Segundo Encuentro de Cosmología y Gravitación en Concepción
COSMOCONCE 2012

Dark Energy Models with Lagrange Multipliers Revisited

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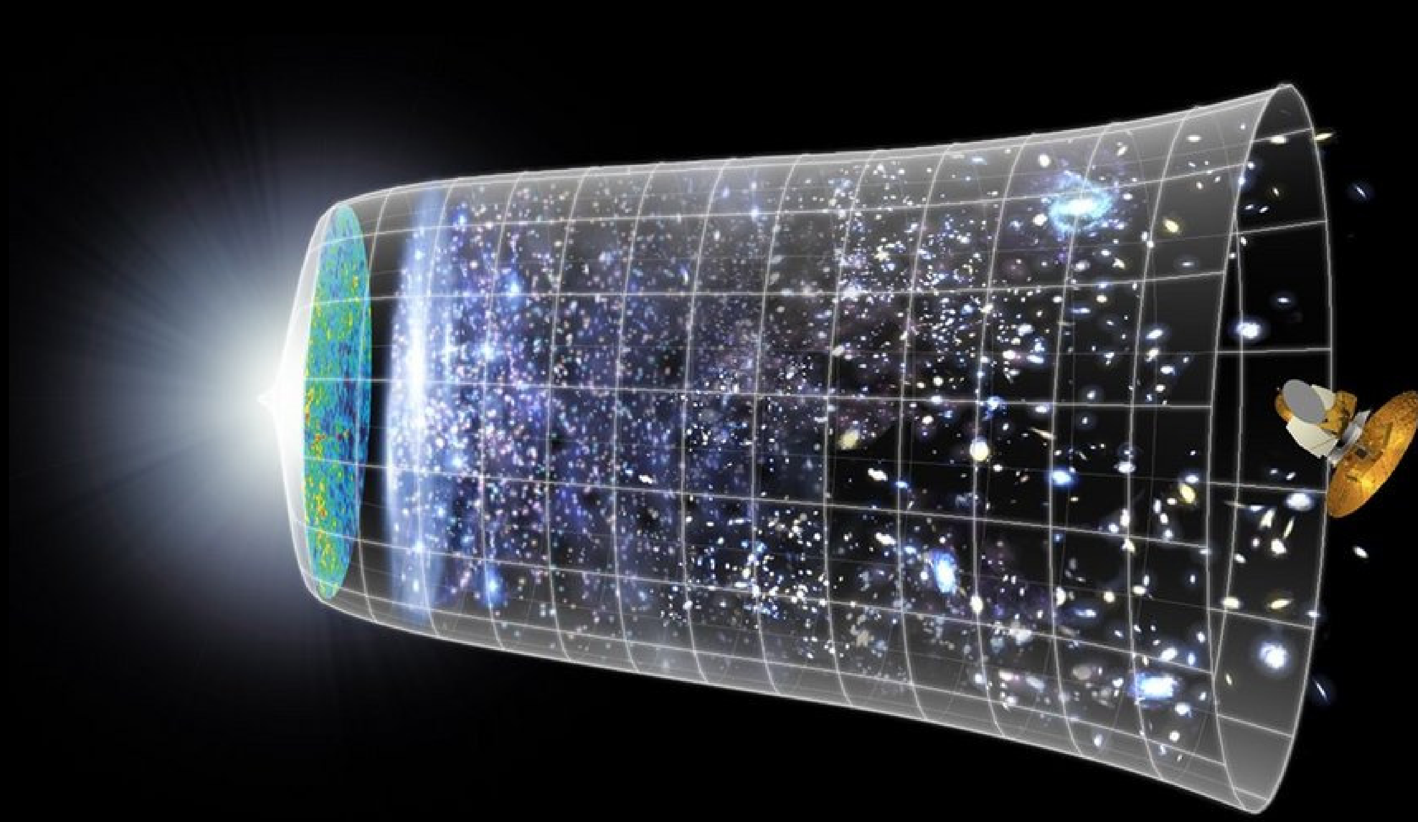
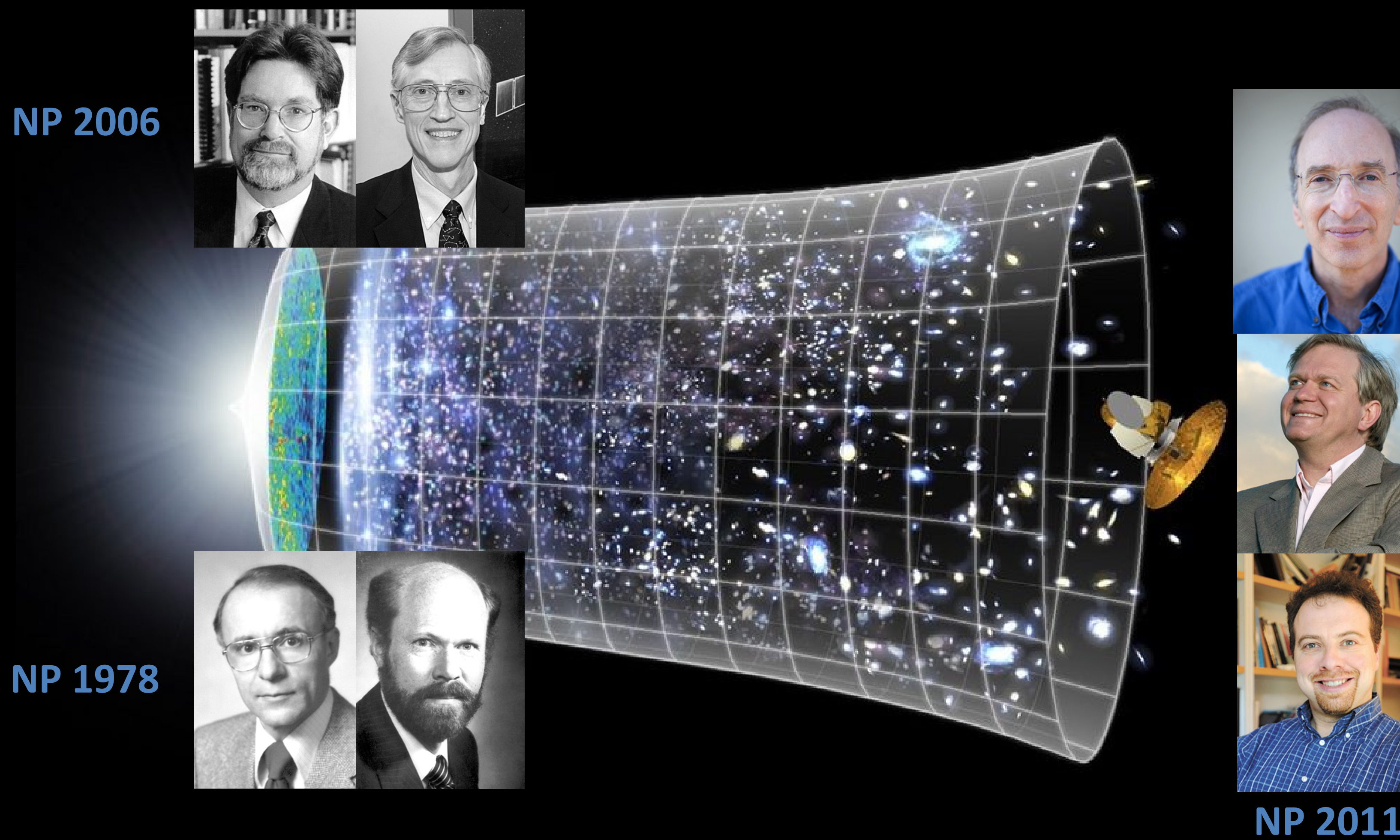
The Universe



