Warm $\frac{\lambda}{4}\phi^4$ inflation in light of Planck 2015 results

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Quinto Encuentro COSMOCONCE
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Outline

The Big-Bang cosmological model

- General Relativity (GR)

$$G_{\mu\nu} = \kappa T_{\mu\nu}, \text{ where } \kappa = 8\pi G$$

- An isotropic and homogeneous expanding universe is described by the Friedmann-Robertson-Walker (FRW) metric

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2 \left(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2 \right) \right].$$

- Energy-momentum tensor for a perfect fluid

$$T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu}$$

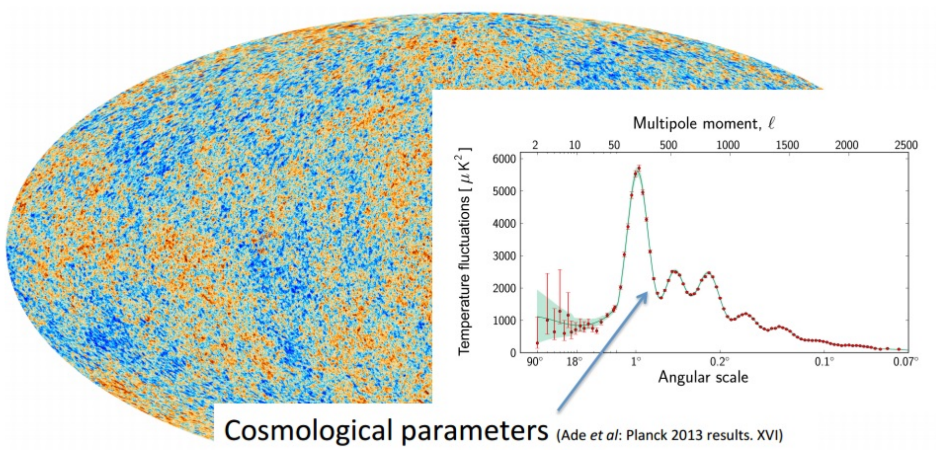
- Friedmann and acceleration equations

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa}{3}\rho - \frac{k}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{6}(\rho + 3P)$$

Observational evidence supporting the Big-Bang cosmological model

- *Hubble's law*
- *Cosmic Microwave Background (CMB)* ($T \sim 2.7K$)
- *Primordial Nucleosynthesis*



$h = 0.674 \pm 0.014$

$\Omega_\Lambda = 0.686 \pm 0.020$

$\Omega_{matter} = 0.314 \pm 0.020$

$\Omega_\kappa = -0.04 \pm 0.05$

Cosmic Inflation

Some Big-Bang cosmological model shortcomings:

- *The flatness problem*
- *The horizon problem*
- *The exotic relics problem*

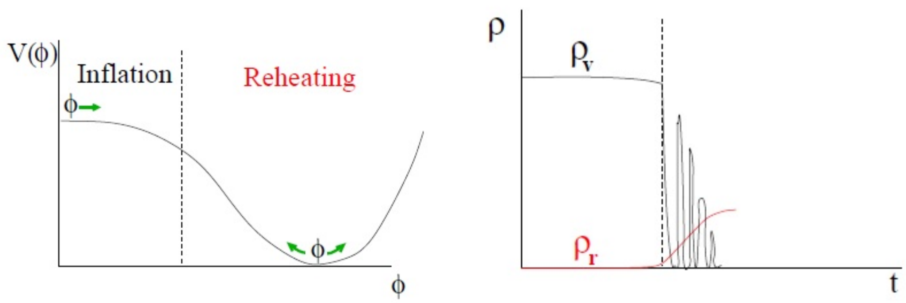
These problems are solved by a phase of acceleration during the very early universe:

$$\ddot{a} > 0 \Rightarrow P < -\rho/3$$

(Starobinsky (1980); Guth (1981); Albrecht, Steinhardt (1982); Linde (1982))

Scalar field inflation

How to get inflation?



