Non Singular Origin of the Universe and its Present Vacuum Energy Density.

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The Anthropic Principle (Weinberg) versus the approach here

- Is interesting to cite this principle, because it is an attempt to connect the existence of the universe as we know it, with life in it, with a bound on the vacuum energy density of the present universe (VEDPU), since a big VEDPU expands the present universe too fast and will not allow galaxy formation and life.
- Problems with this idea: relies on our un-derstanding of life to do this analysis.
- Question: what if instead of life we talk about singularity free universe and what restrictions this imposes on the VEDPU?, is this possible?.

Non singular cosmology and its relation to vacuum energy density

Will be discussed in the context of the TWO MEASURES THEORY (TMT), taking into account also possible quantum corrections due to the zero point fluctuations and we will find the remarkable result that a stable, non singular early universe, in the framework of the so called EMERGENT UNIVERSE SCENARIO is linked to a positive but not too big Vacuum Energy Density for the present universe .
But similar excercise could be attempted for other theories, that is whether the vacuum energy density TODAY is constrained by the requirement of absence of singularty AS THE UNIVERSE ORIGINATED.

We introduce 4 scalar fields in 4-D

Which will play an important geometrical role, by the way, other authors have found that 4 scalar fields in 4-D can be used to define a generally covariant mass term for the graviton . A much simpler use of four scalars in 4D is to define a new MEASURE.

The Basic Idea of the Two Measures Theory (TMT)

The general structure of general coordinate invariant theories is taken usually as

$$S_1 = \int L_1 \sqrt{-g}\, d^4x\,, \tag{1}$$

where $g = \det(g_{\mu\nu})$. The introduction of $\sqrt{-g}$ is required since d^4x by itself is not a scalar but the product $\sqrt{-g}\, d^4x$ is a scalar. Inserting $\sqrt{-g}$, which has the transformation properties of a density, produces a scalar action S_1, as defined by Eq. (1), provided L_1 is a scalar.

In principle, nothing prevents us from considering other densities instead of $\sqrt{-g}$. One construction of such alternative "measure of integration," is obtained as follows: given 4-scalars φ_a $(a = 1, 2, 3, 4)$, one can construct the density

$$\Phi = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_\mu\varphi_a\partial_\nu\varphi_b\partial_\alpha\varphi_c\partial_\beta\varphi_d \tag{2}$$

One can consider both contributions, and allowing therefore both geometrical objects to enter the theory and take as our action

$$S = \int L_1 \sqrt{-g}\, d^4x + \int L_2 \Phi\, d^4x\,. \tag{6}$$

Here L_1 and L_2 are φ_a independent. There is a good reason not to consider nonlinear terms in Φ that mix Φ with $\sqrt{-g}$, for example

$$\frac{\Phi^2}{\sqrt{-g}} \tag{7}$$

appear.

This is because S in Eq. (6) is invariant (up to the integral of a total divergence) under the infinite-dimensional symmetry

$$\varphi_a \rightarrow \varphi_a + f_a(L_2)\,, \tag{8}$$

where $f_a(L_2)$ is an arbitrary function of L_2 if L_1 and L_2 are φ_a independent. Such symmetry (up to the integral of a total divergence) is absent if mixed terms (like (7)) are present. Therefore (6) is considered for the case when no dependence on the measure fields (MF) appears in L_1 or L_2.

Softly Broken Conformal Invariance, simple example

$$S_L = \int L_1 \sqrt{-g}\, d^4x + \int L_2 \Phi\, d^4x\,, \tag{9}$$

$$L_1 = U(\phi)\,, \tag{10}$$

$$L_2 = \frac{-1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)\,, \tag{11}$$

$$R(\Gamma, g) = g^{\mu\nu} R_{\mu\nu}(\Gamma)\,, \quad R_{\mu\nu}(\Gamma) = R^\lambda_{\mu\nu\lambda}\,, \tag{12}$$

$$R^\lambda_{\mu\nu\sigma}(\Gamma) = \Gamma^\lambda_{\mu\nu,\sigma} - \Gamma^\lambda_{\mu\sigma,\nu} + \Gamma^\lambda_{\alpha\sigma}\Gamma^\alpha_{\mu\nu} - \Gamma^\lambda_{\alpha\nu}\Gamma^\alpha_{\mu\sigma}\,. \tag{13}$$

In the variational principle $\Gamma^{\lambda}_{\mu\nu}, g_{\mu\nu}$, the measure fields scalars φ_a and the "matter"-scalar field ϕ are all to be treated as independent variables although the variational principle may result in equations that allow us to solve some of these variables in terms of others.

For the case where the potential terms $U = V = 0$, we have local conformal invariance

$$g_{\mu\nu} \rightarrow \Omega(x) g_{\mu\nu} \tag{14}$$

and φ_a is transformed according to

$$\varphi_a \rightarrow \varphi'_a = \varphi'_a(\varphi_b) , \tag{15}$$

$$\Phi \rightarrow \Phi' = J(x)\Phi , \tag{16}$$

where $J(x)$ is the Jacobian of the transformation of the φ_a fields.

