where the α^m's are arbitrary constants and where n_{max} denotes the greatest index between those of the semigroup elements (namely, $\lambda_{n_{max}}$) whose product with the generators of $\mathfrak{g}$ is contained into the algebra $\mathfrak{g}_S$ (and is thus excluded from the ideal).

D. M. Peñafiel, L. Ravera, 2017

Ejemplo con algebra AdS

Consideremos el algebra $\mathfrak{osp}(2|1) \otimes \mathfrak{sp}(2)$

$$
\begin{aligned}
[J_{ab}, J_{cd}] &= \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}, \\
[J_{ab}, P_c] &= \eta_{bc} P_a - \eta_{ac} P_b, \\
[P_a, P_b] &= J_{ab}, \\
[J_{ab}, Q_\alpha] &= -\frac{1}{2} (\gamma_{ab} Q)_\alpha, \\
[Q_\alpha, P_a] &= -\frac{1}{2} (\gamma_a Q)_\alpha \\
\{Q_\alpha, Q_\beta\} &= -\frac{1}{2} \left[(\gamma^{ab} C)_{\alpha\beta} J_{ab} - 2 (\gamma^a C)_{\alpha\beta} P_a \right].
\end{aligned}
$$

Ejemplo con algebra AdS

$S_E^{\infty} = \{\lambda_\alpha\}_{\alpha=0}^{\infty}$ and perform the multiplication:

$$\mathfrak{g}_S^{\infty} = \{\lambda_\alpha\}_{\alpha=0}^{\infty} \times \left\{ \tilde{J}_{ab}, \tilde{P}_a, \tilde{Q}_\alpha \right\}.$$

Ejemplo con algebra *AdS*

$S_E^{\infty} = \{\lambda_\alpha\}_{\alpha=0}^{\infty}$ and perform the multiplication:

$$\mathfrak{g}_S^{\infty} = \{\lambda_\alpha\}_{\alpha=0}^{\infty} \times \left\{ \tilde{J}_{ab}, \tilde{P}_a, \tilde{Q}_\alpha \right\}.$$

$$\begin{aligned} J_{(ab,2k)} &= \lambda_{2k} \tilde{J}_{ab}, \\ P_{(a,2k+1)} &= \lambda_{2k+1} \tilde{P}_a, \\ Q_\alpha &= \lambda_{2k+1} \tilde{Q}_\alpha \end{aligned}$$

Sector bosónico

$$
\begin{aligned}
\left[J^0_{ab}, J^0_{cd}\right] =& \eta_{bc} J^0_{ad} - \eta_{ac} J^0_{bd} - \eta_{bd} J^0_{ac} + \eta_{ad} J^0_{bc}, \\
\left[J^0_{ab}, Z^2_{cd}\right] =& \eta_{bc} Z^2_{ad} - \eta_{ac} Z^2_{bd} - \eta_{bd} Z^2_{ac} + \eta_{ad} Z^2_{bc}, \\
\left[J^0_{ab}, \mathbf{J}^{\mathbf{n}}_{\mathbf{cd}}\right] =& \eta_{\mathbf{bc}} \mathbf{J}^{\mathbf{n}}_{\mathbf{ad}} - \eta_{\mathbf{ac}} \mathbf{J}^{\mathbf{n}}_{\mathbf{bd}} - \eta_{\mathbf{bd}} \mathbf{J}^{\mathbf{n}}_{\mathbf{ac}} + \eta_{\mathbf{ad}} \mathbf{J}^{\mathbf{n}}_{\mathbf{bc}}, \\
[\mathbf{J}^{\mathbf{n}}_{\mathbf{ab}}, \mathbf{J}^{\mathbf{n}}_{\mathbf{cd}}] =& \eta_{\mathbf{bc}} \mathbf{J}^{\mathbf{n+n}}_{\mathbf{ad}} - \eta_{\mathbf{ac}} \mathbf{J}^{\mathbf{n+n}}_{\mathbf{bd}} - \eta_{\mathbf{bd}} \mathbf{J}^{\mathbf{n+n}}_{\mathbf{ac}} + \eta_{\mathbf{ad}} \mathbf{J}^{\mathbf{n+n}}_{\mathbf{bc}}, \\
\left[J^0_{ab}, P^1_c\right] =& \eta_{bc} P^1_a - \eta_{ac} P^1_b, \\
\left[J^0_{ab}, \mathbf{P}^{\mathbf{s}}_{\mathbf{c}}\right] =& \eta_{\mathbf{bc}} \mathbf{P}^{\mathbf{s}}_{\mathbf{a}} - \eta_{\mathbf{ac}} \mathbf{P}^{\mathbf{s}}_{\mathbf{b}}, \\
\left[\mathbf{J}^{\mathbf{n}}_{\mathbf{ab}}, P^1_c\right] =& \eta_{\mathbf{bc}} \mathbf{P}^{\mathbf{n+1}}_{\mathbf{a}} - \eta_{\mathbf{ac}} \mathbf{P}^{\mathbf{n+1}}_{\mathbf{b}}, \\
[\mathbf{J}^{\mathbf{n}}_{\mathbf{ab}} \mathbf{P}^{\mathbf{s}}_{\mathbf{c}}] =& \eta_{\mathbf{bc}} \mathbf{P}^{\mathbf{n+s}}_{\mathbf{a}} - \eta_{\mathbf{ac}} \mathbf{P}^{\mathbf{n+s}}_{\mathbf{b}}, \\
\left[P^1_a, P^1_b\right] =& Z^2_{ab}, \\
[\mathbf{P}^{\mathbf{s}}_{\mathbf{a}}, \mathbf{P}^{\mathbf{s}}_{\mathbf{b}}] =& \mathbf{J}^{\mathbf{s+s}}_{\mathbf{ab}}, \\
\left[\mathbf{P}^{\mathbf{s}}_{\mathbf{a}}, P^1_b\right] =& \mathbf{J}^{\mathbf{s+1}}_{\mathbf{ab}},
\end{aligned}
$$