- Scalar field $S_\phi = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right)$
- Homogeneus scalar field in a $k = 0$ FRW universe

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad P_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$$

- Equations of motion

$$H^2 = \frac{\kappa}{3} \left(\frac{\dot{\phi}^2}{2} + V(\phi) \right), \quad \ddot{\phi} + 3H\dot{\phi} + V_\phi = 0$$

Scalar field inflation

- Slow-roll motion $\frac{1}{2}\dot{\phi}^2 \ll V(\phi),\ \ \ddot{\phi} \ll 3H\dot{\phi}$

$$P_\phi \simeq -\rho_\phi$$

- Slow-roll equations

$$H^2 \simeq \frac{\kappa}{3}\,V, \quad 3H\dot{\phi} + V_\phi \simeq 0$$

- Slow-roll parameters

$$\epsilon_V = \frac{1}{2\kappa}\left(\frac{V_\phi}{V}\right)^2 \ll 1, \quad \eta_V = \frac{1}{\kappa}\frac{V_{\phi\phi}}{V} \ll 1$$

- Number of e-folds

$$N \equiv \int_{t_*}^{t_{end}} H\,dt$$

Cosmological perturbations

Inflationary observables

Extra bonus: the standard theory of inflation predicts that the large scale structure of the universe can be traced back to quantum vacuum fluctuations of the inflaton during the inflationary expansion.

- Curvature perturbation: $\mathcal{R} = H\delta\phi/\dot{\phi}$, $\delta\phi = \frac{H}{2\pi}$

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_{\mathcal{R}}(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}, \ \mathcal{P}_{\mathcal{R}}(k_*) = amplitude,$$

- Scalar spectral index:

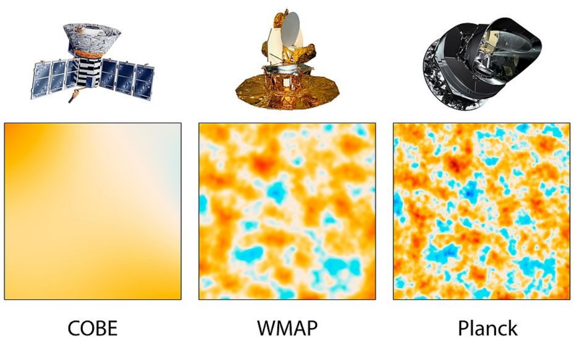
$$n_s - 1 = \frac{d\ln \mathcal{P}_R}{d\ln k} \simeq 2\eta_V - 6\epsilon_V,$$

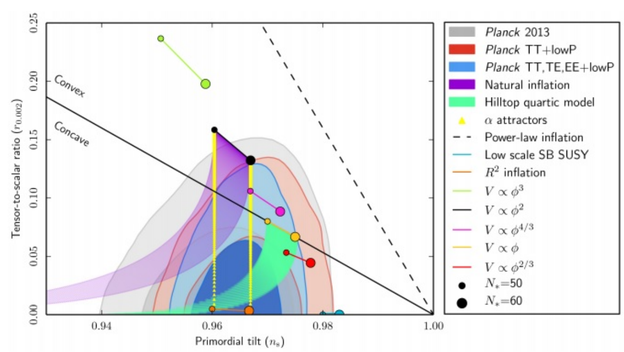
- Tensor-to-scalar ratio:

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} \simeq 16\,\epsilon_V$$

Inflation and CMB observations

Inflationary observables





- Planck 2015 results (P. A. R. Ade *et al.* (2015)):

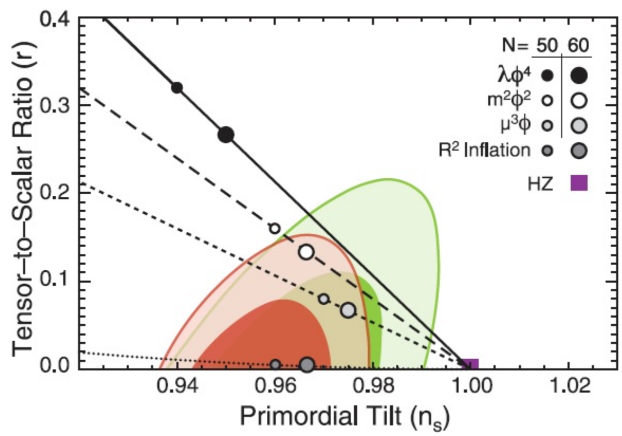
$$\mathcal{P}_{\mathcal{R}}(k_*) = \left(2.142^{+0.049}_{-0.049}\right) \times 10^{-9}$$

$$n_s = 0.9667 \pm 0.004$$

$$r < 0.113$$

$\frac{\lambda}{4}\phi^4$ inflationary model

$$\epsilon_V = 8\frac{M_p^2}{\phi^2},\ \eta_V = 12\frac{M_p^2}{\phi^2}$$
$$\mathcal{P}_{\mathcal{R}} \simeq \frac{2\lambda}{3\pi^2}N^3 \rightarrow \lambda \sim 10^{-14}$$
$$n_s \simeq 1 - \frac{4}{N}$$
$$r \simeq 4(1-n_s)$$