This will be a symmetry in the case $U = V = 0$ if

$$\Omega = J . \tag{17}$$

Notice that J can be a local function of space-time, this can be arranged by performing for the φ_a fields one of the (infinite) possible diffeomorphism in the internal φ_a space.

special exponential form for the U and $\breve{V}$ potentials. Indeed, if we perform the global scale transformation ($\theta = \text{const}$)

$$g_{\mu\nu} \to e^{\theta} g_{\mu\nu} , \tag{18}$$

then (9) is invariant provided $V(\phi)$ and $U(\phi)$ are of the form[25]

$$V(\phi) = f_1 e^{\alpha\phi} , \quad U(\phi) = f_2 e^{2\alpha\phi} \tag{19}$$

and φ_a is transformed according to

$$\varphi_a \to \lambda_{ab} \varphi_b , \tag{20}$$

which means

$$\Phi \to \det(\lambda_{ab}) \Phi \equiv \lambda \Phi \tag{21}$$

such that

$$\lambda = e^{\theta} \tag{22}$$

and

$$\phi \to \phi - \frac{\theta}{\alpha} . \tag{23}$$

4. Spontaneously Broken Scale Invariance

Now we will solve for the scalar

$$\chi = \frac{\Phi}{\sqrt{-g}}$$

$$A^{\mu}_{a}\partial_{\mu} L_2 = 0 \,, \tag{24}$$

where $A^{\mu}_{a} = \varepsilon^{\mu\nu\alpha\beta}\varepsilon_{abcd}\partial_{\nu}\varphi_{b}\partial_{\alpha}\varphi_{c}\partial_{\beta}\varphi_{d}$. Since it is easy to check that $A^{\mu}_{a}\partial_{\mu}\varphi_{a'} = \frac{\delta_{aa'}}{4}\Phi$, it follows that $\det(A^{\mu}_{a}) = \frac{4^{-4}}{4!}\Phi^3 \neq 0$ if $\Phi \neq 0$. Therefore if $\Phi \neq 0$ we obtain that $\partial_{\mu} L_2 = 0$, or that

$$L_2 = \frac{-1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu} \partial_{\mu}\phi \partial_{\nu}\phi - V = M \,, \tag{25}$$

Considering now the variation with respect to $g^{\mu\nu}$, we obtain

$$\Phi\left(\frac{-1}{\kappa} R_{\mu\nu}(\Gamma) + \frac{1}{2}\phi_{,\mu}\phi_{,\nu}\right) - \frac{1}{2}\sqrt{-g}U(\phi)g_{\mu\nu} = 0 \,. \tag{31}$$

Solving for $R = g^{\mu\nu}R_{\mu\nu}(\Gamma)$ from Eq. (31) and introducing in Eq. (25), we obtain

$$M + V(\phi) - \frac{2U(\phi)}{\chi} = 0 \,, \tag{32}$$

a constraint that allows us to solve for χ,

$$\chi = \frac{2U(\phi)}{M + V(\phi)} \,. \tag{33}$$

To get the physical content of the theory, it is best consider variables that have well-defined dynamical interpretation. The original metric does not has a nonzero canonical momenta. The fundamental variable of the theory in the first-order formalism is the connection and its canonical momenta is a function of $\bar{g}_{\mu\nu}$ given by

$$\bar{g}_{\mu\nu} = \chi g_{\mu\nu} \tag{34}$$

and χ given by Eq. (33). Interestingly enough, working with $\bar{g}_{\mu\nu}$ is the same as going to the "Einstein conformal frame." In terms of $\bar{g}_{\mu\nu}$ the non-Riemannian contribution $\Sigma^{\alpha}_{\mu\nu}$ disappears from the equations. This is because the connection can be written as the Christoffel symbol of the metric $\bar{g}_{\mu\nu}$. In terms of $\bar{g}_{\mu\nu}$, the equations of motion for the metric can be written then in the Einstein form (we define $\bar{R}_{\mu\nu}(\bar{g}_{\alpha\beta})$ = usual Ricci tensor in terms of the bar metric $= R_{\mu\nu}$ and $\bar{R} = \bar{g}^{\mu\nu}\bar{R}_{\mu\nu}$)

$$\bar{R}_{\mu\nu}(\bar{g}_{\alpha\beta}) - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R}(\bar{g}_{\alpha\beta}) = \frac{\kappa}{2}T^{\rm eff}_{\mu\nu}(\phi), \tag{35}$$

where

$$T^{\rm eff}_{\mu\nu}(\phi) = \phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}\bar{g}^{\alpha\beta} + \bar{g}_{\mu\nu}V_{\rm eff}(\phi) \tag{36}$$

and

$$V_{\rm eff}(\phi) = \frac{1}{4U(\phi)}(V+M)^2\,. \tag{37}$$

In terms of the metric $\bar{g}^{\alpha\beta}$, the equation of motion of the scalar field ϕ takes the standard general-relativity form

$$\frac{1}{\sqrt{-\bar{g}}}\partial_\mu\left(\bar{g}^{\mu\nu}\sqrt{-\bar{g}}\partial_\nu\phi\right) + V'_{\rm eff}(\phi) = 0\,. \tag{38}$$

Notice that if $V + M = 0$, $V_{\rm eff} = 0$ and $V'_{\rm eff} = 0$ also, provided V' is finite and $U \neq 0$ there. This means the zero cosmological constant state is achieved without any sort of fine-tuning. That is, independently of whether we add to V a constant

Effective Potential for Exponential forms of U and V (scale invariance)

$$V(\phi) := \frac{\left(e^{\phi} - 1\right)^2}{20 e^{2\phi}}$$

$M < 0$

$\varepsilon := 0$

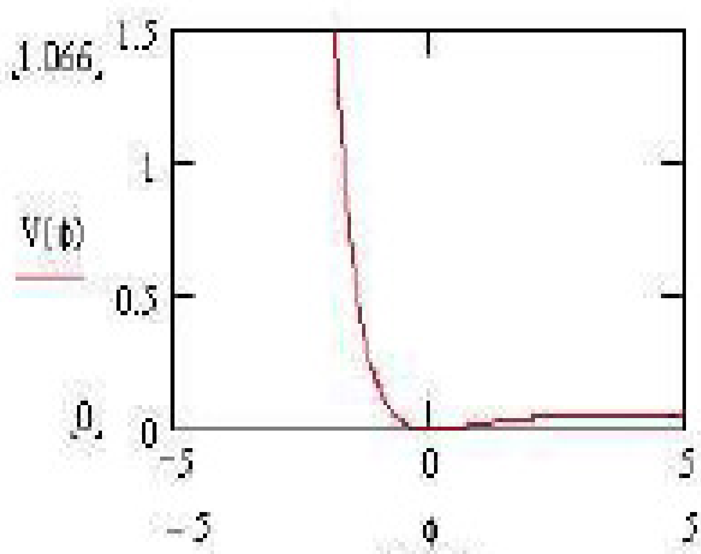

figure1

A comment, before going on (then we ignore this for the rest of talk)