$$
\begin{aligned}
\left[\tilde{\mathbf{P}}^{\mathbf{n}}_{\mathbf{a}}, \tilde{\mathbf{Q}}^{\mathbf{m}}_{\alpha}\right] =& \frac{\mathbf{1}}{\mathbf{2}}\left(\boldsymbol{\Gamma}_{\mathbf{a}}\tilde{\mathbf{Q}}^{\mathbf{m+n}}\right)_{\alpha},\\
\left[\tilde{\mathbf{P}}^{\mathbf{n}}_{\mathbf{a}}, Q_{\alpha}\right] =& \frac{\mathbf{1}}{\mathbf{2}}\left(\boldsymbol{\Gamma}_{\mathbf{a}}\tilde{\mathbf{Q}}^{\mathbf{n+1}}\right)_{\alpha},\\
\left[P_a, \tilde{\mathbf{Q}}^{\mathbf{m}}_{\alpha}\right] =& \frac{\mathbf{1}}{\mathbf{2}}\left(\boldsymbol{\Gamma}_{\mathbf{a}}\tilde{\mathbf{Q}}^{\mathbf{m+1}}\right)_{\alpha},\\
\left[P_a, Q_{\alpha}\right] =& \frac{\mathbf{1}}{\mathbf{2}}\left(\boldsymbol{\Gamma}_{\mathbf{a}}\tilde{\mathbf{Q}}^{\mathbf{2}}_{\alpha}\right)_{\alpha},\\
\left[\tilde{\mathbf{J}}^{\mathbf{n}}_{\mathbf{ab}}, \tilde{\mathbf{Q}}^{\mathbf{m}}_{\alpha}\right] =& \frac{\mathbf{1}}{\mathbf{2}}\left(\boldsymbol{\Gamma}_{\mathbf{a}}\tilde{\mathbf{Q}}^{\mathbf{m+n}}\right)_{\alpha},\\
\left[\mathbf{J}_{\mathbf{ab}}, \tilde{\mathbf{Q}}^{\mathbf{m}}_{\alpha}\right] =& \frac{\mathbf{1}}{\mathbf{2}}\left(\boldsymbol{\Gamma}_{\mathbf{a}}\tilde{\mathbf{Q}}^{\mathbf{m}}\right)_{\alpha},\\
\left[\tilde{\mathbf{J}}^{\mathbf{n}}_{\mathbf{ab}}, \tilde{\mathbf{Q}}_{\alpha}\right] =& \frac{\mathbf{1}}{\mathbf{2}}\left(\boldsymbol{\Gamma}_{\mathbf{a}}\tilde{\mathbf{Q}}^{\mathbf{n+1}}\right)_{\alpha},\\
\left[J_{ab}, Q_{\alpha}\right] =& \frac{1}{2}\left(\Gamma_a Q\right)_{\alpha},\\
\left\{\tilde{\mathbf{Q}}^{\mathbf{n}}_{\alpha}, \tilde{\mathbf{Q}}^{\mathbf{m}}_{\beta}\right\} =& -\frac{\mathbf{1}}{\mathbf{2}}\left[\left(\boldsymbol{\Gamma}^{\mathbf{ab}}\mathbf{C}\right)_{\alpha\beta}\tilde{\mathbf{J}}^{\mathbf{m+n}}_{\mathbf{ab}} - \mathbf{2}\left(\boldsymbol{\Gamma}^{\mathbf{a}}\mathbf{C}\right)_{\alpha\beta}\tilde{\mathbf{P}}^{\mathbf{m+n}}\right],\\
\left\{Q_{\alpha}, \tilde{\mathbf{Q}}^{\mathbf{m}}_{\beta}\right\} =& -\frac{\mathbf{1}}{\mathbf{2}}\left[\left(\boldsymbol{\Gamma}^{\mathbf{ab}}\mathbf{C}\right)_{\alpha\beta}\tilde{\mathbf{J}}^{\mathbf{m+n}}_{\mathbf{ab}} - \mathbf{2}\left(\boldsymbol{\Gamma}^{\mathbf{a}}\mathbf{C}\right)_{\alpha\beta}\tilde{\mathbf{P}}^{\mathbf{m+n}}\right],\\
\left\{Q_{\alpha}, Q_{\beta}\right\} =& -\frac{1}{2}\left[\left(\Gamma^{ab}C\right)_{\alpha\beta}Z_{ab} - 2(\boldsymbol{\Gamma}^{\mathbf{a}}\mathbf{C})_{\alpha\beta}\tilde{\mathbf{P}}^{\mathbf{2}}\right],
\end{aligned}
$$

