

Unitariedad y violacion de la simetria de Lorentz en QED de orden superior

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- 2 Lee-Wick theories
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$$\mathcal{L} = \bar{\psi} (i \not{\partial} - m) \psi,$$

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However, one could ask if it is possible or makes sense to extend these theories in order to contain more time derivatives, such that the Lagrangian turns out to be

$$L = L(x, \dot{x}, \ddot{x}, \dots, x^{(n)}). \quad (1)$$

Ostrogradski studied these kind of theories long time ago, and developed a Hamiltonian framework in order to deal with them. [M. Ostrogradski, *Mem. Acad. St. -Pétersbourg* **VI**, 385 (1850)]

The only requirement is not having constraints in which case one can recover a standard order theory. M.S. Plyushchay: *Mod. Phys. Lett. A4*, 837 (1989)]

It was shown, however, that higher order theories have unbounded energy.

The Pais-Uhlenbeck model

[A. Pais and G. E. Uhlenbeck, Phys. Rev. **79**, 145 (1950)]

The P-U oscillator is, basically, the standard harmonic oscillator with an additional higher time derivative term. To be more precise its equation of motion is

$$gq^{(4)} + \ddot{q} + \omega^2 q = 0, \quad (2)$$

where $q^{(4)}$ is a fourth order time derivative and g can be considered a small coupling constant. The equation of motion is obtained from the Lagrangian:

$$L_{PU} = -\frac{g}{2}\ddot{q}^2 + \frac{1}{2}\dot{q}^2 - \frac{1}{2}\omega^2 q^2. \quad (3)$$

The Pais-Uhlenbeck model

This system can be seen as two standard harmonic oscillators by means of the change of variables,

$$\begin{aligned}q_+ &= (\partial_t^2 - k_-^2)q, \\q_- &= (\partial_t^2 - k_+^2)q.\end{aligned}\tag{4}$$

The Lagrangian with the new variables is

$$L_{PU} = \left(\frac{1}{2} \dot{q}_+^2 - \frac{1}{2} k_+^2 q_+^2 \right) - \left(\frac{1}{2} \dot{q}_-^2 + \frac{1}{2} k_-^2 q_-^2 \right),\tag{5}$$

with $k_{\pm}^2 = \frac{1}{2g}(1 \pm \sqrt{1 - 4g\omega^2})$.

The Pais-Uhlenbeck model

Following the canonical formalism we write the Hamiltonian as:

$$\hat{H}_{PU} = k_- \hat{a}_+^\dagger \hat{a}_+ - k_+ \hat{a}_-^\dagger \hat{a}_- + \frac{1}{2}(k_- - k_+), \quad (6)$$

where \hat{a}_+ , \hat{a}_+^\dagger , \hat{a}_- , \hat{a}_-^\dagger are the standard creation and annihilation operators. The second term produces arbitrary negative energy states as can be seen by acting \hat{a}_-^\dagger on the empty wave function (defined by $\hat{a}_+ \Phi_0 = \hat{a}_- \Phi_0 = 0$)

$$\Phi_0 = N \exp \left[-\frac{\sqrt{1 - 4g\omega^2}}{2(k_+ + k_-)} (k_+ k_- q^2 + \dot{q}^2) + \sqrt{-g\omega^2} q \dot{q} \right]. \quad (7)$$

An alternative proposal to quantize would be to redefine the vacuum $\hat{a}'_0 = \hat{a}_-^\dagger \Phi'_0 = 0$ which stabilizes the theory but it could spoil unitarity when interactions are introduced.

Summarizing:

Extra degrees of freedom

The Lagrangian corresponds to two standard harmonic oscillators with one of them having a relative minus sign respect to the other.

Instabilities

Classically, when adding interactions the system becomes unstable due to the unboundedness of the energy.

Swapping problems: stability for unitarity

However, by a redefinition of the vacuum state, the quantum mechanical problem becomes stable. This last procedure leads to unavoidable negative norm states. These ghosts states could render the theory non unitary when interaction are considered.