In the expression for the effective potential for the scalar field one can obtain instead of strictly zero at the bottom a very small vacuum energy density by saying that the constant of integration M is not perfectly defined, but instead is gaussian - distributed with a very small width, then energy density at bottom is proportional to (energy density in flat region) X (width/average of M), may be justified by wormhole theory.

6. Generation of Two Flat Regions after the Introduction of a R^2 Term

As we have seen, it is possible to obtain a model that through a spontaneous breaking of scale invariance can give us a flat region. We want to obtain now two flat regions in our effective potential. A simple generalization of the action S_L will fix this. What one needs to do is simply consider the addition of a scale invariant term of the form

$$S_{R^2} = \epsilon \int \left(g^{\mu\nu} R_{\mu\nu}(\Gamma)\right)^2 \sqrt{-g}\, d^4x \,. \tag{45}$$

The total action being then $S = S_L + S_{R^2}$.[38] In the first-order formalism, S_{R^2} is not only globally scale invariant but also locally scale invariant, that is conformally

The variation of the action with respect to $g^{\mu\nu}$ gives now

$$R_{\mu\nu}(\Gamma)\left(\frac{-\Phi}{\kappa} + 2\epsilon R\sqrt{-g}\right) + \Phi\frac{1}{2}\phi_{,\mu}\,\phi_{,\nu} - \frac{1}{2}\left(\epsilon R^2 + U(\phi)\right)\sqrt{-g}\,g_{\mu\nu} = 0\,. \tag{46}$$

It is interesting to notice that if we contract this equation with $g^{\mu\nu}$, the ϵ terms do not contribute. This means that the same value for the scalar curvature R is obtained as in Sec. 2, if we express our result in terms of ϕ, its derivatives and $g^{\mu\nu}$. Solving the scalar curvature from this and inserting in the other ϵ-independent equation $L_2 = M$, we get still the same solution for the ratio of the measures which was found in the case where the ϵ terms were absent, i.e. $\chi = \frac{\Phi}{\sqrt{-g}} = \frac{2U(\phi)}{M+V(\phi)}$.

metric $\bar{g}_{\mu\nu}$ given by

$$\bar{g}_{\mu\nu} = \left(\frac{\Omega}{\sqrt{-g}}\right) g_{\mu\nu} = (\chi - 2\kappa\epsilon R) g_{\mu\nu} , \tag{47}$$

$\bar{g}_{\mu\nu}$ defines now the "Einstein frame." Equations (46) can now be expressed in the "Einstein form"

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = \frac{\kappa}{2} T^{\rm eff}_{\mu\nu} , \tag{48}$$

where

$$T^{\rm eff}_{\mu\nu} = \frac{\chi}{\chi - 2\kappa\epsilon R}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}\bar{g}^{\alpha\beta}\right) + \bar{g}_{\mu\nu} V_{\rm eff} , \tag{49}$$

$$V_{\rm eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2} . \tag{50}$$

Here it is satisfied that $\frac{-1}{\kappa} R(\Gamma, g) + \frac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V = M$, equation that expressed in terms of $\bar{g}^{\alpha\beta}$ becomes $\frac{-1}{\kappa} R(\Gamma, g) + (\chi - 2\kappa\epsilon R)\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V = M$. This allows us to solve for R and we get

$$R = \frac{-\kappa(V + M) + \frac{\kappa}{2}\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\chi}{1 + \kappa^2\epsilon\bar{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi} . \tag{51}$$

Notice that if we express R in terms of ϕ, its derivatives and $g^{\mu\nu}$, the result is the same as in the model without the curvature squared term, this is not true anymore once we express R in terms of ϕ, its derivatives and $\bar{g}^{\mu\nu}$.

In any case, once we insert (51) into (50), we see that the effective potential (50) will depend on the derivatives of the scalar field now. It acts as a normal scalar field potential under the conditions of slow rolling or low gradients and in the case the scalar field is near the region $M + V(\phi) = 0$.

Potential with 2 flat regions M<0

$$V(\phi) := \frac{\left(e^{\phi} - 1\right)^2}{4\left[\left(e^{\phi} - 1\right)^2 + 5e^{2\phi}\right]}$$

$M < 0$

$\varepsilon > 0$

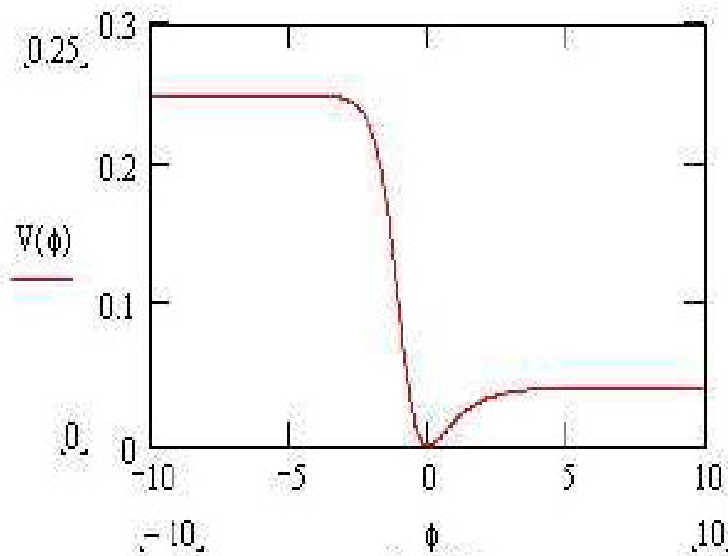

figure2

For M>0, we can get a "quintessential inflation" potential

$$V(\phi) := \frac{\left(e^{\phi} + 1\right)^2}{4\left[\left(e^{\phi} + 1\right)^2 + 5\,e^{2\,\phi}\right]}$$

