$\rho_{de}\left(z=2{,}34\right)/\rho_{de}\left(z=0\right)=-\left(1{,}2\mp 0{,}8\right)$, a violation of WEC?

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General Relativity

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \left(R - 2\Lambda \right) = 8\pi T_{\mu\nu} \longrightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi \left(T_{\mu\nu} - \frac{\Lambda}{8\pi} g_{\mu\nu} \right), \tag{1}$$

where $R_{\mu\nu}$ =Ricci tensor, $g_{\mu\nu}$ =metric, R =Ricci scalar, Λ =cosmological constant, $T_{\mu\nu}$ =energy-momentum tensor. We use units $G = c = 1$.

Cosmology

By using the metric (FLRW, homogeneous and isotropic spacetime)

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right], \tag{2}$$

being a is the cosmic scale factor, k =curvature index ($-1 =$ *open*, $0 =$ *flat and* $+1 =$ *closed*) and

$$T_{\mu\nu} = (\rho + p)\, u_\mu u_\nu + \rho g_{\mu\nu}, \tag{3}$$

where ρ =energy density, p =pressure, we have the Friedmann equation

$$3H^2(z) = \rho - \frac{3k}{a_0^2}(1+z)^2 + \Lambda, \tag{4}$$

and the conservation equation

$$\frac{d\rho(z)}{dz} = 3\left(\frac{\rho(z) + p(z)}{1+z}\right) \longleftrightarrow \dot{\rho} + 3H(\rho + p) = 0, \tag{5}$$

where z is the redshift parameter ($1 + z = a_0/a$), H is the Hubble parameter. And we must give an equation of state, for example, $p = .\omega\rho$ and ω is not necessarily a constant, this is, $\omega = \omega(z)$.

In particular, the ΛCDM model is cosmological constant plus a pressureless fluid ($p_{dm} = 0 \longleftrightarrow \omega_{dm} = 0$, *dust*) and eventually zero curvature

$$3H^2(z) = \rho_{dm0}(1+z)^3 - \frac{3k}{a_0^2}(1+z)^2 + \Lambda, \tag{6}$$

$$\frac{d\rho_{dm}(z)}{dz} = 3\left(\frac{\rho_{dm}(z)+0}{1+z}\right) \longrightarrow \rho_{dm}(z) = \rho_{dm0}(1+z)^3 . \tag{7}$$

Now, we consider the scheme (from Λ to $\Omega_{de}(z)$, to Hell with «old» astronomers!)

$$E^2(z) = \Omega_{de}(z) + \Omega_{dm0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{k0}(1+z)^2 , \tag{8}$$

where $E(z) = H(z)/H_0$, $\Omega_{de}(z) = \rho_{de}(z)/3H_0^2$, $\Omega_{dm0} = \rho_{dm0}/3H_0^2$, $\Omega_{r0} = \rho_{r0}/3H_0^2$ and $\Omega_{k0} = -k/a_0^2H_0^2$, all observable parameters and

$$\frac{d\Omega_i(z)}{dz} = 3\left(\frac{1+\omega_i}{1+z}\right)\Omega_i(z) , \tag{9}$$

$\forall \rho_i$ ($\omega_r = 1/3$, $\omega_{dm} = 0$ and here, $\ddot{a} < 0$. And $\omega_\Lambda = -1 \longrightarrow -1 < \omega_{de} < -1/3$; $\omega_{ph} < -1 \Longrightarrow \ddot{a} > 0$).

* $\Omega_{\Lambda 0}$ (Ω_{de0}) $= 0{,}685$ and $\Omega_{dm0} = 0{,}315$: Planck colaboration (2013), and $\Omega_{k0} = -0{,}05 \pm 0{,}06$: arXiv:1404.0773 [astro-ph.CO].

Energy conditions (links between pressure and density)

$$\mathbf{S}\,(trong)\;\;\mathbf{E}\,(nergy)\;\;\mathbf{C}\,(ondition)\;:$$

$$T_{\mu\nu} f^\mu f^\nu \geqslant \frac{1}{2} T f^\mu f_\mu \;\; and\; f^\mu \; is\; a\; timelike\; vector,$$

$$and\; in\; GR,\; R_{\mu\nu} f^\mu f^\nu \geqslant 0 \longrightarrow \textbf{gravity must be attractive} \longleftrightarrow \;\rho + 3p \geqslant 0\;. \tag{10}$$