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Quantum field theories

In field theory, higher order operators result attractive since they can soften the ultraviolet divergences. [B. Podolsky, *Phys. Rev.* **62**, 68 (1942).].

Recently, higher-order operators have become object of intense study in the description of Lorentz invariance violation. [R. C. Myers and M. Pospelov, *Phys. Rev. Lett.* **90** (2003) 211601; V. A. Kostelecky and M. Mewes, *Phys. Rev. D* **80**, 015020 (2009); A. Kostelecky and M. Mewes, *Phys. Rev. D* **85**, 096005 (2012).]

In this way, they have come to contribute to the phenomenology contained in the extensions of the standard model using normal dimension operators (four dimensions and below) . However, the modifications they introduce are substantially different.

Lee-Wick theories

A Lee-Wick model is a covariant field theory that incorporates higher time derivatives. For example, the Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{g}{\Lambda}\bar{\psi}\square\psi, \quad (8)$$

where g is a dimensionless positive coupling constant and Λ is an ultraviolet energy scale.

One can proceed as before by defining the new fields

$$\begin{aligned} \psi_+ &= \beta(i\partial\!\!\!/ + m_-)\psi, \\ \psi_- &= \beta(i\partial\!\!\!/ - m_+)\psi, \end{aligned} \quad (9)$$

with $\beta = \left(\frac{g/\Lambda}{m_+ + m_-}\right)^{\frac{1}{2}}$ and

$$m_{\pm} = \frac{\mp 1 + \sqrt{1 + 4g\frac{m}{\Lambda}}}{2g}\Lambda, \quad (10)$$

Lee-Wick model

the Lagrangian (8) can be written in terms of these fields as,

$$\mathcal{L} = \bar{\psi}_+(i\partial\!\!\!/ - m_+)\psi_+ - \bar{\psi}_-(i\partial\!\!\!/ + m_-)\psi_-. \quad (11)$$

Here we have written a higher time derivative theory in terms of two decoupled standard fermions. However, the second mode has the wrong sign in front of its Lagrangian density.

The non vanishing anticommutators will be

$$\begin{aligned} \{\psi_+^\alpha(\vec{x}, t), \psi_+^{\dagger\beta}(\vec{y}, t)\} &= -\{\psi_-^\alpha(\vec{x}, t), \psi_-^{\dagger\beta}(\vec{y}, t)\} \\ &= \delta^{\alpha\beta} \delta^3(\vec{x} - \vec{y}). \end{aligned} \quad (12)$$

Note that the minus sign of the anticommutators of the minus fields is responsible for the negative norm states.

Lee-Wick model

Now, decomposing the new fields in terms of plane wave solutions we find

$$\psi_+(\vec{x}, t) = \sum_s \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_+}} \quad (13)$$

$$\times \left[b_+^s(\mathbf{p}) e^{-ip_+ \cdot x} u_+^s(\mathbf{p}) + d_+^{s\dagger}(\mathbf{p}) e^{ip_+ \cdot x} v_+^s(\mathbf{p}) \right],$$

$$\psi_-(\vec{x}, t) = \sum_s \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\sqrt{2E_-}} \quad (14)$$

$$\times \left[b_-^s(\mathbf{p}) e^{-ip_- \cdot x} u_-^s(\mathbf{p}) + d_-^{s\dagger}(\mathbf{p}) e^{ip_- \cdot x} v_-^s(\mathbf{p}) \right].$$

where $p_\pm = (\omega_\pm, \vec{p})$ and $E_\pm = \sqrt{\vec{p}^2 + m_\pm^2}$ and u, v are the eigenpinors satisfying the orthogonality relations

$$u_\pm^{\dagger s} u_\pm^r = v_\pm^{\dagger s} v_\pm^r = 2E_\pm \delta^{sr}, \quad (15)$$