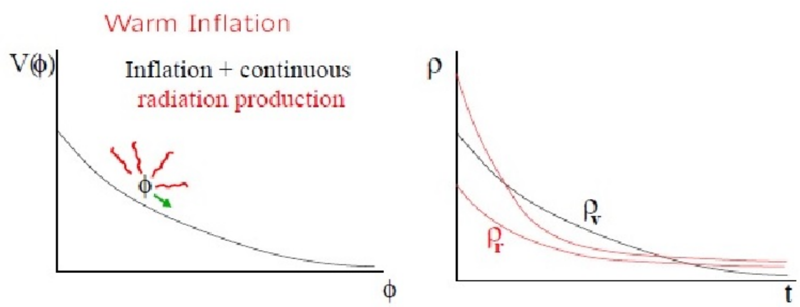


The $\frac{\lambda}{4}\phi^4$ model, which predicts a large value of the tensor-to-scalar ratio r , lies well outside of the joint 95 % CL region in the $n_s - r$ plane for the WMAP 9-year data and is further excluded by CMB data at smaller scales.

..but what if the universe was not supercooled?

Warm inflation

A. Berera and L. Z. Fang PRL74 1995, Berera PRL75 1995



- Interactions of the inflaton with other d.o.f. are important during inflation and generate dissipative terms ⇒ small fraction of the inflaton energy density can be converted to radiation.

$$H^2 = \frac{1}{3M_p^2}(\rho_\phi + \rho_r)$$

$$\dot{\rho}_\phi + 3H(\rho_\phi + P_\phi) = -\Gamma\dot{\phi}^2, \quad \dot{\rho}_r + 4H\rho_r = \Gamma\dot{\phi}^2$$

- $\Gamma = \Gamma(T, \phi)$: dissipative coefficient.
- $\Gamma(\phi, T) = aT^m/\phi^{m-1}$; $m = 3, 1, 0, -1$ (Bastero-Gil et al. 07 (2011))

Warm inflation at work

- During inflation $\rho_\phi \gg \rho_r$, $\dot{\rho}_r \ll 4H\rho_r$, and $\dot{\rho}_r \ll \Gamma\dot{\phi}^2$
- Slow-roll equations

$$H^2 \simeq \frac{1}{3M_p^2} V$$

$$3H\dot{\phi}(1+R) \simeq V_\phi, \quad 4H\rho_r \simeq \Gamma\dot{\phi}^2$$

$R \equiv \frac{\Gamma}{3H}$

- Weak dissipative regime ($R < 1$); Strong dissipative regime ($R > 1$)
- If $\rho_r = C_r T^4$, where $C_r = \pi^2 g_*/30 \;\rightarrow\; T = \left[\frac{\Gamma V_\phi^2}{36 C_r H^3 (1+R)^2}\right]^{1/4}$
- Slow-roll parameters

$$\epsilon_V = \frac{M_p^2}{2}\left(\frac{V_\phi}{V}\right)^2, \quad \eta_V = M_p^2\left(\frac{V_{\phi\phi}}{V}\right)$$
$$\beta = M_p^2\left(\frac{\Gamma_\phi V_\phi}{\Gamma V}\right), \quad \sigma = M_p^2\left(\frac{V_\phi}{\phi V}\right)$$

- Slow-roll conditions: $\epsilon_V \ll 1+R$, $\eta_V \ll 1+R$, $\beta \ll 1+R$, $\sigma \ll 1+R$

Perturbations in warm inflation

- In the presence of a thermal bath, when $T > H$, the quantum fluctuations of the inflaton are dominated by the thermal.
- Power spectrum of the curvature perturbation

$$\mathcal{P}_{\mathcal{R}} \simeq \left(\frac{H}{2\pi}\right)^2 \left(\frac{3H^2}{V_\phi}\right)^2 (1+R)^{5/2} \left(\frac{T}{H}\right)$$

