5º Encuentro COSMOCONCE: Cosmología y Gravitación en Concepción

GSL of Thermodynamics for Holographic Dark Energy and Cosmological Interaction

F. Arévalo , P. Cifuentes, F. Peña
fabiola.arevalo@ufrontera.cl
Departamento de Ciencias Físicas, Facultad de IyC, Universidad de La Frontera.

April 7, 2016, Concepción, Chile.

Content

Cosmology in the context of General Relativity

Einstein Field Eqs.

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = \kappa T_{\mu\nu}$$

$g_{\mu\nu}$ is the metric, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar $T_{\mu\nu}$ is the energy momentum tensor.

Cosmological Principle

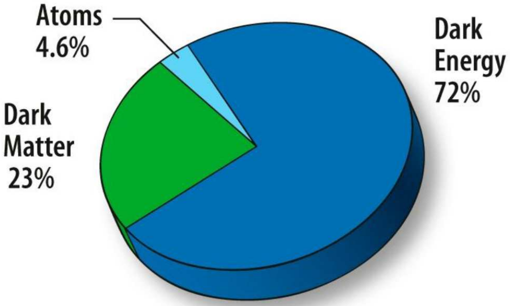
The Universe is homogeneous and isotropic. The most general metric that fulfills this is the **Friedmann-Robertson-Walker(FRW)** metric

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right)$$

$a(t)$ is the scale factor, (r, θ, ϕ) are the spatial coordinates, t is the cosmic time, p the pressure and ρ the total energy density.

Idealized Matter Content

$$T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p\, g_{\mu\nu}$$



TODAY

The SCM is a Cold Dark Matter (CDM) component with no pressure ($p_{DM} = 0$) and a Cosmological constant component ($p_\Lambda = -\rho_\Lambda$)

Equation of Motion ΛCDM

$$\dot\rho_{DM} + 3H\rho_{DM} = 0 \to \rho_{DM} = \rho^0_{DM} a^{-3}$$
$$\dot\rho_\Lambda = 0 \to \rho_\Lambda = \rho^0_\Lambda$$

Defining $\Omega^0_i = \frac{\rho^0_i}{3H^2_0}$

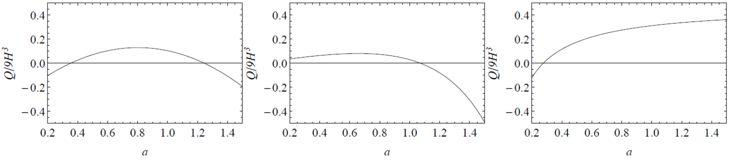
$$r_0 = \frac{\Omega^0_{DM}}{\Omega^0_\Lambda} \approx O(1)$$

Modifications of Dark Energy

- Cosmological Interaction
- Holographic Dark Energy
- Many others...

Interacting Dark Sector

$$\dot\rho_{DE} + 3H(\rho_{DE} + p_{DE}) = -Q,$$
$$\dot\rho_{DM} + 3H(\rho_{DM} + p_{DM}) = Q.$$
$$\text{Ricci-like } \rho_{DE} = 3\alpha H^2 + 3\beta\dot H$$



Outline

Generalized Second Law of Thermodynamics

To examine the thermodynamic behavior of a cosmological scenario, the universe (in our case the interacting dark sector) can be considered as a thermodynamical system with a certain boundary L in a *brief* state of equilibrium.

$$T_{DE} dS_{DE} = p_{DE} dV + dE_{DE}, \quad E_{DE} \equiv \frac{4\pi}{3} L^3 \rho_{DE},$$
$$T_{DM} dS_{DM} = p_{DM} dV + dE_{DM}, \quad E_{DM} \equiv \frac{4\pi}{3} L^3 \rho_{DM}.$$

Its associated temperature is given as $T_h \equiv \frac{1}{2\pi L}$, according to [8] and references therein. The entropy associated with the horizon is $S_h \equiv 8\pi^2 L^2$.

