Cosmic anisotropic doomsday?

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Future singularities at finite time: Big Rip

Phantom Energy and Cosmic Doomsday [PRL 2003]

[R. Caldwell, M. Kamionkowski, N. Weinberg]

$$H(t)^2 = H_0^2\left(\Omega_m a(t)^{-3} + (1-\Omega_m)a(t)^{-3(1+\omega)}\right)$$

$H(t) = \dot{a}(t)/a(t)$ $\quad$ $\boldsymbol{\omega < -1}$: constant

$$a(t) \rightarrow \left(\frac{3(1+\omega)}{2}\left[H_0\sqrt{1-\Omega_m}(t-t_0) + \frac{2}{3(1+\omega)}\right]\right)^{\frac{2}{3(1+\omega)}}$$

$$t_{br} - t_0 \rightarrow \frac{-2}{3(1+\omega)H_0\sqrt{1-\Omega_m}}$$

$$\Rightarrow \begin{cases} a(t_{br}) \rightarrow \infty \\ \rho(t_{br}) \rightarrow \infty \\ p(t_{br}) \rightarrow \infty \end{cases}$$

$a(t)$: scale factor

$H(t)$: expansion rate

H_0 : Hubble parameter

Ω_m : matter density parameter

ω : state parameter

t_0 : the time today

$\rho(t)$: energy density

$p(t)$: pressure

Future singularities at finite time: Big Rip

Phantom Energy and Cosmic Doomsday [PRL 2003]

[R. Caldwell, M. Kamionkowski, N. Weinberg]

$\omega = -3/2$

$\Omega_m = 0.3$

$H_0 = 70 \frac{km}{s\ Mpc}$

$t_{br} \sim 35\ Gyr$

Time	Event
$\sim 10^{-43}$ s	Planck era
$\sim 10^{-36}$ s	Inflation
First Three Minutes	Light Elements Formed
$\sim 10^{5}$ yr	Atoms Formed
~ 1 Gyr	First Galaxies Formed
~ 15 Gyr	*Today*
$t_{rip} - 1$ Gyr	Erase Galaxy Clusters
$t_{rip} - 60$ Myr	Destroy Milky Way
$t_{rip} - 3$ months	Unbind Solar System
$t_{rip} - 30$ minutes	Earth Explodes
$t_{rip} - 10^{-19}$ s	Dissociate Atoms
$t_{rip} = 35$ Gyrs	Big Rip

Types of future singularities

Type I (“Big Rip”): For $t \to t_s, a \to \infty, \rho \to \infty, |p| \to \infty$

Type II (“Sudden”): For $t \to t_s, a \to a_s, \rho \to \rho_s, |p| \to \infty$

Type III (“Big Freeze”): For $t \to t_s, a \to a_s, \rho \to \infty, |p| \to \infty$

Type IV (“Generalized Sudden”): For $t \to t_s, a \to a_s, \rho \to 0, |p| \to 0,$
and higher derivatives of H diverge.

Type V (“w-singularities”): For $t \to t_s, a \to a_s, \rho \to 0, |p| \to 0, \omega \to \infty,$
and higher derivatives of H are regular.

Properties of singularities in (phantom) dark energy universo [PRD 2005]

[Sh. Nojiri, S. Odintsov, Sh. Tsujikawa]

Sources of future singularities

- Phantom Fluids
- Fluids with time dependent equation of state

  I. Brevik, V.V. Obukhov, K.E. Osetrin, A.V. Timoshkin; Mod. Phys. Lett. A (2012)
- Interacting coupled fluids

  I. Brevik, A.V. Timoshkin, Y. Rabochaya; Mod. Phys. Lett. A (2013)
- Bulk viscosity

  M. Cataldo, N. Cruz, S. Lepe; Phys. Lett. B (2005)
- Inhomogeneous and spherically symmetric (evolving wormholes)

  M. Cataldo, P. Meza; Phys. Rev. D (2013)

Bianchi I models (anisotropic)

$$ds^2 = dt^2 - a_1^2(t)dx^2 - a_2^2(t)dy^2 - a_3^2(t)dz^2$$

$$\bar{a} = (a_1 a_2 a_3)^{1/3},$$

$$H = \dot{\bar{a}}/\bar{a}, \qquad H_i = \frac{\dot{a}_i}{a_i}$$

$$H = \frac{1}{3}(H_1 + H_2 + H_3)$$

$$p = \frac{1}{3}(p_1 + p_2 + p_3)$$

$$\begin{aligned} 3H^2 &= \kappa\rho + \frac{\sigma^2}{2}, \\ -2\dot{H} &= \kappa(\rho + p) + \sigma^2, \\ \dot{\rho} + 3H(\rho + p) &= \vec{\sigma}\cdot\vec{\Sigma}, \\ \dot{\vec{\sigma}} + 3H\vec{\sigma} &= \vec{\Sigma} \end{aligned}$$

$\sigma_i = H_i - H$: Shear vector

$\Sigma_i = p_i - p$: Transverse pressure vector

$$\sigma_1 + \sigma_2 + \sigma_3 = 0,$$
$$\Sigma_1 + \Sigma_2 + \Sigma_3 = 0$$

Internal Symmetry in Bianchi type I cosmology [PRD 2003]
[L. Chimento]

