HAWKING RADIATION FROM z=3 AND z=1 LIFSHITZ BLACK HOLES

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where z is the relative scale between time and space dimensions.

The spacetime geometry is given by

$$ds^2 = -\left(\frac{r}{l}\right)^{2z} dt^2 + \frac{l^2}{r^2} dr^2 + \frac{r^2}{l^2} d\overrightarrow{x}^2, \tag{2}$$

metric called the Lifshitz spacetime and this line element is invariant under $(t, r, \overrightarrow{x}) \longrightarrow \left(\lambda^z t, \lambda^{-1} r, \lambda \overrightarrow{x}\right)$.

Then, from the Lagrangian (in $d = 2+1$)

$$L = \frac{1}{2}\int d^3x\sqrt{-g}\left[R - 2\Lambda + 2l^2\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right)\right], \qquad (3)$$

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and by using the Ansatz

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the field equations, besides appropriate asymptotic conditions, turn out to be solved by

$$ds^2 = -\left(\frac{r^2}{l^2}\right)^{2z} \left(1 - \frac{r_+^2}{r^2}\right) dt^2 + \frac{l^2}{r^2} \left(1 - \frac{r_+^2}{r^2}\right)^{-1} dr^2 + \frac{r^2}{l^2} d\overrightarrow{x}^2, \quad (5)$$

so that for $z = 3$, we have

$$ds^2 = -\left(\frac{r^2}{l^2}\right)^6 \left(1 - \frac{r_+^2}{r^2}\right) dt^2 + \frac{l^2}{r^2}\left(1 - \frac{r_+^2}{r^2}\right)^{-1} dr^2 + r^2 d\varphi^2, \qquad (6)$$

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where $-\infty \leq t \leq \infty$, $r \geqslant 0$ and $0 \leq \varphi \leq 2\pi$, and $r_+ = l\sqrt{M}$ is the black hole event horizon and M is an integration constant related to the black hole mass and l is the curvature radius of the Lifshitz spacetimes related to the cosmological constant through $\Lambda = -13/2l^2$.

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* See, Scalar field scattering by a $z = 3$ Lifshitz black hole... **S. Lepe**, J. Lorca, F. Peña and Y. Vásquez, Phys. Rev. D**86** (2012) 066008.

II. Thermodynamics for z=3 Lifshitz black hole

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The thermodynamical quantities of this black hole, i.e., temperature, entropy and mass are, respectively,

$$\begin{aligned} T &= \frac{r_+^3}{2\pi l^4}\,, \quad S = 2\pi r_+ \neq \frac{\pi r_+}{2} = \frac{1}{4}Area\,, \\ \hat{M} &= \frac{r_+^4}{4l^4} = \left(\frac{M}{2}\right)^2, \\ &\longrightarrow \quad TdS = d\hat{M}. \end{aligned} \tag{7}$$

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Whether or not the *Area*/4-law is a universal law which must satisfy all black holes, independent of the dimensionality of the spacetime in which they live, is still an open question.

III. The relativistic Hamilton-Jacobi equation

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(From here we follow: **S. Lepe** and B. Merello, Mod. Phys. Lett. A**29** (2014) 1450187)

$$g^{\mu\nu}\left(\frac{\partial I}{\partial x^{\mu}}\right)\left(\frac{\partial I}{\partial x^{\nu}}\right)+m^2=0, \tag{8}$$

where I is the classical action, m is the mass of the particle and $x^{\mu}=(t,r,\varphi)$.

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where I is the classical action, m is the mass of the particle and $x^{\mu}=(t,r,\varphi)$.

Near the horizon

$$g^{tt}(r)=-\frac{l^{6}}{2r_{+}^{5}(r-r_{+})}\ ,\quad g^{rr}(r)=\frac{2r_{+}}{l^{2}}(r-r_{+})\ ,\quad g^{\varphi\varphi}(r)=\frac{1}{r_{+}^{2}}, \qquad (9)$$

so that by replacing (6) in (5) and by doing $I = -\omega t + R(r) + j\varphi$, where ω and j are the energy and angular momentum of the particle m and $R(r)$ represents the geometric content of the spacetime under consideration, we obtain

$$R(r) = \pm\frac{l^4\omega}{2r_+^3}\int\frac{dr}{r-r_+}\sqrt{1-2r_+\left(\frac{r_+^2}{l^3\omega}\right)^2\left(m^2+\frac{j^2}{r_+^2}\right)(r-r_+)}, \quad (10)$$

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where we rescue the imaginary part (classically forbidden process) which is related to the Boltzmann factor for emision at the Hawking temperature

$$R(r) = \pm \frac{l^4 \omega}{2r_+^3} (i\pi) \longrightarrow I = -\omega t \pm \frac{l^4 \omega}{2r_+^3} (i\pi) + j\varphi,, \qquad (11)$$

The $\pm$ sign in (8) correspond to outgoing and ingoing particles, respectively. Given that in the classical limit all is absorbed by the black hole (with no reflection) we write $\mathrm{Im}\, I$ corresponding to outgoing particles

$$\mathrm{Im}\, I = \frac{\pi l^4 \omega}{2 r_+^3} = \frac{\omega}{4T} = \frac{\pi l \omega}{2^{5/2}} \hat{M}^{-3/4}. \tag{12}$$

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We note that $2\,\mathrm{Im}\, I = \omega T/2$ (non-thermal nature of the emission). For thermal emission $2\,\mathrm{Im}\, I = \omega T$.

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When a particle with energy ω tunnels out, the mass of the black hole changed into $\hat{M} \longrightarrow \hat{M} - \omega \longleftrightarrow l\sqrt{M} \longrightarrow l\sqrt{M - \omega}$, where $l\sqrt{M}$ is the horizon radius before pair-creation and $l\sqrt{M - \omega}$ the horizon radius after pair-creation.

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So,

$$\mathrm{Im}\, I \longrightarrow -\frac{\pi l}{2^{5/2}} \int_{\hat{M}}^{\hat{M}-\omega} d\left(\hat{M}-\omega\right)\left(\hat{M}-\omega\right)^{-3/4} = -\frac{1}{4}\left[\left(1-\frac{\omega}{M}\right)^{1/4} - 1\right] \tag{13}$$

by recalling that $S = 2\pi r_+$.

In the WKB approximation, the tunneling rate (tunneling probability for the classically forbidden trajectory from inside to outside the horizon) is given by $\Gamma \sim \exp\left(-2\,\mathrm{Im}\, I\right)$ so that, in the present case

$$\Gamma \sim \exp\left(\frac{1}{2}\Delta S\right)\,, \tag{14}$$

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If we repeat for the BTZ black hole ($z = 1$-Lifshitz black hole), we find $\Gamma \sim \exp\left(\Delta S\right)$. And, for this black hole, we have $S = A/4$.

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- Nevertheless, we apologize that the difference in the tunneling rates obtained can be a signal to discriminate between black holes which satisfy the *Area*/4-law and those who do not.

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- However, we believe that the thermodynamic properties of these, if they do, must be independent of that choice, as it should be if we accept (assume?) that black holes are thermal objects.

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Whatever it is, the future will say!

Thanks!