Models from Data



There are observational evidence to establish that there have been two accelerating expansion stages in the evolution of our universe

The ΛCDM model

Action (Gravitational Interaction + Matter content)

$$S = \int (\mathcal{L}_G + \mathcal{L}_m)\, d^4x$$

Geometry (Symmetries: Homogeneity + Isotropy):
Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + d\Omega^2 \right)$$

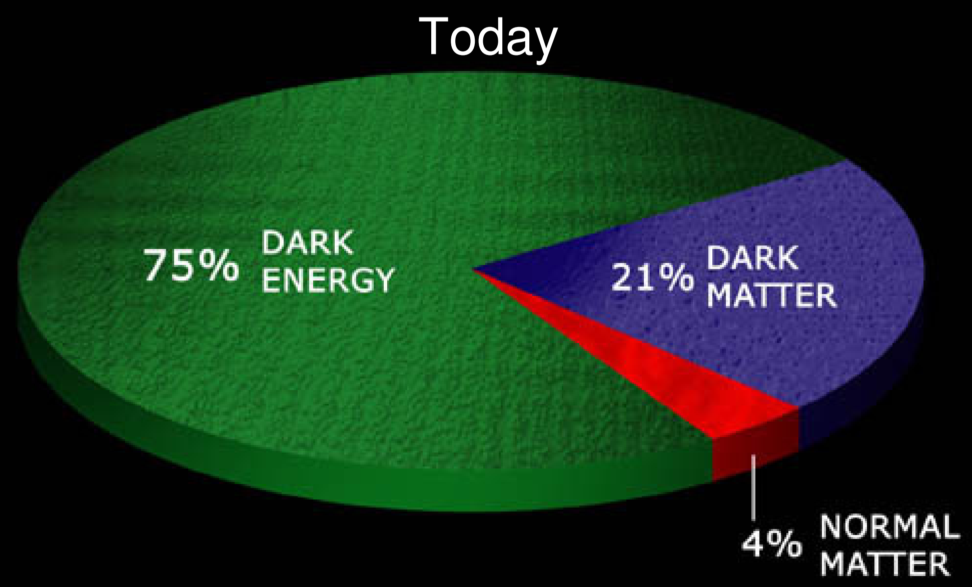
Flat: $k = 0$
Open: $k = -1$
Close: $k = +1$