Non-Standard Maxwell superalgebra

$$
\begin{aligned}
[J_{ab}, J_{cd}] =& \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}, \\
[J_{ab}, P_c] =& \eta_{bc} P_a - \eta_{ac} P_b, \\
[P_a, P_b] =& Z_{ab}, \\
[J_{ab}, Q_\alpha] =& \frac{1}{2} (\Gamma_a Q)_\alpha , \\
[Z_{ab}, Z_{cd}] =& 0, \\
[Z_{ab}, Q_\alpha] =& 0, \\
\{Q_\alpha, Q_\beta\} =& -\frac{1}{2} \left(\Gamma^{ab} C\right)_{\alpha\beta} Z_{ab}.
\end{aligned}
$$

J. Lukierski, Generalized Wigner-Inönü Contractions and Maxwell (Super)Algebras, (2011)

Tensores invariantes

$$
\begin{aligned}
\langle J_{ab} J_{cd} \rangle &= \alpha^0 \langle \tilde{J}_{ab} \tilde{J}_{cd} \rangle = \alpha^0 \tilde{\mu}_0 \left(\eta_{ad} \eta_{bc} - \eta_{ac} \eta_{bd} \right) \equiv \tilde{\alpha}^0 \left(\eta_{ad} \eta_{bc} - \eta_{ac} \eta_{bd} \right), \\
\langle J_{ab} Z_{cd} \rangle &= \alpha^2 \langle \tilde{J}_{ab} \tilde{J}_{cd} \rangle = \alpha^2 \tilde{\mu}_0 \left(\eta_{ad} \eta_{bc} - \eta_{ac} \eta_{bd} \right) \equiv \tilde{\alpha}^2 \left(\eta_{ad} \eta_{bc} - \eta_{ac} \eta_{bd} \right), \\
\langle J_{ab} P_c \rangle &= \alpha^1 \langle \tilde{J}_{ab} \tilde{P}_c \rangle = \alpha^1 \tilde{\mu}_1 \epsilon_{abc} \equiv \tilde{\alpha}^1 \epsilon_{abc}, \\
\langle P_a P_b \rangle &= \alpha^2 \langle \tilde{P}_a \tilde{P}_b \rangle = \alpha^2 \tilde{\mu}_0 \eta_{ab} \equiv \tilde{\alpha}^2 \eta_{ab}, \\
\langle Q_\alpha Q_\beta \rangle &= \alpha^2 \tilde{\mu}_0 C_{\alpha\beta} \equiv \tilde{\alpha}^2 C_{\alpha\beta},
\end{aligned}
$$