The total entropy rate is $\dot{S}_{tot} = \dot{S}_{DE} + \dot{S}_{DM} + \dot{S}_h$

$$\frac{\dot{S}_{tot}}{16\pi^2 L} = \left(1 - \dot{H} L^2\right)\left(\dot{L} - HL\right) + HL.$$

Outline

Horizon related to the length scale $\rho_{DE} \equiv \frac{3c^2}{L^2}$

We write the GSL using $q \equiv -\left(1 + \frac{\dot{H}}{H^2}\right)$ and $r \equiv \frac{\rho_{DM}}{\rho_{DE}} = \frac{H^2 L^2}{c^2} - 1$

$$\frac{H\dot{S}_{tot}}{16\pi^2 c^2} = \left[1 + c^2(1+r)(q+1)\right]\left[\frac{r'}{2} + (1+r)q\right] + (1+r).$$

We can also write it in terms of $P_{DE} = \frac{p_{DE}}{H^2}$ and $\Gamma = \frac{Q}{3H^3}$

$$\frac{S'_{tot}}{4\pi^2 L^2} = 4 + \left[(3 + P_{DE})\, c^2(1+r) + 2\right] \times \left[1 + (1+r)\,(P_{DE} + \Gamma)\right],$$

Using auxiliary functions

$$f(a) \equiv -P_{DE} - \frac{1}{(1+r)}, \; ,$$

$$g(a) \equiv \frac{-4}{(1+r)\big((3+P_{DE})c^2(1+r)+2\big)},$$

$$\frac{S'_{tot}}{4\pi^2 L^2} = -\frac{4}{g(a)}\left(\Gamma - f(a) - g(a)\right)$$

If $g(a) > 0$, then we obtain $\Gamma < f(a) < f(a) + g(a)$. On the other hand if $g(a) < 0$, then we obtain $f(a) > \Gamma > f(a) + g(a)$.

Outline

Known horizon and the GSL

There are known horizon in the literature for holographic dark energy scenarios, we shall analyze briefly the GSL on three of them:
The Hubble horizon L_H in the FRW flat universe is given by

$$L_H \equiv H^{-1} \rightarrow H\frac{\dot{S}_H}{16\pi^2} = \left(H^{-2}\dot{H}\right)^2$$

The cosmological event horizon L_E is defined as

$$L_E \equiv a\int_t^\infty \frac{dt}{a} \rightarrow \frac{\dot{S}_E}{16\pi^2 L_E} = 2c^2(1+r)^2 r' - c^2(1+r) - 1$$

Now, we consider a linear combination of L_H and L_E assumed by [9], while investigating thermodynamics and the phantom barrier,

$$L_{HE} \equiv \lambda_H L_H + \lambda_E L_E \rightarrow \frac{\dot{S}_{HE}}{16\pi^2 L_{HE}} = \left[1 - c^2\frac{r'}{2} + c^2(r+1)\right](\lambda_H - \lambda_E) + \left[(\lambda_H - \lambda_E)^2 + 1\right] c\sqrt{r+1} ,$$

Outline

Ricci Dark Energy scenario

We consider that since the dark energy is related to the horizon in an holographic context, and Ricci and Ricci-like scenarios consider the dark energy as an specific Ansatz, we shall study the possibility of using such Ansatz as an expression for a surface representing the horizon in a thermodynamical context.

The Ricci dark energy

$$\rho_{DE} \equiv 3c^2(2H^2 + \dot{H}) \quad \longleftrightarrow \quad p_{DE} = H^2 - \frac{2}{3c^2}\rho_{DE}$$

The auxiliary parameters are f(a)=$-1 + \frac{2-c^2}{c^2(1+r)}$, g(a)=-$\frac{1}{c^2(1+r)^2}$, then

- if $c^2 > \frac{2}{2+r}$, then interaction could be negative
- if $c^2 < \frac{(1+2r)}{(2+r)(1+r)}$, the interaction Γ should always be positive at late times.

