Holographic recipe for type-B Weyl anomalies

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joint work with F. Bugini

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Aim of the talk

- Overview of holographic computation of CFT Weyl/trace/conformal anomalies:

  one of the most robust entries of the AdS/CFT dictionary.

- Unveil a recipe to read off type-B Weyl anomaly from bulk Lagrangian:

  arguably the simplest prescription, illustrated by explicit examples (if time permits).

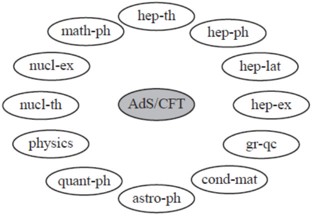
Outline

19 years of AdS/CFT duality

- Maldacena's conjecture: string-/M-theory in AdS_{n+1} $\Leftrightarrow$ CFT_n on the conformal boundary

  realization of holographic principle ['t Hooft & Susskind] & string of the large-N gauge theory ['t Hooft]

- Modern Times: plenty of extrapolations, top-down & bottom-up



The AdS/CFT duality spans all physics arXivs.

[AdS/CFT Duality User Guide, Makoto Natsuume]

Perhaps most impressive: AdS/QB "Holographic Photosynthesis" [arXiv:1603.09107 [hep-th]]
Word of caution: 'solution in search of a problem' [arXiv:1211.0004 [physics.pop-ph]]

Excerpts from the dictionary in the 'canonical' case

II B superstring @ $AdS_5 \times S^5$ $\Leftrightarrow$ $\mathcal{N}$=4 SU(N) SYM @ $Mink_4$

- isometries [AdS & S] $\Leftrightarrow$ global symmetries [conformal & R-symmetry]
- Planck length [L_{Planck} / L_{AdS}] $\Leftrightarrow$ rank of gauge group [$N^{-1/4}$]
- string size [L_{string} / L_{AdS}] $\Leftrightarrow$ 't Hooft coupling [$\lambda^{-1/4}$]

Classical SuGra [L_{Planck} & $L_{string} \to 0$] $\Leftrightarrow$ Planar & Strong [N & $\lambda >> 1$]

- sugra fields [dilaton, KK desc., graviton] $\Leftrightarrow$ gauge-inv. operators [YM F^2, single-traces, EM tensor]
- mass $\Leftrightarrow$ scaling dimension [e.g. scalar field $m^2 = \Delta(\Delta - n)$]

GKP/W calculational prescription $Z_{AdS} = Z_{CFT}$ [Gubser+Klebanov+Polyakov/ Witten '98]

- boundary behavior ($x \to 0$) of a scalar in Poincaré patch

$$ds^2 = \frac{L_{AdS}^2}{x^2}\{dx^2 + d\vec{y}^2\} \quad , \quad \hat{\phi} \sim \mathcal{A}\, x^{\Delta_-} + \mathcal{B}\, x^{\Delta_+}$$

- CFT interpretation: $\mathcal{A} \equiv J \to$ source , $\mathcal{B} \equiv \langle O_{\Delta_+} \rangle \to$ VEV of dual operator

- GKP/W prescription: CFT generating functional = gravitational partition function (saddle)

$$\boxed{Z_{CFT}[J] = e^{-S_{grav}[\hat{\phi} \sim J]}}$$

In particular, control on the boundary metric g would render the CFT energy-momentum $\langle T_{\mu\nu} \rangle$ and its (anomalous!) trace $\langle T \rangle$.

But on the boundary CFT, prescribe boundary metric g up to conformal trafos. $g \to e^{2\omega} g$ {orbit = conformal class $[g]$}

Subtlety: boundary metric & energy-momentum tensor

Resort to the Fefferman-Graham construction in conformal geometry [Fefferman+Graham'84]

- Bulk Poincaré-Einstein metric $\hat{g}$ with conformal infinity $(\mathcal{M}_n, [g])$. A representative g in the conformal class $[g]$ is associated to a defining function x so that (for even n)

$$\hat{g} = x^{-2}\left(dx^2 + g_x\right)$$

$$g_x = g + g_{(2)} \cdot x^2 + \text{even powers} + g_{(n)} \cdot x^n + h \cdot x^n \cdot \log x + ...$$

(i) $g_{(2i)}$ for $2i < n$ and trace of $g_{(n)}$ locally determined by g ;
(ii) traceless part of $g_{(n)}$ is divergence-free ;
(iii) h is traceless and locally determined by g .

- For a bulk Poincaré-Einstein $\hat{Ric} = -n\,\hat{g}$ the action is proportional to the volume (but needs regularization, e.g. with IR-cutoff ϵ)

$$I_{EH} = -\frac{\hat{R} - 2\hat{\Lambda}}{16\pi G_N} \cdot Vol_{\hat{g}}(\{x > \epsilon\}) = \frac{n}{8\pi G_N} \cdot Vol_{\hat{g}}(\{x > \epsilon\})$$

Volume anomaly and Q-curvature

- Volume asymptotics

$$Vol_{\hat{g}}(\{x > \epsilon\}) = \frac{c_0}{\epsilon^n} + \frac{c_2}{\epsilon^{n-2}} + (even\, powers) + \mathcal{L} \cdot \log \frac{1}{\epsilon} + \mathcal{V} + o(1)$$

- Renormalized volume $\mathcal{V}$ is anomalous, i.e. not a conformal invariant with respect to the conformal class $[g]$ of boundary metrics, and $\mathcal{L}$ is the (integrated) volume anomaly.