$M > 0.$

$\varepsilon > 0$

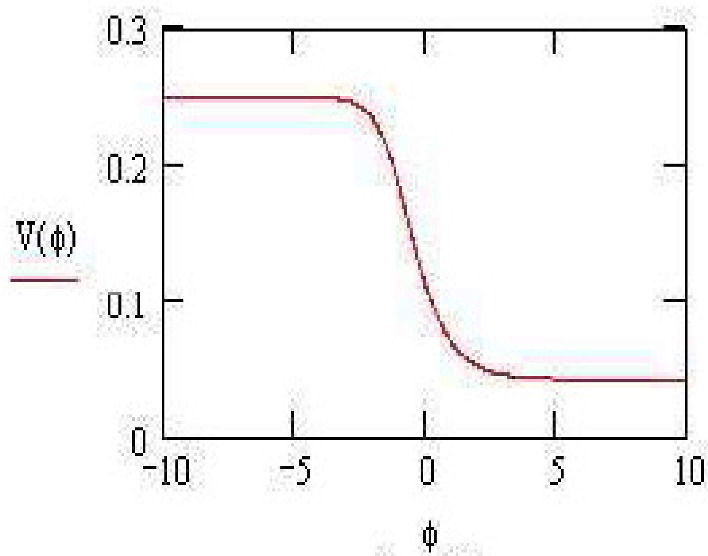

figure3

Second, for asymptotically large but negative values of the scalar field, we have

$$V_{\text{eff}}(\phi \rightarrow -\infty) \rightarrow \frac{1}{4\epsilon\kappa^2} \,. \tag{54}$$

- Another flat region is also obtained for the other extreme, the dilaton field goes to plus infinity. If the constant of integration M is negative, there is a local minimum at zero for the effective potential, we will consider this theory , but generalized somewhat to allow for the effect of zero point fluctuations, for this we consider first

in more details the Einstein frame

we see that the original metric does not have a canonically conjugated momentum (this turns out to be zero), in contrast, the canonically conjugated momentum to the conection turns out to be a function exclusively of $\overline{g}_{\mu\nu}$, this Einstein metric is therefore a genuine dynamical canonical variable, as opposed to the original metric. There is also a lagrangian formulation of the theory which uses $\overline{g}_{\mu\nu}$, as we will see in the next section, what we can call the action in the Einstein frame. In this frame we can quantize the theory for example and consider contributions without reference to the original frame, thus possibly considering breking the TMT structure of the theory through quantum effects, but such breaking will be done "softly" through the introduction of a cosmological term only. Surpringly, the remaining structure of the theory, reminiscent from the original TMT structure will be enough to control the strength of this additional cosmological term once we demand that the universe originated from a non singular and stable emergent state.

V. GENERALIZING THE MODEL TO INCLUDE EFFECTS OF ZERO POINT FLUCTUATIONS

The effective energy-momentum tensor can be represented in a form like that of a perfect fluid

$$T_{\mu\nu}^{eff} = (\rho + p)u_\mu u_\nu - p\bar{g}_{\mu\nu}, \qquad \text{where} \qquad u_\mu = \frac{\phi_{,\mu}}{(2X)^{1/2}} \tag{46}$$

here $X \equiv \frac{1}{2}\bar{g}^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}$. This defines a pressure functional and an energy density functional. The system of equations obtained after solving for χ, working in the Einstein frame with the metric $\bar{g}_{\mu\nu}$ can be obtained from a "k-essence" type effective action, as it is standard in treatments of theories with non linear kinetic tems or k-essence models[56]-[59]. The action from which the classical equations follow is,

$$S_{eff} = \int \sqrt{-\bar{g}} d^4x \left[-\frac{1}{\kappa}\bar{R}(\bar{g}) + p\left(\phi, R\right) \right] \tag{47}$$

$$p = \frac{\chi}{\chi - 2\kappa\epsilon R} X - V_{eff} \tag{48}$$

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2}$$

where it is understood that,

$$\chi = \frac{2U(\phi)}{M + V(\phi)}.$$

We have two possible formulations concerning R: Notice first that $\bar{R}$ and R are different objects, the $\bar{R}$ is the Riemannian curvature scalar in the Einstein frame, while R is a different object. This R will be treated in two different ways:

1. First order formalism for R. Here R is a lagrangian variable, determined as follows, R that appear in the expression above for p can be obtained from the variation of the pressure functional action above with respect to R, this gives exactly the expression for R that has been solved already in terms of X, ϕ, etc.

2. Second order formalism for R. R that appear in the action above is exactly the expression for R that has been solved already in terms of X, ϕ, etc. The second order formalism can be obtained from the first order formalism by solving algebraically R from the eq. obtained by variation of R , and inserting back into the action.

In contrast to the simplified models studied in literature[56]-[59], it is impossible here to represent $p(\phi, X; M)$ in a factorizable form like $\tilde{K}(\phi)\tilde{p}(X)$. The scalar field effective Lagrangian can be taken as a starting point for many considerations.