The total entropy rate is given as

$$\frac{H\dot{S}_R}{8\pi^2 c^2(1+r)} = 2c^2(1+r)\left[1+(r+1)(1+\Gamma)\right] - 2(1+2r)$$

Ricci Dark Energy

Now, we consider the entropy change without the interaction explicitly, but rather in terms of r and r', given as

$$\frac{H\dot{S}_R}{16\pi^2} = \left[1 - \frac{1}{c^2} + 2(1+r)\right]\left[\frac{r'}{2} - \frac{1}{c^2} + (1+r)\right] + (1+r).$$

We can write the pressure as $P_{DE} = -\left[\frac{2}{c^2(1+r)} - 1\right]$, that indicates that the evolution of r could be determined by the evolution of P_{DE}, relation obtained from the holographic context. Then the interaction can be written as

$$\Gamma = \frac{r' + r\left[\frac{2}{c^2} - (1+r)\right]}{(1+r)^2}, \tag{1}$$

a function of (r, r'). In our case the sign of Γ remains undetermined so far. Using r as a function of P_{DE} and its derivative in the interaction, we obtain

$$\Gamma = \frac{c^2}{2}\left[P'_{DE} - P_{DE}\left(\frac{2}{c^2} - 1 + P_{DE}\right)\right], \tag{2}$$

Observational Constraints for Ricci DE

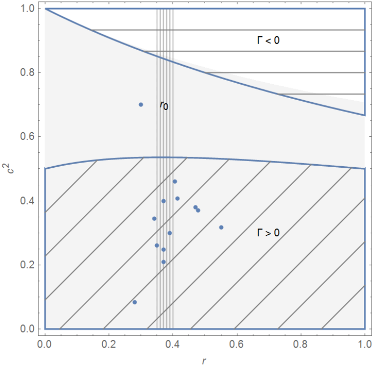


Figure : Phase-space of (r, c^2) considering zones where the interaction Q respects the GSL for Ricci Dark Energy. The shaded area is where the pressure is negative at late times. The dots are pairs of (r_0, c^2) obtained with observational data from [**?**] and references therein, and the vertical lines for r_0 are indicated for comparison.

Ricci-like Dark Energy scenario

Ricci-like dark energy

$$\rho_{DE} \equiv 3(\alpha H^2 + \beta \dot{H}) \quad \longleftrightarrow \quad p_{DE} = -\frac{2}{3\beta}\rho_{DE} + (2\alpha - 3\beta)H^2/\beta$$

Considering this equation in terms of r, we obtain

$$\frac{H\dot{S}_{RI}}{8\pi^2(1+r)} = 2 + \left[1 - \frac{1}{\beta}\left(1 - \alpha(1+r)\right)\right]\left[1 - \frac{2}{\beta} + (1+r)\left(\frac{2\alpha - 3\beta}{\beta} + \Gamma\right)\right]$$

the auxiliary parameters are
$f(a) = \frac{2-\beta}{\beta(1+r)} - \frac{2\alpha-3\beta}{\beta}$, $g(a) = \frac{-2\beta/(1+r)}{\beta-1+\alpha(1+r)}$
The sign of interaction in Ricci-like HDE can be negative or positive while respecting GSL, according to the ranges obtained from replacing (f, g) inequalities in terms of r, α and β.

Ricci-like Dark Energy

$$\beta^2 H \frac{\dot{S}_{RI}}{16\pi^2} = \beta^2(r+1) + [\beta - (1 - \alpha(r+1))] \left[\frac{\beta}{2} r' - 1 + (\alpha - \beta)(1+r)\right]$$

