Pure Gauss-Bonnet holographic superconductor

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arXiv:1507.02309(PRD), 1511.02541(PRD), 1308.2976(EJPC)
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Quinto Encuentro COSMOCONCE, Concepción

April 7-8, 2016

Plan of the talk

- Motivation: Pure Lovelock gravity
- Properties of Pure Lovelock theories
- Black hole solutions and their thermodynamics
- Scalar field coupled to Pure Lovelock gravity
- Holographic phase transition
- Final remarks

Motivation: Pure Lovelock gravity

- Higher-order curvature terms can be seen as high-energy corrections of Einstein gravity in the strong regime
- **Lanczos-Lovelock gravity** – natural extension of Gen. Rel. to higher $D = d + 1$ dimensions, second order field equations in $g_{\mu\nu}$

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- **Our motivation in the context of AdS/CFT**:

\- *dynamics of gravity described by purely higher-order terms*

Properties of Pure Lovelock theories

Lanczos-Lovelock action

I	$= \int d^{d+1}x \sum_{k=0}^{[d/2]} \alpha_k \mathcal{L}_k(R)$	family of theories $\{\alpha_k\}$
$\mathcal{L}_k$	$= \frac{1}{2^k} \sqrt{-g}\ \delta^{\mu_1 \cdots \mu_{2k}}_{\nu_1 \ldots \nu_{2k}}\ R^{\nu_1 \nu_2}_{\mu_1 \mu_2} \cdots R^{\nu_{2k-1} \nu_{2k}}_{\mu_{2k-1} \mu_{2k}}$	dim. continued Euler density

$$\mathcal{L} = \quad \alpha_0 + \alpha_1 R + \alpha_2 \left(R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} \right) + \cdots$$

cosmological constant + Einstein-Hilbert + Gauss-Bonnet + $\cdots$

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Equations of motion

$$0 = L^\mu_\nu \equiv -\sum_k \frac{\alpha_k}{2^{k+1}}\,\delta^{\mu\mu_1\cdots\mu_{2k}}_{\nu\nu_1\cdots\nu_{2k}}\, R^{\nu_1\nu_2}_{\mu_1\mu_2}\cdots R^{\nu_{2k-1}\nu_{2k}}_{\mu_{2k-1}\mu_{2k}}$$

$$\sim R^\mu_\nu - \frac{1}{2}R\delta^\mu_\nu + \Lambda\delta^\mu_\nu + \alpha H^\mu_\nu + \cdots$$

L^μ_ν = Lovelock tensor: symmetric, satisfies the Bianchi identity

$$\frac{\alpha_0}{\alpha_1} = -2\Lambda\,, \qquad \frac{\alpha_2}{\alpha_1} = \alpha\,, \qquad \alpha_1 = \frac{1}{16\pi G}\,, \qquad \ldots$$

Properties of Pure Lovelock theories

Gravities with constant curvature vacua

- Maximally symmetric, empty space solution with constant curvature

$$\boxed{R^{\mu\nu}_{\alpha\beta} = \lambda \delta^{\mu\nu}_{\alpha\beta}} \qquad \lambda = \pm\frac{1}{\ell^2}$$

- Equation of motion

$$P(\lambda) = \sum_k^{k_{\max}} \frac{\alpha_k}{(d-2k-1)!}\,\lambda^k = 0 \quad \Rightarrow \quad \text{a set } \{\lambda_{\text{eff}}\}$$

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Special choice of $\{\alpha_k\}$

- For some critical values of α_k, the vacuum λ_{eff} is degenerate and the metric is not fully fixed by the field equations [Wheeler '86]
- Pure Lovelock (PL) gravity has a (unique) non-degenerate vacuum
- General Relativity is a special case of PL gravity

Properties of Pure Lovelock theories

Only two couplings in the action, α_0 and α_p $(p \geq 1)$

[Crisostomo,Troncoso, Zanelli '00; Cai,Ohta '06]

$$I = \pm \frac{1}{2\kappa} \int d^{d+1}x \left(\frac{1}{2^p}\, \delta^{\mu_1 \cdots \mu_{2p}}_{\nu_1 \cdots \nu_{2p}}\, R^{\nu_1 \nu_2}_{\mu_1 \mu_2} \cdots R^{\nu_{2p-1} \nu_{2p}}_{\mu_{2p-1} \mu_{2p}} - 2\Lambda \right)$$