Lee-Wick model

The Hamiltonian of the theory can be written in terms of the standard creation and annihilation operators for the fields ψ_{\pm} as

$$H = \sum_s \int d^3 p \left(E_+ (b_+^{s\dagger}(\vec{p}) b_+^s(\vec{p}) + d_+^{s\dagger}(\vec{p}) d_+^s(\vec{p})) \right. \\ \left. + E_- (b_-^{s\dagger}(\vec{p}) b_-^s(\vec{p}) + d_-^{s\dagger}(\vec{p}) d_-^s(\vec{p})) \right), \quad (16)$$

and,

$$\{b_{\pm}^s(\vec{p}), b_{\pm}^{r\dagger}(\vec{k})\} = \pm (2\pi)^3 \delta^{sr} \delta^3(\vec{p} - \vec{k}), \\ \{d_{\pm}^s(\vec{p}), d_{\pm}^{r\dagger}(\vec{k})\} = \pm (2\pi)^3 \delta^{sr} \delta^3(\vec{p} - \vec{k}), \quad (17)$$

are the nonvanishing anticommutators of creation and annihilation operators for particles (b) and antiparticles (d) of spin s and r . Here is evident the positivity of the energy spectrum and the indefiniteness of Fock space.

Summarizing again

The theory doubles the number of modes, the new modes correspond to negative norm states, and the theory can always be defined with positive energies.

[T. D. Lee and G. C. Wick, Nucl. Phys. B 9, 209 (1969); T. D. Lee, G. C. Wick, Phys. Rev. D2, 1033-1048 (1970)]

By introducing interactions the wrong sign may cause the loss of unitarity. However, it has been shown that with a suitable prescription for the propagators it is possible to maintain unitarity controlled.

[R. E. Cutkosky, P. V. Landshoff, D. I. Olive, J. C. Polkinghorne, Nucl. Phys. B12, 281-300 (1969).]

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The photon Myers-Pospelov model

The Myers-Pospelov Lagrangian density for photons is given by

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{\xi}{2M_P}n_\mu\epsilon^{\mu\nu\lambda\sigma}A_\nu(n\cdot\partial)^2F_{\lambda\sigma}, \quad (18)$$

where n is a four-vector defining a preferred reference frame, M_P is the Planck mass and ξ is a dimensionless parameter.

The equations of motion derived from the Lagrangian (18) are

$$\partial_\mu F^{\mu\nu} + g\epsilon^{\nu\alpha\lambda\sigma}n_\alpha(n\cdot\partial)^2F_{\lambda\sigma} = 4\pi j^\nu, \quad (19)$$

where we have introduced a source j^ν and defined $g = \xi/M_P$.

The photon Myers-Pospelov model

In terms of the physical fields

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t} - \nabla A_0, \quad (20)$$

$$\vec{B} = \nabla \times \vec{A}, \quad (21)$$

we can rewrite Eq.(19) as

$$\begin{aligned} \nabla \cdot \vec{E} + 2g(n \cdot \partial)^2(\vec{n} \cdot \vec{B}) &= 4\pi\rho, \\ -\frac{\partial \vec{E}}{\partial t} + \nabla \times \vec{B} + 2g(n \cdot \partial)^2(n_0 \vec{B} - (\vec{n} \times \vec{E})) &= 4\pi\vec{j}, \end{aligned} \quad (22)$$

together with the usual identities

$$\begin{aligned} \nabla \cdot \vec{B} &= 0, \\ \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0. \end{aligned} \quad (23)$$

Polarization vectors and dispersion relation

To start, let us consider the two-dimensional hyperplane orthogonal to k^μ and n^μ

$$e^{\mu\nu} = \eta^{\mu\nu} - \frac{(n \cdot k)}{D} (n^\mu k^\nu + n^\nu k^\mu) + \frac{k^2}{D} n^\mu n^\nu + \frac{n^2}{D} k^\mu k^\nu, \quad (24)$$

where

$$D(k, n) = (n \cdot k)^2 - n^2 k^2. \quad (25)$$

We note that the tensor $e_{\mu\nu}$ is an orthogonal projector, for instance it reduces well to the transverse delta $\delta_{ij}^T = \delta_{ij} - \frac{k_i k_j}{|\vec{k}|^2}$ when the preferred four-vector has only a temporal component. Also, by direct calculation it can be checked that $e^{\mu\nu} n_\nu = e^{\mu\nu} k_\nu = 0$.