But, we can have quintessence fields: $-1 < \omega < -1/3$ and phantom fields: $\omega_{ph} < -1$ **such that gravity ceases to be attractive.** In our neighborhood «we feel» attractive gravity but at large scales «we feel» repulsive gravity.

$$\mathbf{N}\,(ull)\;\;\mathbf{E}\,(nergy)\;\;\mathbf{C}\,(ondition)\;:$$
$$T_{\mu\nu}k^{\mu}k^{\nu}\geqslant 0\;\;and\;\;k^{\mu}\;\;is\;a\;null\;vector,$$
$$and\;in\;GR,\;R_{\mu\nu}k^{\mu}k^{\nu}\geqslant 0\;\;(defines\;the\;structure\;of\;light\;cones)\longleftrightarrow p\geqslant -\rho\;, \tag{11}$$

and the phantoms violate this energy condition: $p < -\rho$ $(\omega_{ph} < -1)$.

$$\mathbf{W}\,(eak)\;\;\mathbf{E}\,(nergy)\;\;\mathbf{C}\,(ondition)\;:$$
$$T_{\mu\nu}V^{\mu}V^{\nu}\geqslant 0\;\;and\;\;V^{\mu}\;\;is\;a\;timelike\;vector\;\;\longleftrightarrow \rho\geqslant 0, \tag{12}$$

inviolable!, at least in classical physics (General Relativity is a classical theory).

$$\mathbf{D}\,(o\,\mathrm{mín}\,ant)\;\;\mathbf{E}\,(nergy)\;\;\mathbf{C}\,(ondition)\;:$$
$$T_{\mu\nu}V^{\mu}V^{\nu}\geqslant 0\;\;and\;\;V^{\mu}\;\;is\;a\;timelike\;vector$$
$$+T^{\mu\nu}V_{\nu}\;is\;a\;non-space\;like\;vector\longleftrightarrow |p|\leq\rho, \tag{13}$$

and the phantoms violate also this energy condition: $p < -\rho$ $(\omega_{ph} < -1)$.

Negative pressure (men of little faith!)

- Negative effective magnetic pressure in turbulent convection, arXiv:1104.4541[astro-ph.SR],
- Detection of negative effective magnetic pressure instability in turbulence simulations, arXiv:1109.1270 [astro-ph.SR],
- An Accelerating Universe from Dark Matter Interactions with Negative Pressure, arXiv:hep-ph/0211397 [astro-ph.SR],
- On the Cosmological Constant Problems and the Astronomical Evidence for a Homogeneous Energy Density with Negative Pressure,

  arXiv:astro-ph/0203330,
- ... and many more.

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Observational data

[1] T. Delubac et al, arXiv:1404.1801 [astro-ph.CO] and arXiv:1411.1074 [astro-ph.CO]

Here, the framework is given by $E^2(z) = \Omega_{de}(z) + \Omega_{dm0}(1+z)^3$.

[2] V. H. Cárdenas, arXiv:1405.5116 [astro-ph.CO],

Here, a CPL parametrization: $\omega(z) = \omega(0) + \acute{\omega}(0)\, z\,(1+z)^{-1}$ is used for $\Omega_{de}(z)$ under the framework given in (8).

The surveys are BAO (Baryon Acoustic Oscillations) and BOSS (Baryon Oscillation Spectroscopic Survey), mainly.

(a) The problem. Neglecting radiation and curvature, from (8) we have

$$\Omega_{de}(z) = E^2(z) - \Omega_{dm0}(1+z)^3, \qquad (14)$$

and from [1,2]

$$\frac{\Omega_{de}(2{,}34)}{\Omega_{de0}} = \frac{E^2(2{,}34) - \Omega_{dm0}(1+2{,}34)^3}{1-\Omega_{dm0}} = -(1{,}2 \mp 0{,}8) = -\Delta < 0, \qquad (15)$$

and we have a heavy problem with WEC!

***** BAO (from SDSS collaboration) ... $2{,}1 \le z \le 3{,}5$ (quasars).

(b) Curvature? Neglecting radiation, from (8) we have

$$\Omega_{de}(z) = E^2(z) - \Omega_{dm0}(1+z)^3 - \Omega_{k0}(1+z)^2, \tag{16}$$
$$= E^2(z) - \Omega_{dm0}(1+z)^3 + |\Omega_{k0}|(1+z)^2,$$
$$\longrightarrow \frac{\Omega_{de}(2{,}34)}{\Omega_{de0}} = \frac{E^2(2{,}34) - \Omega_{dm0}(1+2{,}34)^3 + |\Omega_{k0}|(1+2{,}34)^2}{1-\Omega_{dm0}+|\Omega_{k0}|}, \tag{17}$$

whether we are considering a closed universe ($k=1 \Longrightarrow \Omega_{k0} < 0$). Is this an argument that solves the problem given in (15)?