- Scalar spectral index

$$n_s \simeq 1 + \frac{1}{1+R}\left[-(2-5A_R)\epsilon_V - 3A_R\eta_V + (2+4A_R)\sigma\right],$$

where $A_R = \frac{R}{1+7R}$.
- Tensor-to-scalar ratio

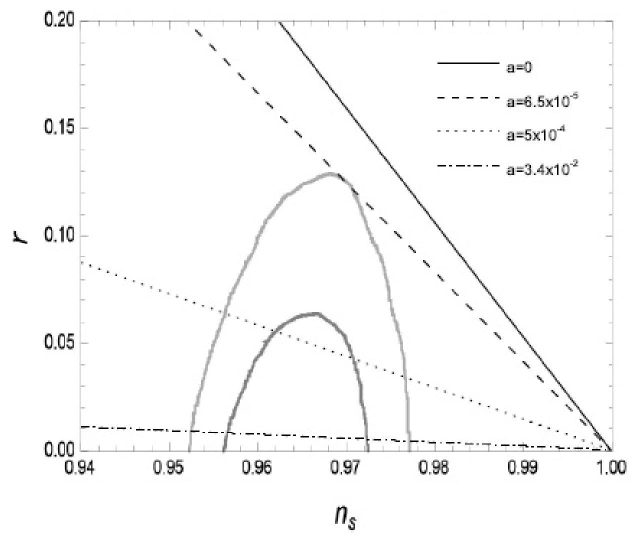
$$r \simeq \left(\frac{H}{T}\right)\frac{16\epsilon_V}{(1+R)^{5/2}}.$$

Dynamics of warm $\frac{\lambda}{4}\phi^4$ inflation

Weak dissipative regime

- We study the case $m = 1$, $\Gamma \propto T$
- Slow-roll parameters

$$\epsilon_V = 8\frac{M_p^2}{\phi^2}, \ \eta_V = 12\frac{M_p^2}{\phi^2}, \ \beta = 0$$
$$\sigma = 4\frac{M_p^2}{\phi^2}$$



- Perturbations

$$\mathcal{P}_{\mathcal{R}} \simeq \left(\frac{\lambda\sqrt{a}N^3}{6\sqrt{70}\pi^3}\right)^{2/3}$$
$$n_s \simeq 1 - \frac{1}{N}$$
$$r \simeq \frac{4\sqrt{14}}{625\sqrt{5a}}(1 - n_s)$$

$$6.5 \times 10^{-5} < a < 3.4 \times 10^{-2}$$
$$10^{-15} < \lambda < 10^{-13}$$

Dynamics of warm $\frac{\lambda}{4}\phi^4$ inflation

Strong dissipative regime

- We study the case $m = 1$, $\Gamma \propto T$
- Slow-roll parameters

$$\epsilon_V = 8\frac{M_p^2}{\phi^2},\ \eta_V = 12\frac{M_p^2}{\phi^2},\ \beta = 3\frac{M_p^2}{\phi^2}$$

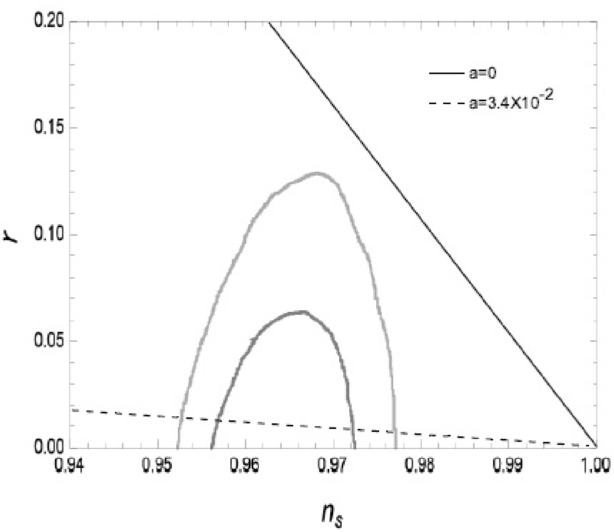
$$\sigma = 4\frac{M_p^2}{\phi^2},\ \left(\phi_{end} = \frac{(6^7 35\lambda)^{1/4}}{a} M_p\right)$$