Gravitational constant $[\kappa] = (\text{length})^{d+1-2p}$

Cosmological constant $[\Lambda] = (\text{length})^{-2p}$

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Gravitational constant	$[\kappa] = (\text{length})^{d+1-2p}$
Cosmological constant	$[\Lambda] = (\text{length})^{-2p}$

- Effective (A)dS radius $\quad P(\lambda) \propto \lambda^p - \frac{2(d-2p-1)!\Lambda}{(d-1)!}$
- In odd dimensions there is a unique (A)dS vacuum
- In even dimensions, for $p = 1$ there is a unique vacuum and for $p > 1$ there is one dS and one AdS vacuum, they are never degenerate

Properties of Pure Lovelock theories

Degrees of freedom N^*

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- Number of d.o.f. depends on the background, $0 \leq N^* \leq \frac{D(D-3)}{2}$
  AdS background: $N^* = \frac{D(D-3)}{2}$ [Dadhich,Durka,Merino,O.M.'16]

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PL gravity does not have ghosts

- Einstein-GB gravity in AdS space has two AdS vacua and one has ghosts [Boulware, Deser '85]
- Ghosts in PL gravity are analyzed by looking at the effective action around a background, $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
- Background is dS ($\epsilon = +1$) or AdS ($\epsilon = -1$)

Properties of Pure Lovelock theories

- Linearized equation of motion

$$\boxed{\delta L^{\mu}_{\nu} = \frac{\epsilon^{p-1} p\,(d-2)!}{\ell^{2p-2}\,(d-2p)!}\,\delta G^{\mu}_{\nu}}$$

L^{μ}_{ν} = Lovelock tensor; G^{μ}_{ν} = Einstein tensor;

- Sign of the kinetic term: ϵ^{p-1}

  (gives ghost-free $I_{\text{eff}}[h]$ in the (A)dS background)

- Einstein-Hilbert AdS $\quad \frac{1}{2\kappa}\int (R-2\Lambda)$
- Pure Gauss-Bonnet AdS $\quad -\frac{1}{2\kappa}\int (RR-2\Lambda)$

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Now we can move on and look at the black hole solutions

Black hole solutions in PL AdS gravity

- **Pure Lovelock gravity coupled to Maxwell field**

$$\begin{aligned} L_{\mu\nu}(R) &= T_{\mu\nu}(F) \\ \nabla_\mu F^{\mu\nu} &= 0 \end{aligned}$$

- **Static, spherically symmetric BH ansatz**

$$g_{\mu\nu}\, dx^\mu dx^\nu = -f(r)\, dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2_{d-1}$$
$$A = \phi(r)\, dt$$

- **General asymptotically AdS solution**

- Electrically neutral [Cai '06] $\qquad f(r) = k + \frac{r^2}{\ell^2}\left(1 - \frac{M_0 \ell^{2p}}{r^d}\right)^{\frac{1}{p}}$

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- Electrically charged

$$f(r) = k + \frac{r^2}{\ell^2}\left(1 - \frac{M_0 \ell^{2p}}{r^d} + \frac{(-1)^{p-1}\rho^2 \ell^{2p}}{c_p\, e^2 r^{2d-2}}\right)^{\frac{1}{p}} \qquad k = 0, \pm 1$$

$$\phi(r) = \mu - \frac{\rho}{r^{d-2}}\,, \qquad c_p = \frac{2(d-1)(d-3)!}{(d-2p)!}$$

Black hole solutions in PGB AdS gravity

Existence of the singularity $R \sim (M_0/r^d)^{1/p}$ $(M_0 \neq 0, p > 0)$

Existence of the horizon $f(r_+) = 0, \quad p > 1$

- Spherical horizons ($k = 1$) *do not exist*
- Non-compact horizons exist: planar and hyperbolic ($k = 0, -1$)

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Planar black holes, electrically neutral, $p > 1$

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Hyperbolic geometry $\frac{r_+^{2d-2}}{\ell^{2p}} - r_+^{2d-2p-2} - M_0 r_+^{d-2} + \frac{(-1)^{p-1}\rho^2}{c_p e^2} = 0$

Black hole solutions in PGB AdS gravity

Asymptotic behavior is Schwarzschild-AdS (or RN-AdS)

$$f \longrightarrow -1 + \frac{r^2}{\ell^2} - \frac{M}{r^{d-2}} + \frac{(-1)^p Q^2}{c_p\, r^{2d-4}}\,, \qquad r \longrightarrow \infty$$

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We choose 4D quantum field theory $\leftrightarrow$ 5D AdS gravity