(c) The bet: interacting dark energy-dark matter?

$$(1+z)\frac{d\rho_{de}}{dz} = 3(1+\omega_{de})\rho_{de} + Q/H, \tag{18}$$
$$(1+z)\frac{d\rho_{dm}}{dz} = 3\rho_{dm} - Q/H. \tag{19}$$

and $Q > 0$ (observational data, Xiao-Dong et al, JCAP **1312** (2013) 001). From (19) we can write

$$d\ln\left[\rho_{dm}\left(1+z\right)^{-3}\right] = -\left(\frac{Q}{H\rho_{dm}}\right) d\ln\left(1+z\right), \tag{20}$$

and if we consider the Ansatz $Q = \lambda_1 H\rho_{dm}$ [arXiv:1012.3904], we have

$$\bar{\Omega}_{dm}\left(z\right) = \bar{\Omega}_{dm0}\left(1+z\right)^{3-\lambda_1}, \tag{21}$$

and

$$E\left(z\right) = \bar{\Omega}_{de}\left(z\right) + \bar{\Omega}_{dm}\left(z\right), \tag{22}$$

and by doing $\bar{\Omega}_{de}\left(0\right) = \Omega_{de0}$ and $\bar{\Omega}_{dm}\left(0\right) = \Omega_{dm0}$, we can write

$$\frac{\bar{\Omega}_{de}\left(\tilde{z}\right)}{\Omega_{de0}} = -\Delta + \left(\frac{\Omega_{dm0}}{1-\Omega_{dm0}}\right)\left(1+\tilde{z}\right)^{3}\left[1-\left(1+\tilde{z}\right)^{-\lambda_1}\right], \tag{23}$$

where $\tilde{z} = 2{,}34$ and $\Delta = 1{,}2 \mp 0{,}8 > 0$. If $\lambda_1 > 0$, the second term r.h.s. is one positive. Is this an argument that solves the problem given in (15)?

By considering in (20) the Ansatz $Q = \lambda_2 H \rho_{de}$ [arXiv:1012.3904], we have

$$d \ln \left[\rho_{dm} \left(1+z\right)^{-3} \right] = \frac{1}{r\left(z\right)} d \ln \left(1+z\right)^{-\lambda_2}, \tag{24}$$

where $r\left(z\right) = \Omega_{dm}\left(z\right)/\Omega_{de}\left(z\right)$ (coincidence parameter). We take, for instance, the Ansatz (like-CPL)

$$r\left(z\right) = r\left(0\right) + \acute{r}\left(0\right) z \left(1+z\right)^{-1}, \tag{25}$$

and in this case the solution of (24) is

$$\Omega_{dm}(z) = \Omega_{dm0} \left(1+z\right)^{3-\lambda_2/[r(0)+\acute{r}(0)]}, \tag{26}$$

and then? To be continued.
* The Ansatz $Q = \lambda_1 H \rho_{dm}$ is not good given that the curvature perturbation is not stable (is stable if $\omega_{de} < -1$: phantom fields). The curvature perturbation can always be stable if $Q = \lambda_2 H \rho_{de}$ (here, $-1 < \omega_{de} < 0$: quintessence fields). Pay attention here, the current observational data $\omega\left(0\right)$ is not conclusive about $\omega\left(0\right) > -1$ or $\omega\left(0\right) < -1$ (cosmological constant is $\omega = -1$, always).

The bet.

As already said, **interacting fluids** (modified ΛCDM)!

V. H. Cárdenas (UV) and S. Lepe (PUCV), work in progress. The idea is «to obtain» the interaction from the first principles (not as an input given at hand). For example, thermodynamic (entropy), the Holographic Principle, entropic approach, ... **And we must have in account the phantom fields, why no? Is a motivating thought!**

Finally, **the reference** E. Abdalla, E. G. M. Ferreira, J. Quintin and Bin Wang, arXiv:1424.2777v1 [astro-ph.CO] 8 Dec 2014, **gives us the reason!** Here, λ_1 and λ_2 are both positive parameters (BOSS survey) **and**

$$\Omega_{de}(z) > 0 \Leftrightarrow \mathbf{WEC}, \tag{27}$$

as it should be!

Thanks a lot