In particular, the quantization of the model can proceed from (47) and additional terms could be generated by radiative corrections. We will focus only on a possible cosmological term in the Einstein frame added (due to zero point fluctuations) to (47), which leads then to the new action

$$S_{eff,\Lambda} = \int \sqrt{-\bar{g}} d^4x \left[-\frac{1}{\kappa} \bar{R}(\bar{g}) + p(\phi, R) - \Lambda \right] \tag{51}$$

This addition to the effective action leaves the equations of motion of the scalar field unaffected, but the gravitational equations aquire a cosmological constant. Adding the Λ term can be regarded as a redefinition of $V_{eff}(\phi, X; M)$

$$V_{eff}(\phi, R) \rightarrow V_{eff}(\phi, R) + \Lambda \tag{52}$$

As we will see the stability of the emerging Universe imposses interesting constraints on Λ

After introducing the Λ term, we get from the variation of R the same value of R, unaffected by the new Λ term, but as one can easily see then R does not have the interpretation of a curvature scalar in the original frame since it is unaffected by the new source of energy

ANALYSIS OF THE EMERGENT UNIVERSE SOLUTIONS

We now want to consider the detailed analysis of The Emerging Universe solutions and in the next section their stability in the TMT scale invariant theory. We start considering the cosmological solutions of the form (in the Einstein frame),

$$ds^2 = dt^2 - a(t)^2(\frac{dr^2}{1-r^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)), \phi = \phi(t) \tag{53}$$

We will consider a scenario where the scalar field ϕ is moving in the extreme right region $\phi \to \infty$, in this case the expressions for the energy density ρ and pressure p are given by,

$$\rho = \frac{A}{2}\dot{\phi}^2 + 3B\dot{\phi}^4 + C \tag{54}$$

and

$$p = \frac{A}{2}\dot{\phi}^2 + B\dot{\phi}^4 - C \tag{55}$$

It is interesting to notice that all terms proportional to $\dot{\phi}^4$ behave like "radiation", since $p_{\dot{\phi}^4} = \frac{\rho_{\dot{\phi}^4}}{3}$ is satisfied. here the constants A, B and C are given by,

$$A = \frac{f_2}{f_2 + \kappa^2 \epsilon f_1^2}, \tag{56}$$

$$B = \frac{\epsilon \kappa^2}{4(1 + \kappa^2 \epsilon f_1^2 / f_2)} = \frac{\epsilon \kappa^2}{4} A, \tag{57}$$

$$C = \frac{f_1^2}{4 f_2 (1 + \kappa^2 \epsilon f_1^2 / f_2)} = \frac{f_1^2}{4 f_2} A + \Lambda. \tag{58}$$

It will be convenient to "decompose" the constant Λ into two pieces,

$$\Lambda = -\frac{1}{4\kappa^2\epsilon} + \Delta\lambda \tag{59}$$

since as $\phi \to -\infty$, $V_{eff} \to \Delta\lambda$. Therefore $\Delta\lambda$ has the interesting interpretation of the vacuum energy density in the $\phi \to -\infty$ vacuum. As we will see, it is remarkable that the stability and existence of non singular emergent universe implies that $\Delta\lambda > 0$, and it is bounded from above as well.

The equation that determines such static universe $a(t) = a_0 = constant$, $\dot{a} = 0$, $\ddot{a} = 0$ gives rise to a restriction for $\dot{\phi}_0$ that have to satisfy the following equation in order to guarantee that the universe be static, because $\ddot{a} = 0$ is proportional to $\rho + 3p$, we must require that $\rho + 3p = 0$, which leads to

$$3B\dot{\phi}_0^4 + A\dot{\phi}_0^2 - C = 0, \tag{60}$$

This equation leads to two roots, the first being

$$\dot{\phi}_1^2 = \frac{\sqrt{A^2 + 12BC} - A}{6B} .$$

The second root is:

$$\dot{\phi}_2^2 = \frac{-\sqrt{A^2 + 12BC} - A}{6B} . \tag{62}$$

It is also interesting to see that if the discriminant is positive, the first solution has automatically positive energy density, if we only consider cases where $C > 0$, which is required if we want the emerging solution to be able to turn into an inflationary solution eventually. One can see that the condition $\rho > 0$ for the first solution reduces to the inequality $w > (1 - \sqrt{1 - w})/2$, where $w = -12BC/A^2 > 0$, since we must have $A > 0$, otherwise we get a negative kinetic term during the inflationary period, and as we will see in the next section, we must have that $B < 0$ from the stability of the solution, and as long as the discriminant is positive, i.e. $0 < w < 1$, it is always true that this inequality is satisfied.

STABILITY OF THE STATIC SOLUTION

We will now consider the perturbation equations. Considering small deviations of ϕ the from the static emerging solution value $\dot{\phi}_0$ and also considering the perturbations of the scale factor a, we obtain, from Eq. (54)

$$\delta\rho = A\dot{\phi}_0\delta\dot{\phi} + 12B\dot{\phi}_0^3\delta\dot{\phi} \tag{63}$$

at the same time $\delta\rho$ can be obtained from the perturbation of the Friedmann equation

$$3(\frac{1}{a^2} + H^2) = \kappa\rho \tag{64}$$

and since we are perturbing a solution which is static, i.e., has $H = 0$, we obtain then

$$-\frac{6}{a_0^3}\delta a = \kappa\delta\rho \tag{65}$$

we also have the second order Friedmann equation

$$\frac{1 + \dot{a}^2 + 2a\ddot{a}}{a^2} = -\kappa p \tag{66}$$

For the static emerging solution, we have $p_0 = -\rho_0/3$, $a = a_0$, so

$$\frac{2}{a_0^2} = -2\kappa p_0 = \frac{2}{3}\kappa\rho_0 = \Omega_0\kappa\rho_0 \tag{67}$$