5D Pure Gauss-Bonnet black hole, $d = 4$, $p = 2$, $\Lambda = \frac{12}{\ell^4}$

$$\begin{aligned} f(r) &= -1 + \frac{r^2}{\ell^2}\sqrt{1 - \frac{M_0\,\ell^4}{r^4} - \frac{\rho^2\,\ell^4}{6e^2\,r^6}} \\ \phi(r) &= \mu - \frac{\rho}{r^{d-2}} \end{aligned}$$

There is only one horizon $r_+^2 > 0$

Hawking temperature $$T = \frac{1}{4\pi}\left(\frac{2r_+^3}{\ell^4} + \frac{\rho^2}{6e^2}\frac{1}{r_+^3}\right)$$

There are no extremal PGB AdS black holes (only one r_+)

Black hole solutions in PGB AdS gravity

Black hole thermodynamics

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$$B_{\text{gravity}} = \frac{3}{4\kappa} \int\limits_{r\to\infty} d^4x \sqrt{-h}\, \delta^{j_1\cdots j_4}_{i_1\cdots i_4} K^{i_1}_{j_1} \delta^{i_2}_{j_2} \left(\mathcal{R}^{i_3 i_4}_{j_3 j_4} - K^{i_3}_{j_3} K^{i_4}_{j_4} + \frac{1}{3\ell^2} \delta^{i_3}_{j_3} \delta^{i_4}_{j_4} \right)$$

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  $K_{ij} =$ extrinsic curvature; $\mathcal{R}^i{}_{jkl}(h) =$ intrinsic curvature of ∂M
- The solution satisfies the quantum statistical relation

  $$G = TI^E = U + \mu Q - TS$$
- Thermodynamic internal energy $U = \frac{V_3}{\kappa}\left(\frac{9k^2}{2} + 3M\right)$ is Noether E
- Thermodynamic electric charge $Q = \frac{V_3 \rho}{\kappa e^2}$ is $U(1)$ Noether charge
- Entropy in Lovelock gravities is known $S = \frac{24\pi V_3}{\kappa} r_+$

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We can turn now to a holographic field theory.

Application of Gauge/Gravity duality

AdS gravity in $d+1$ dimensions	$\leftrightarrow$	CFT in d dimensions
Non-extremal black hole in AdS_{d+1} gravity	$\leftrightarrow$	Thermal QFT_d

- Mathematically, this is an equivalence of quantum partition functions

of two theories

$$\begin{aligned} Z_{\mathrm{AdS}}[\phi \to \phi_{(0)}] &\simeq Z_{\mathrm{QFT}}[\phi_{(0)}] \\ I_{\mathrm{ren}}[\phi_{(0)}] &\simeq I_{\mathrm{eff}}[\phi_{(0)}] \end{aligned}$$

Gravitational bulk fields	$\phi = \{g_{\mu\nu}, A_\mu, \psi, \ldots\}$
Boundary conditions	$\phi \to \phi_{(0)}$
Regularized (finite) classical action	$I_{\mathrm{ren}}[\phi_{(0)}]$ (gravity)
Quantum effective action	$I_{\mathrm{ren}}[\phi_{(0)}]$ (QFT)

Holographic superconductors

- Calculation of the CFT operators $\tau^{ij}, J^i, \mathcal{O}, \ldots$ (energy-momentum tensor, electric charge current, scalar operator, ...)

$$\boxed{\delta I_{\text{ren}} = \int d^d x \sqrt{-g_{(0)}} \left(\frac{1}{2} \tau^{ij} \delta g_{(0)ij} + J^i \, \delta A_{i(0)} + \mathcal{O} \, \delta\phi_{(0)} + \cdots \right)}$$

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Comments

- Strong coupling QFT results are obtained from classical gravity approximation
- It is not necessary to know the local CFT action to obtain physical information about the system – it is all contained in the quantum effective action

Holographic superconductors

- **Identification of macroscopic quantities**

- QFT electric current (1-point function) $\langle J^i \rangle = \frac{1}{\sqrt{|g_{(0)}|}} \frac{\delta I_{\text{ren}}}{\delta A_{i(0)}}$

- QFT transport coefficients = n-point functions

  Ohm law $J^i(\omega) = \sigma^{ij}(\omega) E_j \Rightarrow$ Conductivity

  (the linear response of a system to a temperature gradient & electric field)

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- **Role of the scalar field**