P.K. Concha, O. Fierro, E.K. Rodríguez, P. Salgado, Chern-Simons Su- pergravity in D = 3 and Maxwell superalgebra (2015)

Hidden AdS-Lorentz Superalgebra

$$
\begin{aligned}
[J_{ab}, J_{cd}] &= \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}, \\
[J_{ab}, Z_{cd}] &= \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \\
[Z_{ab}, Z_{cd}] &= \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \\
[Q_\alpha, Z_{ab}] &= -(\gamma_{ab} Q)_\alpha - (\gamma_{ab} Q')_\alpha, \\
[J_{ab}, P_c] &= \eta_{bc} P_a - \eta_{ac} P_b, \\
[Q_\alpha, P_a] &= -i(\gamma_a Q)_\alpha - i(\gamma_a Q')_\alpha, \\
[P_a, P_b] &= -Z_{ab}, \quad [J_{ab}, Q_\alpha] = -(\gamma_{ab} Q)_\alpha, \quad \left[J_{ab}, Q'_\alpha\right] = -(\gamma_{ab} Q')_\alpha, \\
[Z_{ab}, P_c] &= \eta_{bc} P_a - \eta_{ac} P_b, \\
\{Q_\alpha, Q_\beta\} &= -i(\gamma^a C)_{\alpha\beta} P_a - \frac{1}{2}(\gamma^{ab} C)_{\alpha\beta} Z_{ab}, \\
\left\{Q_\alpha, Q'_\beta\right\} &= \left\{Q'_\alpha, Q'_\beta\right\} = 0, \quad \left[Q'_\alpha, Z_{ab}\right] = 0, \quad \left[Q'_\alpha, P_a\right] = 0,
\end{aligned}
$$

Ecuaciones de Maurer-Cartan

$$
\begin{aligned}
R^{ab} &= 0, \\
DV^{a} &= \frac{i}{2}\bar{\Psi}\gamma^{a}\Psi - eB^{ab}V_{b}, \\
D\Psi &= \frac{i}{2}e\gamma_{a}\Psi V^{a} + \frac{e}{4}\gamma_{ab}\Psi B^{ab}, \\
DB^{ab} &= \frac{1}{2}\bar{\Psi}\gamma^{ab}\Psi - eB^{ac}B_{c}{}^{b} + eV^{a}V^{b}, \\
D\eta &= \frac{i}{2}\gamma_{a}\Psi V^{a} + \frac{1}{4}\gamma_{ab}\Psi B^{ab},
\end{aligned}
$$