- Now, the integrated volume anomaly is the integral of *Branson's Q − curvature*#
[Graham+Zworski]:

$$\mathcal{L} = \int_{\mathcal{M}^n} Q_n$$

- Plug in the on-shell Lagrangian density to read off the CFT_n *trace anomaly*

$$\boxed{\langle T \rangle \; = \; coeff \times Q_n}$$

More on Q-curvature in F.Bugini's talk. For recent progress, e.g. recurrences, cf. [Juhl and Fefferman+Graham] .

Holographic trace anomaly [Henningson+Skenderis'99]

- $n = 2:$ $\frac{3L_{AdS}}{2G_N} = c$ ('real' holography!) [Brown+Henneaux'86]

$$\langle T \rangle = c \cdot R$$

- $n = 4:$ $\frac{L^3_{AdS}}{G_N} = N^2$ (again 'real' holography!) . Pure Ricci (no $Riem^2$) so that $a = c$

$$\langle T \rangle \equiv -a\,E_4 + c\,W^2 = N^2 \cdot \{Ric^2 - \frac{1}{3}R^2\}$$

- $n = 6:$ $\frac{L^5_{AdS}}{G_N} = N^3$. Again Pure Ricci.

$$\langle T \rangle \equiv a\,E_6 + c_1\,trW^3 + c_2\,tr'\,W^3 + c_3\,(W\Box W + \ldots) = N^3 \cdot \{Ric^3 + \ldots\}$$

Beyond EH : $c \neq a$

- O(N) correction from open strings and closed unoriented: $\widehat{Riem}^2$ [Blau+Gava+Narain]
- General quadratic Lagrangian: $\alpha \cdot \widehat{R}^2 + \beta \cdot \widehat{Ric}^2 + \gamma \cdot \widehat{Riem}^2$ [Nojiri+Odintsov and Schwimmer+Theisen]
- Violation of the KSS bound $\eta/s \geq 1/4\pi$: quasi-topological gravity w/ cubic and quartic curv. inv. [Myers et al.]

(A) Universal result for Type-A (Euler term): read off directly from the Lagrangian at the 'true AdS vacuum' :) [Imbimbo+Schwimmer+Theisen+Yankielowicz]

(B) Whereas for Type-B (pure Weyl terms): several scattered results :([Myers et al.; Parnachev et al.; Rong-Xin Miao; ...]
One needs to compute in each dimension for: 3 quadratic, 11 cubic, ...

A useful trick ...but still no pattern

(5 to 4 dims)

$$\langle T \rangle = -a\, E_4 + c\, W^2$$

(i)evaluate at the round sphere (conformally flat) to get a : bulk AdS! [Imbimbo et al.]

(ii)evaluate at a Ricci-flat metric to get c : bulk can be reconstructed!

(7 to 6 dims)

$$\langle T \rangle = a\, E_6 + c_1\, I_1 + c_2\, I_2 + c_3\, I_3 .$$

(i)evaluate at the round sphere (conformally flat) to get a : bulk AdS! [Imbimbo et al.]

(ii)evaluate at a Ricci-flat metric and two more to get all $c's$: bulk can be reconstructed in each case!

We can do all above computations in a single step for a generic boundary Einstein metric: this is one of the exceptional cases where the bulk metric can be fully reconstructed (no obstruction)!

Our recipe:

(i) Evaluate bulk action in a Poincaré metric (possibly w/ modified AdS radius): pure-Ricci terms will only contribute to the volume (traceless-Ricci and Cotton vanish) and collect the 'deviations' which are pure-Weyl.

(ii) Go to the basis where one trades I_3 by the Fefferman-Graham invariant $\Phi_{n+1} = -W\Box W + 16PWW$:

- (5 to 4 dims)

$$\{\widehat{1}, \widehat{W^2}, \widehat{I}_1, \widehat{I}_2, \widehat{\Phi}_5, \ldots\}$$

  and now read off the anomaly $\mathcal{A}_4$

$$\boxed{\mathcal{A}_4(\{\widehat{1}, \widehat{W^2}, \widehat{I}_1, \widehat{I}_2, \widehat{\Phi}_5, \ldots\}) = \{Q_4, W^2, 0, 0, 0, \ldots\}}$$

- (7 to 6 dims)

$$\{\widehat{1}, \widehat{I}_1, \widehat{I}_2, \widehat{\Phi}_7, \widehat{W^4} \ldots\}$$

  and now read off the anomaly $\mathcal{A}_6$

$$\boxed{\mathcal{A}_6(\{\widehat{1}, \widehat{I}_1, \widehat{I}_2, \widehat{\Phi}_7, \widehat{W^4} \ldots\}) = \{Q_6, I_1, I_2, \Phi_6, 0, \ldots\}}$$

MANY THANKS FOR YOUR ATTENTION