$\psi = 0$, Black hole solution, $T \geq T_c$, Normal phase of a superconductor, G_0
$\psi \neq 0$, Hairy black hole, $T \leq T_c$, Superconducting phase, $G \leq G_0$

- **Thermodynamic free energy** G

- Calculated from the Euclidean action, I^E_{ren}

Scalar field coupled to gravity

We need an order parameter, complex scalar field $\hat{\Psi} = \frac{1}{\sqrt{2}}\Psi(x)\, e^{i\theta(x)}$

Stückelberg holographic superconductor [Franco et al. '09]

$\mathcal{F}(\Psi) > 0$, non-minimal $U(1)$ coupling

$$I_{\text{Stück}} = -\frac{1}{2\kappa}\int d^5x\sqrt{-g}\left[-\frac{1}{4e^2}F^2 - \frac{1}{2}(\partial\Psi)^2 - \frac{m^2}{2}\Psi^2 - \frac{1}{2}\mathcal{F}\,(\partial\theta - A)^2\right]$$

Has an interesting phase transition structure if $\mathcal{F} = \Psi^2 + c_3\Psi^3 + c_4\Psi^4$

Total action $\quad I = I_{\text{PGB}} + I_{\text{Maxwell}} + I_{\text{Stück}} + B$

Equations of motion

$$\begin{aligned} L_{\mu\nu} &= T_{\mu\nu}(F,\Psi) \\ \nabla_\nu F^{\nu\mu} &= -e^2\mathcal{F}(\Psi)\,(\nabla^\mu\theta - A^\mu) \\ (\Box - m^2)\,\Psi &= \frac{1}{2}\mathcal{F}'(\Psi)\,(\nabla\theta - A)^2 \end{aligned}$$

Gauge fixing the local $U(1)$ symmetry: $\theta(x) = 0$

Scalar field coupled to gravity

Black hole ansatz: $k = -1$, static, spherically symmetric

$$
\begin{aligned}
g_{\mu\nu}\, dx^{\mu} dx^{\nu} &= -f(r)\, dt^2 + \frac{dr^2}{f(r)N(r)} + r^2 d\Omega^2_{k=-1} \\
A_{\mu} &= \phi(r)\, \delta^t_{\mu} \\
\Psi &= \Psi(r)
\end{aligned}
$$

Equations of motion

$$
\begin{aligned}
0 =& -\frac{6fN'(fN-k)}{r^3} + \frac{1}{2}\, fN\, \Psi'^2 + \frac{\mathcal{F}\phi^2}{2f} \\
0 =& -\frac{6Nf'(fN-k)}{r^3} + \Lambda + \frac{N\phi'^2}{4e^2} - \frac{\mathcal{F}\phi^2}{4f} + \frac{1}{4}\, m^2\Psi^2 - \frac{1}{4}\, fN\Psi'^2 \\
0 =& \frac{3Nf\,\Psi'}{r} + \frac{N'f\,\Psi'}{2} + Nf'\,\Psi' + Nf\,\Psi'' - m^2\Psi + \frac{\phi^2\mathcal{F}'}{2f} \\
0 =& \frac{3N\phi'}{r} + \frac{\phi' N'}{2} + N\phi'' - \frac{e^2\mathcal{F}\phi}{f}
\end{aligned}
$$

It is difficult to find an exact solution when $\Psi \neq 0$

Scalar field coupled to gravity

Boundary conditions

i) Behavior at the horizon $(r = r_+)$

$$\begin{aligned} f(r_+) &= 0 \\ f'(r_+) &= \frac{4\pi T}{\sqrt{N(r_+)}} = \text{ finite} \\ N(r_+) &= \text{ finite} \neq 0 \\ \phi(r_+) &= 0 \\ \Psi'(r_+) &= \frac{m^2}{4\pi T}\frac{\Psi(r_+)}{\sqrt{N(r_+)}} = \text{ finite} \end{aligned}$$

ii) Behavior at the boundary $(r \to \infty)$

$$\begin{aligned} f &\simeq -1 + \frac{r^2}{\ell^2} - \frac{M_0\ell^2}{2r^2} \\ fN &\simeq -1 + \frac{r^2}{\ell^2} - \frac{M\ell^2}{2r^2} \\ N &= \frac{fN}{f} \simeq 1 + \frac{(M_0 - M)\ell^4}{2r^4} \\ \phi &\simeq \mu - \frac{\rho}{r^2} \\ \Psi &\simeq \frac{\Psi_-}{r^{\Delta_-}} + \frac{\Psi_+}{r^{\Delta_+}} \end{aligned}$$