where we have chosen to express our result in terms of Ω_0, defined by $p_0 = (\Omega_0 - 1)\rho_0$, which for the emerging solution has the value $\Omega_0 = \frac{2}{3}$. Using this in (65), we obtain

$$\delta\rho = -\frac{3\Omega_0\rho_0}{a_0}\delta a \tag{68}$$

and equating the values of $\delta\rho$ as given by (63) and (68) we obtain a linear relation between $\delta\dot{\phi}$ and δa, which is,

$$\delta\dot{\phi} = D_0\delta a \tag{69}$$

where

$$D_0 = -\frac{3\Omega_0\rho_0}{a_0\dot{\phi}_0(A + 12B\dot{\phi}_0^2)} \tag{70}$$

we now consider the perturbation of the eq. (66). In the right hand side of this equation we consider that $p = (\Omega - 1)\rho$, with

$$\Omega = 2\Big(1 - \frac{U_{eff}}{\rho}\Big), \tag{71}$$

where,

$$U_{eff} = C + B\,\dot{\phi}^4$$

and therefore, the perturbation of the eq. (66) leads to,

$$-\frac{2\delta a}{a_0^3} + 2\frac{\delta\ddot{a}}{a_0} = -\kappa\delta p = -\kappa\delta((\Omega - 1)\rho) \tag{73}$$

to evaluate this, we use (71), (72) and the expressions that relate the variations in a and $\dot{\phi}$ (69). Defining the "small" variable β as

$$a(t) = a_0(1 + \beta) \tag{74}$$

we obtain,

$$2\ddot{\beta}(t) + W_0^2\beta(t) = 0\,, \tag{75}$$

where,

$$W_0^2 = \Omega_0\,\rho_0 \left[\frac{24\,B\,\dot{\phi}_0^2}{A + 12\,\dot{\phi}_0^2\,B} - 6\frac{(C + B\,\dot{\phi}_0^4)}{\rho_0} - 3\kappa\Omega_0 + 2\kappa\right], \tag{76}$$

notice that the sum of the last two terms in the expression for W_0^2, that is $-3\kappa\Omega_0 + 2\kappa$

vanish since $\Omega_0 = \frac{2}{3}$, for the same reason, we have that $6\frac{(C+B\,\dot\phi_0^4)}{\rho_0} = 4$, which brings us to

$$W_0^2 = \Omega_0\,\rho_0\left[\frac{24\,B\,\dot\phi_0^2}{A + 12\,\dot\phi_0^2\,B} - 4\right], \tag{77}$$

For the stability of the static solution, we need that $W_0^2 > 0$, where $\dot\phi_0^2$ is defined either by E. (61) ($\dot\phi_0^2 = \dot\phi_1^2$) or by E. (62) ($\dot\phi_0^2 = \dot\phi_2^2$). If we take E. (62) ($\dot\phi_0^2 = \dot\phi_2^2$) and use this in the above expression for W_0^2, we obtain,

$$W_0^2 = \Omega_0\,\rho_0\left[\frac{4\sqrt{A^2+12BC}}{-2\sqrt{A^2+12BC} - A}\right], \tag{78}$$

to avoid negative kinetic terms during the slow roll phase that takes place following the emergent phase, we must consider $A > 0$, so, we see that the second solution is unstable and will not be considered further.

Now in the case of the first solution, E. (61) ($\dot\phi_0^2 = \dot\phi_1^2$), then W_0^2 becomes

$$W_0^2 = \Omega_0\,\rho_0\left[\frac{-4\sqrt{A^2+12BC}}{2\sqrt{A^2+12BC} - A}\right], \tag{79}$$

so the condition of stability becomes $2\sqrt{A^2+12BC}-A<0$, or $2\sqrt{A^2+12BC}<A$, squaring both sides and since $A>0$, we get $12BC/A^2<-3/4$, which means $B<0$, and therefore $\epsilon<0$, multiplying by -1, we obtain, $12(-B)C/A^2>3/4$, replacing the values of A,B,C, given by (56) we obtain the condition

$$\Delta\lambda>0, \tag{80}$$

Now there is the condition that the discriminant be positive $A^2+12BC>0$

$$\Delta\lambda<\frac{1}{12(-\epsilon)\kappa^2}\left[\frac{f_2}{f_2+\kappa^2\epsilon f_1^2}\right], \tag{81}$$

since $A=\left[\frac{f_2}{f_2+\kappa^2\epsilon f_1^2}\right]>0$, $B<0$, meaning that $\epsilon<0$, we see that we obtain a positive upper bound for the energy density of the vacuum as $\phi\to-\infty$, which must be positive, but not very big.

VIII. THE VACUUM STRUCTURE OF THE THEORY. EVOLUTION OF THE UNIVERSE, FROM ITS NON SINGULAR ORIGINS TO ITS PRESENT SLOWLY ACCELERATING STATE AT $\phi \rightarrow -\infty$, CROSSING "BARRIERS".

For the discussion of the vacuum structure of the theory, we start studying V_{eff} for the case of a constant field ϕ, given by,

$$V_{eff} = \frac{(f_1 e^{\alpha\phi} + M)^2}{4(\epsilon\kappa^2(f_1 e^{\alpha\phi} + M)^2 + f_2 e^{2\alpha\phi})} + \Lambda \tag{82}$$

This is necessary, but not enough, since as we will see, the consideration of constant fields ϕ alone can lead to missleading conclusions, in some cases, the dependence of V_{eff} on the kinetic term can be crucial to see if and how we can achieve the crossing of an apparent barrier.