Polarization vectors and dispersion relation

Let us introduce the quantity

$$\epsilon^{\mu\nu} = D^{-1/2} \epsilon^{\mu\alpha\rho\nu} n_\alpha k_\rho, \quad (26)$$

having the property

$$\epsilon^{\mu\alpha} \epsilon^\nu{}_\alpha = e^{\mu\nu}, \quad (27)$$

and

$$\epsilon^{\mu\alpha} = e^{\nu\alpha} \epsilon^\mu{}_\nu. \quad (28)$$

Now, any vector J_μ can be written in terms of its \pm components. These are,

$$J_\mu^{(\pm)} = P_{\mu\nu}^{(\pm)} J^\nu, \quad (29)$$

where we define the orthogonal projector

$$P_{\mu\nu}^{(\pm)} = \frac{1}{2} (e_{\mu\nu} \pm i \epsilon_{\mu\nu}). \quad (30)$$

Polarization vectors and dispersion relation

In the frame of the hyperplane we can always select a basis consisting of two real orthonormal four-vectors $e_\mu^{(a)}$ such that

$$e_{\mu\nu} = - \sum_{a=1,2} e_\mu^{(a)} e_\nu^{(b)},$$

$$\eta^{\mu\nu} e_\mu^{(a)} e_\nu^{(b)} = -\delta^{ab}, \quad (31)$$

and the associated complex polarized four-vectors basis

$$\varepsilon_\mu^{(\lambda)} = P_{\mu\nu}^{(\lambda)} e_\mu^{(a)} = \frac{1}{2}(e_\mu^{(1)} + i\lambda e_\mu^{(2)}), \quad (32)$$

where $\lambda = \pm$. To find the dispersion relation let us consider the gauge field in the circular base. Namely,

$$A_\mu(x) = \sum_\lambda \int d^3k \left(\tilde{A}^{(\lambda)}(k) \varepsilon_\mu^{(\lambda)}(k) e^{-ik \cdot x} + \tilde{A}^{(\lambda)*}(k) \varepsilon_\mu^{*(\lambda)}(k) e^{ik \cdot x} \right). \quad (33)$$

Polarization vectors and dispersion relation

which by substitution in the equation of motion produces the two expressions

$$\begin{aligned}(G^{(+)})^{-1}\tilde{A}^{(+)} &\equiv (k^2 + 2g(n \cdot k)^2\sqrt{D})\tilde{A}^{(+)} = 4\pi j^+, \\(G^{(-)})^{-1}\tilde{A}^{(-)} &\equiv (k^2 - 2g(n \cdot k)^2\sqrt{D})\tilde{A}^{(-)} = 4\pi j^-. \end{aligned} \tag{34}$$

Solving the determinant the dispersion relation reads

$$G = (k^2)^2 - 4g^2(n \cdot k)^4 ((n \cdot k)^2 - n^2 k^2) = 0, \tag{35}$$

in agreement with the work [C. M. Reyes, L. F. Urrutia, J. D. Vergara, *Phys. Rev.* **D78**, 125011 (2008); C. M. Reyes, L. F. Urrutia and J. D. Vergara, *Phys. Lett. B* **675**, 336 (2009).]

Minimal extension

There are two possible ways to obtain a standard time derivative theory and still have dimension five operators. The first one is to consider a purely timelike four-vector $n = (1, 0, 0, 0)$, for which the positive solutions are

$$\omega_T^{(\lambda)} = \frac{|\vec{k}|}{\sqrt{1 + 2g\lambda|\vec{k}|}}, \quad (36)$$

where λ labels the circular polarization vectors introduced before. It is clear that the solution $\omega^{(-)}$ remains real only in the region defined by $|\vec{k}| < 1/(2g)$. For higher momenta the negative mode becomes complex introducing instabilities in the theory.