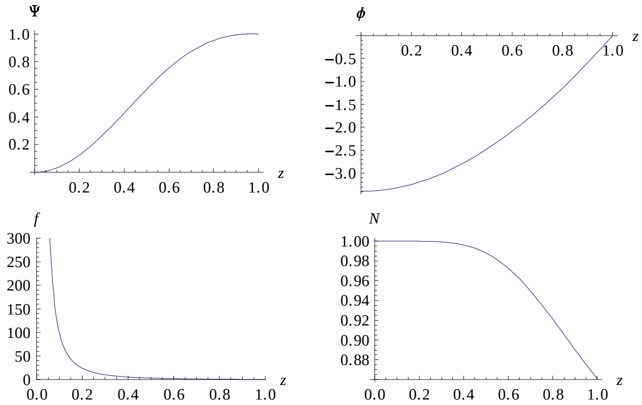
Comments on the choice of the fall-off of the scalar

- Boundary conditions: $\delta\Psi_- = 0$ (*Dirichlet boundary conditions*) or $\delta\Psi_+ = 0$ (*Neumann boundary conditions*)
- Ψ is finite if $\Delta_\pm \geq 0$ (the dual operator is relevant or marginal)
- The dual CFT is unitary in the Breitenlohner-Freedman window, $-4 \leq m^2\ell^2 \leq -3$ or $(2 \leq \Delta_+ \leq 3)$
- We will set $m^2\ell^2 = -3$ that corresponds to $\Delta_- = 1$ and $\Delta_+ = 3$, and choose the Neumann boundary condition where $\Psi_- = 0$. Then the response operator is Ψ_+ so that

$$\boxed{\Psi(r) \simeq \frac{\langle \mathcal{O}_+ \rangle}{r^3}}$$

Scalar field coupled to gravity

Hairy black hole, minimal coupling, $z = \frac{r_+}{r}$, $0 < z \leq 1$

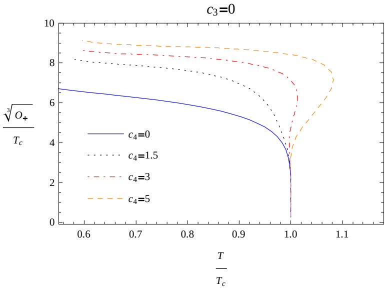


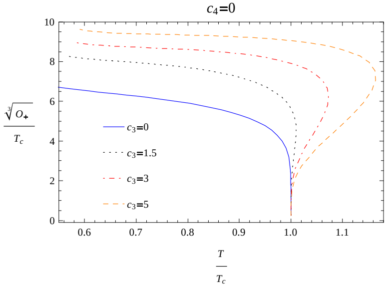
We have to interpret these results

Exact solution: One solution $\Psi = 0$ for all T

Numerical solution: $\Psi \simeq \frac{\langle \mathcal{O}_+ \rangle}{r^3} \neq 0$ when $T \leq T_c = 0,09196\mu$ $(\forall c_n)$

$$\frac{\sqrt[3]{\langle \mathcal{O}_+ \rangle}}{T_c} = \text{dimensionless quantity}$$





Discussion of numerical results

- **Minimal coupling**: $(c_3 = c_4 = 0)$ a possible **second order phase transition**, when the VEV increases monotonously
- **Cubic or quartic interaction**: phase transition changes its character from second order to **first order phase transition** ; VEV does not increase monotonously, there exists a metastable state around $(c_3, c_4) = (0, 2.4)$, $(1.5, 0)$
- **Below** T_c**, two possible phases**, but only the one with lower thermodynamic free energy will be physically realized.

Holographic phase transition

PROBE LIMIT

Scalar field is decoupled from the gravitational background, $e \longrightarrow \infty$

Boundary terms

$$
\begin{aligned}
I_{\text{matter}} &= \frac{1}{4\kappa e^2}\int d^5x\sqrt{\gamma}\,r^3\left(-\phi'^2 + f\Psi'^2 + m^2\Psi^2 - \frac{1}{f}\phi^2\mathcal{F}\right)\\
\delta I_{\text{matter}} &= \frac{1}{2\kappa e^2}\int d^4x\sqrt{\gamma}\,r^3\left(-\phi'\delta\phi + f\Psi'\delta\Psi\right)
\end{aligned}
$$