- Perturbations

$$\mathcal{P}_{\mathcal{R}} \simeq \left[\frac{4\lambda N^3}{125\pi^{8/3}}\left(\frac{2}{315}\right)^{1/3}\right]^{3/4}$$

$$n_s \simeq 1 - \frac{45}{28N}$$

$$r \simeq 8.5 \times 10^{-9}\frac{\pi^{10/3}}{a^4}(1 - n_s)$$



$$a > 3.4 \times 10^{-2}$$

$$\lambda \sim 10^{-15}$$

Probes of non-gaussianities and observational constraints

- The properties of the curvature perturbation $\mathcal{R}$ can be probed by its statistical measures such as the power spectrum $P_{\mathcal{R}}$ and bispectrum $B_{\mathcal{R}}$

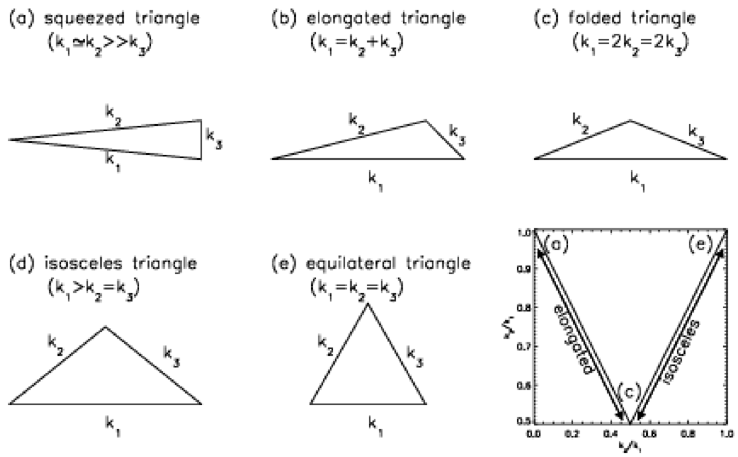
$$\begin{aligned}
\left\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\right\rangle &= (2\pi)^3 P_{\mathcal{R}}(k_1)\delta^3\left(\mathbf{k}_1+\mathbf{k}_2\right)\\
\left\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\right\rangle &= (2\pi)^3 B_{\mathcal{R}}(k_1,k_2,k_3)\delta^3\left(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3\right).
\end{aligned}$$

- The bispectrum can be used to examine the non-gaussianity in the density fluctuations. The normalised amount of non-gaussianity in the bispectrum is described by a non-linearity function f_{NL}, defined by

$$f_{NL}(k_1,k_2,k_3) = \frac{5}{6}\frac{B(k_1,k_2,k_3)}{P(k_1)P(k_2)+P(k_2)P(k_3)+P(k_3)P(k_1)}$$

Probes of non-gaussianities and observational constraints

Shapes of non-gaussianities



For a single scalar field in cold inflation in the "squeezed triangle limit"(J. M. Maldacena (2003))

$$f_{NL} \simeq \frac{5}{12}(n_s - 1)$$

Non-gaussianities in the strong regime of warm inflation

- In warm inflation, both the radiation and the inflaton fluctuate.
- The non-linear coupling between these fluctuations acts as a source of non-gaussianity.
- In the strong regime of warm inflation, the amount of non-gaussianity in the bispectrum, measured by the non-linearity function f_{NL} in the equilateral limit, is given by

$$f_{NL}^{\text{equil}} \simeq -15 \ln\left(1 + \frac{R}{14}\right) - \frac{5}{2}$$

Non-gaussianities in the strong regime of warm inflation

- For our model we have that

$$f_{NL}^{\text{equil}} \simeq -15\ln\left(1+61.3a^2\right)-\frac{5}{2}$$

- Using the constraint on f_{NL} found by Planck (2015)

$$f_{NL}^{\text{equil}} = -4 \pm 43 \;\rightarrow\; a = 0.04 + 0.48$$

Conclusions

- Dissipation in warm inflation causes a friction term Γ in the inflaton's equation of motion, leading to particle and radiation production with inflationary expansion.
- We have studied how an inflaton decay rate proportional to the temperature works in warm inflation with the quartic potential.
- The model predicts a negligible tensor-to-scalar ratio in the strong dissipative regime, while in the weak dissipative regime the tensor-to-scalar ratio can be large enough to be observed.
- We have obtained, in both dissipative regimes regimes, constraints on the parameters of the model from the Planck 2015 data, where we have considered the constraint on the n_s-r plane and primordial non-gaussianities, through f_{NL}, and by other hand ,the condition the model evolves according to the the weak (strong) dissipative regime.
- Warm inflation can rescue the quartic potential that in standard inflation is ruled out by the data.

THANK YOU FOR LISTENING!