Minimal extension

The second alternative is to consider a purely spacetime background. In this case the dispersion relations reads

$$\omega_S^{(\lambda)} = (k^2 + 2g^2(n \cdot k)^4 + \lambda(n \cdot k)^3 (1 + g^2 \vec{n}^4 (\vec{n} \cdot \vec{k})^2)^{1/2})^{1/2}. \quad (37)$$

When the vector n pick ups both a time and space components, then the theory is a higher order theory. For example for a lightlike n we have from the dispersion relation

$$(G^{(\lambda)})^{-1} = \omega^2 - \vec{k}^2 + 2g\lambda(n_0\omega - \vec{n} \cdot \vec{k})^3 = 0. \quad (38)$$

Higher order theory

Without loss of generality we can consider $n = (1, 0, 0, 1)$ in which case the exact solutions are

$$\begin{aligned}\omega_0^{(\lambda)} &= -\frac{1 - 6g\lambda k_z}{6g\lambda} - \frac{-1 + 12g\lambda k_z}{3 \times 2^{2/3}g\lambda\Delta^{(\lambda)}} + \frac{\Delta^{(\lambda)}}{6 \times 2^{1/3}g\lambda}, \\ \omega_1^{(\lambda)} &= -\frac{1 - 6g\lambda k_z}{6g\lambda} + \frac{(1 + i\sqrt{3})(-1 + 12g\lambda k_z)}{6 \times 2^{2/3}g\lambda\Delta^{(\lambda)}} - \frac{(1 - i\sqrt{3})\Delta^{(\lambda)}}{12 \times 2^{1/3}g\lambda}, \\ \omega_2^{(\lambda)} &= -\frac{1 - 6g\lambda k_z}{6g\lambda} + \frac{(1 - i\sqrt{3})(-1 + 12g\lambda k_z)}{6 \times 2^{2/3}g\lambda\Delta^{(\lambda)}} - \frac{(1 + i\sqrt{3})\Delta^{(\lambda)}}{12 \times 2^{1/3}g\lambda},\end{aligned}\tag{39}$$

where

$$\begin{aligned}\Delta^{(\lambda)} &= \left(-2 + 108g^2\vec{k}^2 + 36g\lambda k_z - 108g^2k_z^2 \right. \\ &\quad \left. + \sqrt{(-2 + 108g^2\vec{k}^2 + 36g\lambda k_z - 108g^2k_z^2)^2 + 4(-1 + 12g\lambda k_z)^3} \right)^{1/3}.\end{aligned}$$

Unitarity

In order to review the optical theorem let us consider the S -matrix

$$S = 1 + iT. \quad (40)$$

Unitarity of the S -matrix, implies the equation

$$-i(T - T^\dagger) = T^\dagger T. \quad (41)$$

Taking the matrix elements of the above equation, we must show that

$$2 \operatorname{Im} \mathcal{M}(\text{in} \rightarrow \text{out}) = \sum_f \int d\Pi_f \mathcal{M}^*(\text{out} \rightarrow f) \mathcal{M}(\text{in} \rightarrow f), \quad (42)$$

where the sum runs over all possible final-state particles allowed by energy and momentum conservation. Any violation of unitarity will show up as a contradiction of equation (42).