In this context by choosing a horizon we are inherently choosing a pressure, as a function of r and the model parameters

$$P_{DE} = -\left[2/(1+r) - (2\alpha - 3\beta)\right]/\beta.$$

The result obtained indicates that the evolution of r could be determined by the evolution of P_{DE}, relation obtained from the holographic context. Then, the interaction can be written as

$$\Gamma = \frac{\beta\, r' + r\,(2 - (2\alpha - 3\beta)(1+r))}{\beta(1+r)^2},$$

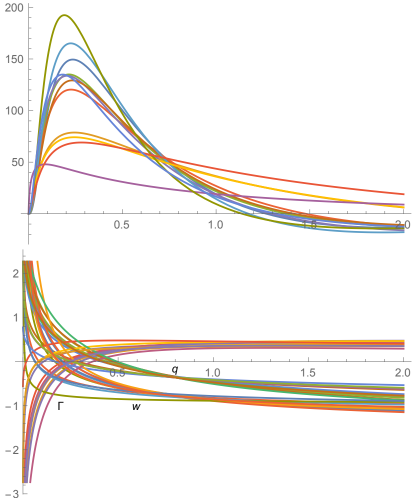
Using $r(P_{DE})$ and its derivative in the interaction

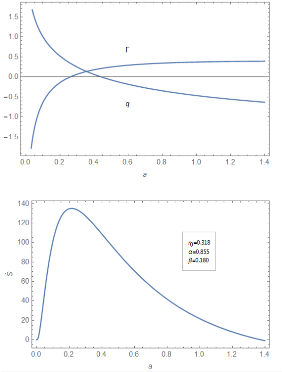
$$\Gamma = \left(\beta P'_{DE} - P_{DE}\,(2(1-\alpha) + \beta(3 + P_{DE}))\right)/2.$$

It is equivalent to study the expression for interaction described by r as it is the interaction as a function of the pressure P_{DE}.

Observational Constraints for Ricci-like

We studied several cases for the entropy rate function





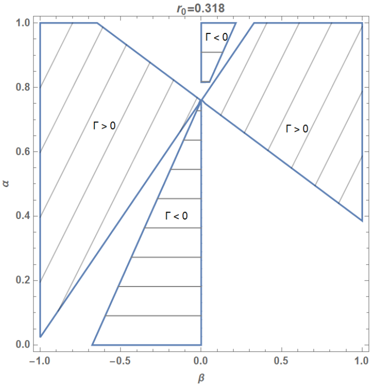


Figure : The graph on the left represents the relation between the GSL and the interaction for a specific r_0 in the phase space of (α, β) and the graph on the right is the entropy change and the top is the deceleration parameter and interaction for a specific holographic model, case III of [2].

Final Remarks

We concluded that

- We propose the HDE Ansatz as an horizon -like radius to study features of the cosmological interaction using thermodynamics.
- We consider the interaction with an unknown sign, generalizing previous studies where the interaction is generally given and positive
- The variation with respect to the cosmic time of the total entropy can be positive or negative depending on the horizon chosen when this horizon includes a term of $\dot{H}$ that inherently determines the pressure.
- For these cases the cosmic coincidence problem is alleviated, given that when the pressure is a variable function, r is also a variable function.
- The Ricci-like HDE is allowed to be negative and respect GSL but only for a certain range, noticeable in Fig 2
- More work in sign-changeable interactions could be explored

References

[1] L. P. Chimento, Phys. Rev. D **81**, 043525 (2010).

[2] F. Arevalo, P. Cifuentes, S. Lepe and F. Pena, Astrophys. Space Sci. **352**, 899 (2014)

[3] S. Lepe, F. Pena and F. Torres, Phys. Rev. D **91**, no. 2, 024023 (2015)

[4] H.Wei, Nucl.Phys. B845 (2011) 381-392.

[5] Y. -H. Li and X. Zhang, "Running coupling: Does the coupling between dark energy and dark matter change sign ..."'

[6] M. Khurshudyan, arXiv:1302.1220 [gr-qc].

[7] F. Arevalo, A. P. R. Bacalhau and W. Zimdahl, Class. Quant. Grav. **29**, 235001 (2012)

[8] M. Jamil, E. N. Saridakis and M. R. Setare, Phys. Rev. D **81**, 023007 (2010)

[9] H. M. Sadjadi and M. Honardoost, Phys. Lett. B **647**, 231 (2007)

Thank you for your attention