Grand canonical ensemble: $\delta\phi = 0$

Neumann boundary conditions for the scalar: $\delta\Psi' = 0$

$$
\begin{aligned}
B_{\text{matter}} &= -\frac{1}{2\kappa e^2}\int\limits_{r_B} d^4x\sqrt{\gamma}\,r^3 f\Psi\Psi'\\
\delta(I_{\text{matter}} + B_{\text{matter}}) &= -\frac{1}{2\kappa e^2}\int d^4x\sqrt{\gamma}\,r^3\left(\phi'\delta\phi + f\Psi\delta\Psi'\right)
\end{aligned}
$$

$\Rightarrow$ **Boundary problem is well-defined**

Holographic phase transition

- Euclidean on-shell evaluation of the action:

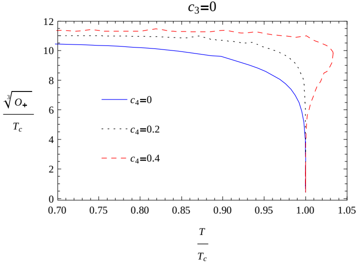
$$I^E_{\rm bulk} + B^E_{\rm matter} = \frac{V_3}{4\kappa e^2 T}\left[\left(r^3 f \Psi\Psi' + r^3\phi\phi'\right)\Big|_{r_+}^{\infty} - \int_{r_+}^{\infty} dr\, \frac{r^3\phi^2\Psi\mathcal{F}'}{2f}\right]$$

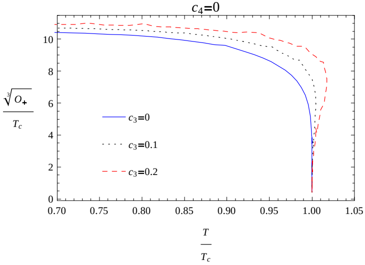
- There is a non-local term (due to a probe limit)
- Scalar field needs the countertem [Franco et al. '09] , but it does not contribute when $\Psi_- = 0$
- Finite free energy, to be calculated numerically

$$\begin{aligned} G &= T\left(I^E_{\rm bulk} + B^E_{\rm matter}\right) \\ &= \frac{V_3}{2\kappa e^2}\left(\rho\mu - \int_{r_+}^{\infty} dr\, \frac{r^3\phi^2\Psi\mathcal{F}'}{4f}\right) \end{aligned}$$

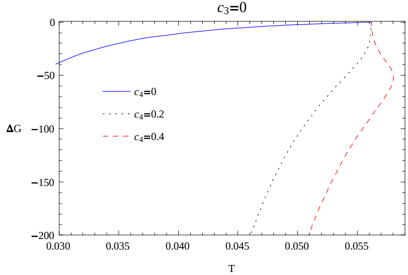
- Numerical results: $T_c \simeq 0.05615\mu$
- $G_{\text{superconducting}} \leq G_{\text{normal}}$, the phase transition will occur
- Results similar to Einstein Gravity

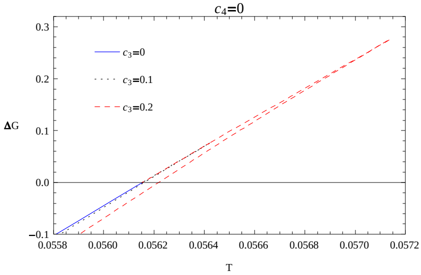
Condensation





Change of the free energy $\Delta G = G_{\text{superconducting}} - G_{\text{normal}}$





Final remarks

Summary

- Pure Lovelock gravity does not have ghosts in AdS background
- Neutral and charged PL AdS black holes have hyperbolic geometry
- Gravitational action can be renormalized
- With Stückelberg scalar and Maxwell fields coupled, there is a charged, hairy black hole solution below some T_c
- The critical temperature is lower compared to EH or EGB cases
- With the increase of the Stückelberg parameters, the hairy solution becomes stronger while the critical temperature is not affected.
- In the probe limit, the free energy decreases below T_c and there is a phase transition or first or second order

Final remarks

More issues to understand better

- Including the backreaction, the free energy becomes divergent below T_c when the mass of Ψ is equal to the upper limit of the BF bound
- We have to understand the divergence of G analytically (log modes)
- Change of the boundary conditions for the scalar
- Calculation of the transport coefficients,
- Meissner effect,

....

Final remarks

More issues to understand better

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- We have to understand the divergence of G analytically (log modes)
- Change of the boundary conditions for the scalar
- Calculation of the transport coefficients,
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....

THANK YOU!