For a constant field ϕ the limiting values of V_{eff} are (now that we added the constant Λ):

First, for asymptotically large positive values, ie. as $\alpha\phi \rightarrow \infty$, we have $V_{eff} \rightarrow \frac{f_1^2}{4(\epsilon\kappa^2 f_1^2 + f_2)} + \Lambda$.

Second, for asymptotically large but negative values of the scalar field, that is as $\alpha\phi \rightarrow -\infty$, we have: $V_{eff} \rightarrow \frac{1}{4\epsilon\kappa^2} + \Lambda = \Delta\lambda$.

In these two asymptotic regions ($\alpha\phi \rightarrow \infty$ and $\alpha\phi \rightarrow -\infty$) an examination of the scalar field equation reveals that a constant scalar field configuration is a solution of the equations, as is of course expected from the flatness of the effective potential in these regions.

Notice that in all the above discussion it is fundamental that $M \neq 0$. If $M = 0$ the potential becomes just a flat one, $V_{eff} = \frac{f_1^2}{4(\epsilon\kappa^2 f_1^2 + f_2)} + \Lambda$ everywhere (not only at high values of $\alpha\phi$).

Finally, there is a minimum at $V_{eff} = \Lambda$ if $M < 0$. In summary, and if $f_2 > 0$, $A > 0$, we have that there is a hierarchy of vacua ,

$$V_{eff}(\alpha\phi \rightarrow -\infty) = \Delta\lambda < V_{eff}(min, M < 0) = \Lambda < V_{eff}(\alpha\phi \rightarrow \infty) = C \tag{83}$$

where $C = \frac{f_1^2}{4\, f_2(1+\kappa^2\epsilon f_1^2/f_2)} + \Lambda = \frac{f_1^2}{4f_2} A + \Lambda$. notice that we assume above that $f_1 > 0$ and $M < 0$, but $f_1 < 0$ and $M > 0$ would be indistinguishable from that situation, that is, the important requirement is $f_1/M < 0$. We could have a scenario where we start the non singular emergent universe at $\phi \rightarrow \infty$ where $V_{eff}(\alpha\phi \rightarrow \infty) = \frac{f_1^2}{4(\epsilon\kappa^2 f_1^2 + f_2)} + \Lambda$, which then slow rolls, then inflates [22] and finally gets trapped in the local minimum with energy density $V_{eff}(min, M < 0) = \Lambda$, that was the picture favored in [22], while here we want to argue that the most attractive and relevant description for the final state of our Universe is realized after inflation in the flat region $\phi \to -\infty$, since in this region the vacuum energy density is positive and bounded from above, so its a good candidate for our present state of the Universe. It remains to be seen howevere whether a smooth transition all the way from $\phi \to \infty$ to $\phi \to -\infty$ is possible.

In order to discuss the possiblility of transition to $\phi \to -\infty$. In our case, since we are interested in a local minimum between $\phi \to \infty$ or $\phi \to -\infty$, we can take M of either sign.

Taking for definitness $f_1 > 0$, $f_2 > 0$, $A > 0$, $\epsilon < 0$, we see that there will be a point, defined by $\epsilon\kappa^2(f_1 e^{\alpha\phi} + M)^2 + f_2 e^{2\alpha\phi} = 0$ where the effective potential for a constant field ϕ, then V_{eff} as given by (82), will spike to ∞, go then down to $-\infty$ and then asymptotically approach its possitive asymptotic value at $\phi \to -\infty$. This has the appearence of a potential barrier. However, this is deceptive, such barrier may exist for constant ϕ, but can be avoided by considering time dependence, say for no space dependence and $\dot{\phi}^2$ given by

$$\dot{\phi}^2 = -\frac{1}{\epsilon\kappa^2} \tag{84}$$

which has a solution in the real domain for $\epsilon < 0$. For this case R (which is not a Riemannian curvature), as given by (44) diverges. In this case then

$$V_{eff} = \frac{\epsilon R^2 + U}{(\chi - 2\kappa\epsilon R)^2} + \Lambda \rightarrow \frac{1}{4\epsilon\kappa^2} + \Lambda = \Delta\lambda \tag{85}$$

that is, for this value of $\dot{\phi}^2$, regardless of the value of the scalar field, the value of V_{eff} becomes degenerate with its value for constant and arbitrarily negative ϕ, which is our candidate vacuum for the present state of the Universe. Therefore there is no barrier that prevents from us reaching arbitrarily negative ϕ from any point in field space in this model.

IX. DISCUSSION, THE CREATION OF THE UNIVERSE AS A "THRESHOLD EVENT" FOR ZERO PRESENT VACUUM ENERGY DENSITY

We have considered a non singular origin for the Universe starting from an Einstein static Universe, the so called "emergent universe" scenario, in the framework of a theory which uses two volume elements $\sqrt{-g}d^4x$ and Φd^4x, where Φ is a metric independent density, used as an additional measure of integration. Also curvature, curvature square terms and for scale invariance a dilaton field ϕ are considered in the action. The first order formalism was applied. The integration of the equations of motion associated with the new measure gives rise to the spontaneous symmetry breaking (S.S.B) of scale invariance (S.I.). After S.S.B. of S.I., using the the Einstein frame metric, it is found that a non trivial potential for the dilaton is generated. One could question the use of the Einstein frame metric $\overline{g}_{\mu\nu}$ in contrast to the original metric $g_{\mu\nu}$. In this respect, it is interesting to see the role of both the original metric and that of the Einstein frame metric in a canonical approach to the first order formalism. Here we see that the original metric does not have a canonically conjugated momentum (this turns out to be zero), in contrast, the canonically conjugated momentum to the conection turns out to be a function exclusively of $\overline{g}_{\mu\nu}$, this Einstein metric is therefore a genuine dynamical canonical variable, as opposed to the original metric.