Unitarity

We will consider the modified QED Lagrangian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{g}{2}n_\mu\epsilon^{\mu\nu\lambda\sigma}A_\nu(n\cdot\partial)^2F_{\lambda\sigma} - e\bar{\psi}\gamma^\mu A_\mu\psi,$$

and now we will check unitarity by means of the optical theorem

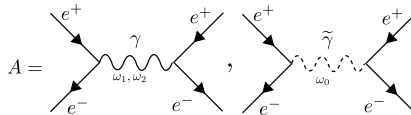


Figura : In-out diagrams for electron-positron scattering to order e^2 . The first diagram corresponds to perturbative fields and the other to the ghost field

Unitarity

Therefore, let us consider the amplitude

$$A = (-ie)^2 \bar{u}(p_1) \gamma^\mu v(p_2) \bar{v}(p_2) \gamma^\nu u(p_1) G_{\mu\nu}(k), \quad (43)$$

where the photon propagator is

$$G_{\mu\nu}(k) = \sum_{\lambda} P_{\mu\nu}^{(\lambda)} G^{(\lambda)}. \quad (44)$$

And we have to compare with

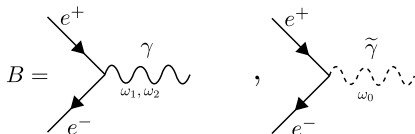


Figura : Physical vertex diagrams to order e^2 for the perturbative and ghost photon fields

Unitarity

We must show that

$$2\text{Im}[A] = B^\dagger B$$

Hence, let us start with A

$$\int \frac{d^4 k}{(2\pi)^4} \delta(p_1 + p_2 - k) A = \sum_\lambda \int \frac{dk^0}{(2\pi)} \int \frac{d^3 \vec{k}}{(2\pi)^3} \delta(p_1 + p_2 - k) \\ \times (-ie)^2 \bar{u}(p_1) \gamma^\mu v(p_2) \bar{v}(p_2) \gamma^\nu u(p_1) P_{\mu\nu}^{(\lambda)} \frac{1}{k^2 + 2g\lambda(n \cdot k)^3}. \quad (45)$$

Unitarity

For the imaginary part only the propagator poles contribute. We can replace the denominator in the the above expression by

$$\frac{1}{k^2 + 2g\lambda(n \cdot k)^3} \rightarrow \delta(f(\omega^\lambda)) \equiv \delta(k^2 + 2g\lambda(n \cdot k)^3) \quad (46)$$

Hence

$$2\text{Im}[A] = \sum_{\lambda} \int \frac{d^3k}{(2\pi)^3} \delta(p_1 + p_2 - k) \times (-ie)^2 \bar{u}(p_1) \gamma^\mu v(p_2) \bar{v}(p_2) \gamma^\nu u(p_1) P_{\mu\nu}^{(\lambda)} \frac{1}{f'(\omega_1^{(\lambda)})}. \quad (47)$$

We are interested in the regions of energy for the electrons and positrons where the effective theory is valid and therefore by energy conservation we have excluded the ghost contribution.

Unitarity

On the other hand, since we are excluding ghost fields to be stable and asymptotic states the second diagram of B do not contribute.

With the correct normalization for the polarization sum, that is the one considered in the field expansion

$$A_\mu(x) = \sum_\lambda \int d^4x \varepsilon_\mu^\lambda \delta(k^2 + 2g\lambda(n \cdot k)^3) e^{-ik \cdot x}$$

we have

$$\begin{aligned} 2Im[A] &= \sum_\lambda \int \frac{d^3k}{(2\pi)^3} \delta(p_1 + p_2 - k) M^{\mu\dagger} P_{\mu\nu}^{(\lambda)} M^\nu \frac{1}{f'(\omega_1^{(\lambda)})} \\ &= \int \frac{d^3k}{(2\pi)^3} \delta(p_1 + p_2 - k) M^{\mu\dagger} M^\nu \sum_\lambda \varepsilon_\mu^{\lambda*} \varepsilon_\nu^\lambda \\ &= \int \frac{d^3k}{(2\pi)^3} \delta(p_1 + p_2 - k) B^\dagger B \end{aligned} \quad (48)$$

Conclusions

- We have shown that for special choices of the background n the theory can be maintained of standard derivative theory.
- We have proved unitarity at tree level in the higher derivative sector of the M-P model
- Next work should be to analyze unitarity at one loop. We believe that the Cutkosky prescription can be implemented but a correct $i\epsilon$ prescription for the propagators needs to be determined.

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