Hidden Maxwell Superalgebra

$$
\begin{aligned}
[J_{ab}, J_{cd}] &= \eta_{bc} J_{ad} - \eta_{ac} J_{bd} - \eta_{bd} J_{ac} + \eta_{ad} J_{bc}, \\
[J_{ab}, Z_{cd}] &= \eta_{bc} Z_{ad} - \eta_{ac} Z_{bd} - \eta_{bd} Z_{ac} + \eta_{ad} Z_{bc}, \\
\left[J_{ab}, \tilde{Z}_{cd}\right] &= \eta_{bc} \tilde{Z}_{ad} - \eta_{ac} \tilde{Z}_{bd} - \eta_{bd} \tilde{Z}_{ac} + \eta_{ad} \tilde{Z}_{bc}, \\
[Z_{ab}, Z_{cd}] &= \eta_{bc} \tilde{Z}_{ad} - \eta_{ac} \tilde{Z}_{bd} - \eta_{bd} \tilde{Z}_{ac} + \eta_{ad} \tilde{Z}_{bc}, \\
[Q_\alpha, Z_{ab}] &= -\left(\gamma_{ab}\Sigma\right)_\alpha, \quad [\Sigma_\alpha, Z_{ab}] = 0, \qquad [J_{ab}, P_c] = \eta_{bc} P_a - \eta_{ac} P_b, \\
[Q_\alpha, P_a] &= -i(\gamma_a \Sigma)_\alpha, \quad [\Sigma_\alpha, P_a] = 0, \qquad [P_a, P_b] = -\tilde{Z}_{ab}, \\
[J_{ab}, Q_\alpha] &= -\left(\gamma_{ab} Q\right)_\alpha, \quad [J_{ab}, \Sigma_\alpha] = -(\gamma_{ab}\Sigma)_\alpha, \qquad [Z_{ab}, P_c] = 0, \\
\{Q_\alpha, Q_\beta\} &= -i(\gamma^a C)_{\alpha\beta} P_a - \frac{1}{2}(\gamma^{ab} C)_{\alpha\beta} Z_{ab}, \qquad \{\Sigma_\alpha, \Sigma_\beta\} = 0, \\
\{Q_\alpha, \Sigma_\beta\} &= -2(\gamma^{ab} C)_{\alpha\beta} \tilde{Z}_{ab}, \\
\left[\tilde{Z}_{ab}, P_c\right] &= \left[Q_\alpha, \tilde{Z}_{ab}\right] = \left[\Sigma_\alpha, \tilde{Z}_{ab}\right] = \left[\tilde{Z}_{ab}, \tilde{Z}_{cd}\right] = 0.
\end{aligned}
$$

Hidden Maxwell Superalgebra

$$
\begin{aligned}
R^{ab} &= 0, \\
DV^{a} &= \frac{i}{2}\bar{\Psi}\gamma^{a}\Psi, \\
D\Psi &= 0, \\
DB^{ab} &= \frac{1}{2}\bar{\Psi}\gamma^{ab}\Psi, \\
D\tilde{B}^{ab} &= \alpha e\bar{\Psi}\gamma^{ab}\Phi + \beta e B^{ac}B_{c}{}^{b} + \gamma e V^{a}V^{b}, \\
D\Phi &= \frac{i}{2}\delta\gamma_{a}\Psi V^{a} + \frac{1}{2}\varepsilon\gamma_{ab}\Psi B^{ab},
\end{aligned}
$$

B^{ab} y $\tilde{B}^{ab}$ son las 1-formas duales de los generadores Z_{ab} y $\tilde{Z}_{ab}$, respectivamente, y Φ es la 1-forma espinorial dual al generador fermionico extra nilpotente Σ_{α}.

Mapa

$$
\begin{tikzcd}
\text{Non-standard Maxwell} & & \text{Hidden Maxwell} & \\
& \text{Maxwell} \arrow[ul] & & \\
AdS \arrow[uu, "(S_E^{(\infty)} \times \mathfrak{g}) \ominus \mathcal{I}"] \arrow[ur] \arrow[d, "(S_E^{(\infty)} \times \mathfrak{g}) \ominus \mathcal{I}"] & & \text{Hidden } AdS\text{-Lorentz} \arrow[ul] \arrow[uu, "S_E^{(\infty)} \ominus \mathcal{I}"] \arrow[r, "\mathfrak{g} \ominus \mathcal{I}"] \arrow[d, "(S_E^{(\infty)} \times \mathfrak{g}) \ominus \mathcal{I}"] & AdS\text{-Lorentz} \\
\text{Poincaré} & & \text{Hidden Poincaré} \arrow[ll, "\mathfrak{g} \ominus \mathcal{I}"'] &
\end{tikzcd}
$$

Figure 3.1: Map between different superalgebras linked through infinite S-expansion and/or ideal subtraction.

Algunas observaciones

1. Obtención de Invariantes topológicos.
2. En (In)finite extensions of algebras from their Inönü-Wigner contractions, Oleg Khasanov1 and Stanislav Kuperstein, establecen que con cada contracción de IW de un álgebra a otra se puede extender la última de manera infinita.
3. No ceros en la S-expansión.
4. Semejanza con el trabajo de A Grassmann Path From AdS_3 to Flat Space, Chethan KRISHNAN, Avinash RAJU and Shubho ROY donde interpretan el radio AdS $1/l$ como una variable de Grasmann del cual resulta un mapeo formal entre gravedad en AdS_3 y un espacio plano.

¡Gracias!