Kasner spacetimes (vacuum)

$$ds^2 = dt^2 - \sum_i \left(1 + \frac{1}{2}\sqrt{6}\sigma_0 t\right)^{\frac{2}{3}\left(1+\sqrt{6}\,\frac{\sigma_{i0}}{\sigma_0}\right)} dx^i$$

$$\sigma_i = \frac{\sigma_{i0}}{\bar{a}^3} \qquad \sigma_0 = \pm\sqrt{\sigma_{10}^2 + \sigma_{20}^2 + \sigma_{30}^2}$$

$$q_i = \frac{1}{3}\left(1 + \sqrt{6}\,\frac{\sigma_{i0}}{\sigma_0}\right) \quad \Rightarrow \quad \begin{aligned} q_1 + q_2 + q_3 &= 1 \\ q_1^2 + q_2^2 + q_3^2 &= 1 \end{aligned}$$

contracting scenario

$$\bar{a}(t) = \left(1 + \frac{1}{2}\sqrt{6}\sigma_0 t\right)^{\frac{1}{3}}$$

$$H(t) = \frac{\sigma_0}{\sqrt{6}}\left(1 + \frac{1}{2}\sqrt{6}\sigma_0 t\right)^{-1}$$

Finite time future singularity

$$t_{vr} = -\frac{2}{\sqrt{6}\sigma_0}$$

$$\sigma_0 < 0$$

Just one of the scale factors may exhibit a future singularity, while the other two do not!
All the directional expansion rates and the average expansión rate diverge.

Biianchi I (LRS) + isotropic fluid

$$ds^2 = dt^2 - a_1^2(t)dx_1^2 - a_1^{2\alpha}(t)(dx_2^2 + dx_3^2)$$

$$ds^2 = dt^2 - (1 + (1 + 2\alpha)H_0 t)^{\frac{2}{1+2\alpha}} dx_1^2 - (1 + (1 + 2\alpha)H_0 t)^{\frac{2\alpha}{1+2\alpha}} (dx_2^2 + dx_3^2)$$

$$\rho = p = \frac{\alpha(\alpha + 2)H_0^2}{(1 + (1 + 2\alpha)H_0 t)^2} > 0$$ stiff fluid

contracting scenario

$$\bar{a}(t) = (1 + (1 + 2\alpha)H_0 t)^{\frac{1}{3}}$$

$$H(t) = \frac{(1 + 2\alpha)H_0}{3(1 + (1 + 2\alpha)H_0 t)}$$

Finite time future singularity

$$t_{rs} = -\frac{1}{(1 + 2\alpha)\,H_0}$$

$$H_0 > 0 \;\&\; \alpha < -2$$

Just one of the scale factors may exhibit a future singularity, while the other two do not!
All the directional expansion rates and the average expansión rate diverge.
Density and pressure also diverge.

Bianchi I + stiff fluid

$$ds^2 = dt^2 - \sum_i \left(1 \pm \frac{H_0}{\lambda} t\right)^{\frac{2}{3}\left(1 \pm \frac{2\sigma_{i0}}{\sqrt{12\rho_0 + 6\sigma_0^2}}\right)} dx^i$$

$$\lambda = \frac{1}{3} + \frac{2\sigma_{10}}{\sqrt{12\rho_0 + 6\sigma_0^2}}$$

$$\rho = p = \frac{36 H_0^2 \rho_0}{\left(\sqrt{12\rho_0 + 6\sigma_0^2} + 6\sigma_{10}\right)^2 \left(1 \pm \frac{H_0}{\lambda} t\right)^2} > 0$$

contracting scenario

$$\bar{a}(t) = \left(1 \pm \frac{H_0}{\lambda} t\right)^{\frac{1}{3}}$$

$$H(t) = \frac{\pm H_0}{3(\lambda \pm H_0 t)}$$

Finite time future singularity

$$t_{rs} = \mp \frac{\lambda}{H_0}$$

Just one of the scale factors may exhibit a future singularity, while the other two do not!
All the directional expansion rates and the average expansión rate diverge.
Density and pressure also diverge.

Bianchi I (LRS) + anisotropic fluid

$$p_1 = \omega_1 \rho$$

$$p_2 = p_3 = \omega_3 \rho$$

$$\lambda = \frac{1+\alpha}{\alpha^2\omega_1 + \alpha^2 + 2\alpha\omega_1 + \alpha + 1}$$

$$ds^2 = dt^2 - \left(1 + \frac{H_0}{\lambda} t\right)^{2\lambda} dx_1^2 - \left(1 + \frac{H_0}{\lambda} t\right)^{2\alpha\lambda} (dx_2^2 + dx_3^2)$$

$$\rho = \frac{\alpha(\alpha+2)H_0^2}{\left(1 + \frac{H_0}{\lambda}\right)^2} > 0$$

$$\omega_3 = \frac{1 + 2\alpha\omega_1 - \alpha}{1+\alpha}$$

expanding scenario

$$\bar{a}(t) = \left(1 + \frac{H_0}{\lambda} t\right)^{\frac{\lambda(1+2\alpha)}{3}}$$

$$H(t) = \frac{H_0(1+2\alpha)\lambda}{3(\lambda + H_0 t)}$$

Finite time future singularity

$$t_{rs} = -\frac{\lambda}{H_0}$$

$$H_0 > 0, \lambda < 0$$

$$\alpha > 0 \Rightarrow \omega_1 < 0 \ \& \ \omega_3 < 0$$

The three scale factors and the average exhibit a future singularity for $\alpha > 0$
All the directional expansion rates and the average expansion rate diverge.
Density and pressure also diverge.

Final Remarks

- Bianchi type I cosmologies may evolve to finite-time singularities, because of the anisotropy of space.

- For Kasner vacuum solution, axysymmetric Bianchi I with a stiff fluid and Bianchi I with stiff fluid, we find contracting Big-Rip scenarios

- Finally, for an axysymmetric space we find an expanding anisotropic Big-Rip scenario for $\omega_1 < 0$ and $\omega_3 < 0$.

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Thanks!