There is also a lagrangian formulation of the theory which uses $\overline{g}_{\mu\nu}$, what we can call the action in the Einstein frame. In this frame we can quantize the theory for example and consider contributions without reference to the original frame, thus possibly considering breaking the TMT structure of the theory, but such breaking will be done "softly" through the introduction of a cosmological term only. In previous studies, we have found that the TMT structure of the theory, where neither the lagrangian L_1that couples to $\sqrt{-g}$, or L_2, that couples to Φ depend on the measure fields, is protected by an infinite dimensional symmetry $\varphi_a \to \varphi_a + f_a(L_2)$, where $f_a(L_2)$ is an arbitrary function of L_2. The additional cosmological term, introduced here in the Einstein frame, does not have a representation of this form in the original frame, therefore breaking the TMT structure (therefore the infinite dimensional symmetry would be also broken by quantum effects). Surpringly, the remaining terms of the theory, reminiscent from the original TMT structure will be enough to control the strength of this additional cosmological term once we demand that the universe originated from a non singular and stable emergent state.

In the Einstein frame we argue that the cosmological term parametrizes the zero point fluctuations.

The resulting effective potential for the dilaton contains two flat regions, for $\phi \to \infty$ relevant for the non singular origin of the Universe, followed by an inflationary phase and then transition to $\phi \to -\infty$, which in this paper we take as describing our present Universe. An intermediate local minimum is obtained if $f_1/M < 0$, the region as $\phi \to \infty$ has a higher energy density than this local minimum and of course of the region $\phi \to -\infty$, if $A > 0$ and $f_2 > 0$. $A > 0$ is also required for satisfactory slow roll in the inflationary region $\phi \to \infty$ (after the emergent phase). The dynamics of the scalar field becomes non linear and these non linearities are instrumental in the stability of some of the emergent universe solutions, which exists for a parameter range of values of the vacuum energy in $\phi \to -\infty$, which must be positive but not very big, avoiding the extreme fine tuning required to keep the vacuum energy density of the present universe small. A sort of solution of the Cosmological Constant Problem, where an a priori arbitrary cosmological term is restricted by the consideration of the nonsingular and stable emergent origin for the universe.

Notice then that the creation of the universe can be considered as a "threshold event" for zero present vacuum energy density, that is a threshold event for $\Delta\lambda = 0$ and we can learn what we can expect in this case by comparing with well known threshold events. For example in particle physics, the process $e^+ + e^- \to \mu^+ + \mu^-$, has a cross section of the form (ignoring the mass of the electron and considering the center of mass frame, E being the center of mass energy of each of the colliding e^+ or e^-),

$$\sigma_{e^+ + e^- \to \mu^+ + \mu^-} = \frac{\pi \alpha^2}{6E^2} \left[2 + \frac{m_\mu^2}{E^2} \right] \sqrt{\frac{E^2 - m_\mu^2}{E^2}}$$

for $E > m_\mu$ and exactly zero for $E < m_\mu$. We see that exactly at threshold this cross section is zero, but at this exact point it has a cusp, the derivative is infinite and the function jumps as we slightly increase E. By analogy, assuming that the vacuum energy can be tuned somehow (like the center of mass energy E of each of the colliding particles in the case of the annihilation process above), we can expect zero probability for exactly zero vacuum energy density $\Delta\lambda = 0$, but that soon after we build up any positive $\Delta\lambda$ we will then able to create the universe, naturally then, there will be a creation process resulting in a universe with a small but positive $\Delta\lambda$ which represents the total energy density for the region describing the present universe, $\phi \to -\infty$.

One challenge would be to in fact calculate from this approach the probability of creating the universe with a given vacuum energy density of the vacuum for the region describing the present universe, $\phi \to -\infty$, the same way we calculate the probability of the process $e^+ + e^- \to \mu^+ + \mu^-$. This will give us the probability of a given present vacuum energy density.

Other virtues of the model

- The late phase of the universe, with a massless scalar field has been shown not to have the 5th force problem in these generic type of models for matter with densities much higher than the vacuum energy density (infamous problem of Quintessence models). E.G. & A. Kaganovich, Annals of Physics, **323:866-882,2008**. e-Print: **arXiv:0704.1998** [gr-qc]

Speculations now: 1. Creating an Emergent Universe?

- One may ask the question: how is it possible to discuss the "creation of the universe" in the context of the "emergent universe"?. After all, the Emergent Universe basic philosophy is that the universe had a past of infinite duration. However, that most simple notion of an emergent universe with a past of infinite duration has been recently challenged by Mithani and Vilenkin, at least in the context of a special model. They have shown that a completle stable emergent universe, although completly stable classically, could be unstable under a tunneling process to collapse. On the other hand, an emergent universe can indeed be created from nothing by a tunneling process as well.

2. Emergent universe as intermediate state

an emerging universe could last for a long time provided it is classically stable, that is where the constraints on the cosmological constant for the late universe discussed here come in. If it is not stable, the emergent universe will not provide us with an appropriate "intermediate state“ connecting the creation of the universe with the present universe. The existence of this stable intermediate state provides in our picture the reason for the universe to prefer a very small vacuum energy density at late times, since universes that are created, but do not make use of the intermediate classically stable emergent universe will almost immediately recollapse, so they will not be "selected". Finally, it could be that we arrive to the emergent solution not by quantum creation, but just by classical evolution from something else, from here we go on to inflation.

Conclusions

- Scale invariant Two Measure model with curvature square implies dilaton potential with two flat regions, one for inflation, other for present accelerated universe (e.g. Quint. Infl.).
- Arbitrary CC in Einstein frame controlled by non singular and stable origin of universe, implies VEDPU>0 and not big.
- Other features: No fifth force problem, possibility of creating Universe gives zero VEDPU=0 as threshold point.

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