Energy - momentum Tensor: Perfect Fluid

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} = \mathrm{diag}(\rho, -p, -p, -p)$$

Rad.: $p = \rho/3$
Dust: $p = 0$
CC: $p = -\rho$

The Matter Content



We do not know what are the constituent parts of dark components

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$$S = \int (\mathcal{L}_G + \mathcal{L}_m)\, d^4x$$

Geometry (Symmetries: Homogeneity + Isotropy):
Friedmann-Robertson-Walker metric

$$ds^2 = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + d\Omega^2 \right)$$

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PROBLEMS

Cosmological Constant

Coincidence Problem

Models for dark energy include…

- More general gravity theory
- Anisotropic or inhomogeneous models
- Interaction between different components
- Viscous cosmological models
- Exotic Fluids: Chaplygin gas

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Dust of Dark Energy

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We introduce a novel class of field theories where energy always flows along timelike geodesics, mimicking in that respect dust, yet which possess non-zero pressure. This theory comprises two scalar fields, one of which is a Lagrange multiplier enforcing a constraint between the other's field value and derivative. We show that this system possesses no wave-like modes but retains a single dynamical degree of freedom. Thus, the sound speed is always identically zero on all backgrounds. In particular, cosmological perturbations reproduce the standard behaviour for hydrodynamics in the limit of vanishing sound speed. Using all these properties we propose a model unifying Dark Matter and Dark Energy in a single degree of freedom. In a certain limit this model exactly reproduces the evolution history of ΛCDM, while deviations away from the standard expansion history produce a potentially measurable difference in the evolution of structure.

Given that DM and DE manifest themselves so far only through their gravitational interaction, it is appealing to consider a unified version of the dark sector.

The Action

Lagrange multipler

Potential

$$S = \int d^4x\sqrt{-g}\left(\overbrace{-\frac{R}{2\kappa^2}}^{\text{Gravity}} + \underbrace{K(\psi, X) + \lambda\left(X - \frac{1}{2}\mu^2(\psi)\right)}_{\text{Matter Content}}\right)$$

Ricci scalar

$\frac{1}{2}g^{\alpha\beta}\nabla_\alpha\psi\nabla_\beta\psi$

The Equations of Motion

$$\boxed{G^{\nu}_{\mu} = \kappa^2 T^{\nu}_{\mu}; \quad \nabla_{\nu} T^{\nu}_{\mu} = 0 \quad \text{and} \quad X = \frac{1}{2}\mu^2(\psi)}$$

Einstein Eq. Conservation Eq. Constraint

$$T^{\nu}_{\mu} = \mathrm{diag}(\rho, -p, -p, -p)$$

Perfect Fluid Form

It is assumed that the dynamics of our present universe is dominated by only one dark component:

$$\rho = \mu^2(K_X + \lambda) - K$$
$$p = K(\psi, \mu^2(\psi))$$

$$\omega = \frac{p}{\rho}$$

State parameter

Dynamical Equations

FRW metric:

$$\boxed{ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right)}$$

We can recover the dynamics of the dark sector when $K = X$

$$\boxed{\begin{aligned} \omega' &= 3\omega\left(1+\omega-\epsilon\sqrt{\frac{-6\omega}{1-k\chi}}\right) \\ \chi' &= \chi(1+3\omega) \\ \epsilon' &= -\epsilon^2(\Gamma-1)\sqrt{\frac{-6\omega}{1-k\chi}} \end{aligned}}$$

$$\chi = \frac{3}{\rho a^2}$$

$$\epsilon = -\left(\frac{\mu_\psi}{\mu}\right)$$

$$\Gamma = \frac{\mu_{\psi\psi}\mu}{\mu_\psi^2}$$

Primes denote derivative respect to log (a) and subscript derivatives respect to ψ

Nearly Flat Scalar Field Potential

We consider a nearly flat scalar field potential i.e.: $\epsilon \ll 1$ and nearly constant, then

$$\begin{aligned} \omega' &= 3\omega\left(1+\omega-\sqrt{\frac{\omega}{\omega_f}}\frac{(1+\omega_f)}{\sqrt{1-k\chi}}\right) \\ \chi' &= \chi(1+3\omega) \end{aligned}$$

Where $\epsilon = \frac{3}{2\sqrt{6}}\frac{1+\omega_f}{\sqrt{-\omega_f}}$ and ω_f has to be close to -1

For a constant potential, $\epsilon = 0$ and $\omega_f = -1$, the critical points are all independent of the spatial curvature

Dynamical System Analysis
Critical Points

- $\chi_c = 0 \text{ and } \omega_c = \omega_f$
- $\chi_c = 0 \text{ and } \omega_c = 0$

This critical points are independent of the curvature of spatial sections

- $\omega_c = -\dfrac{1}{3} \quad \text{and} \quad \chi_c = \dfrac{10 + \frac{3}{\omega_f} + 3\omega_f}{4k} \qquad k \neq 0$

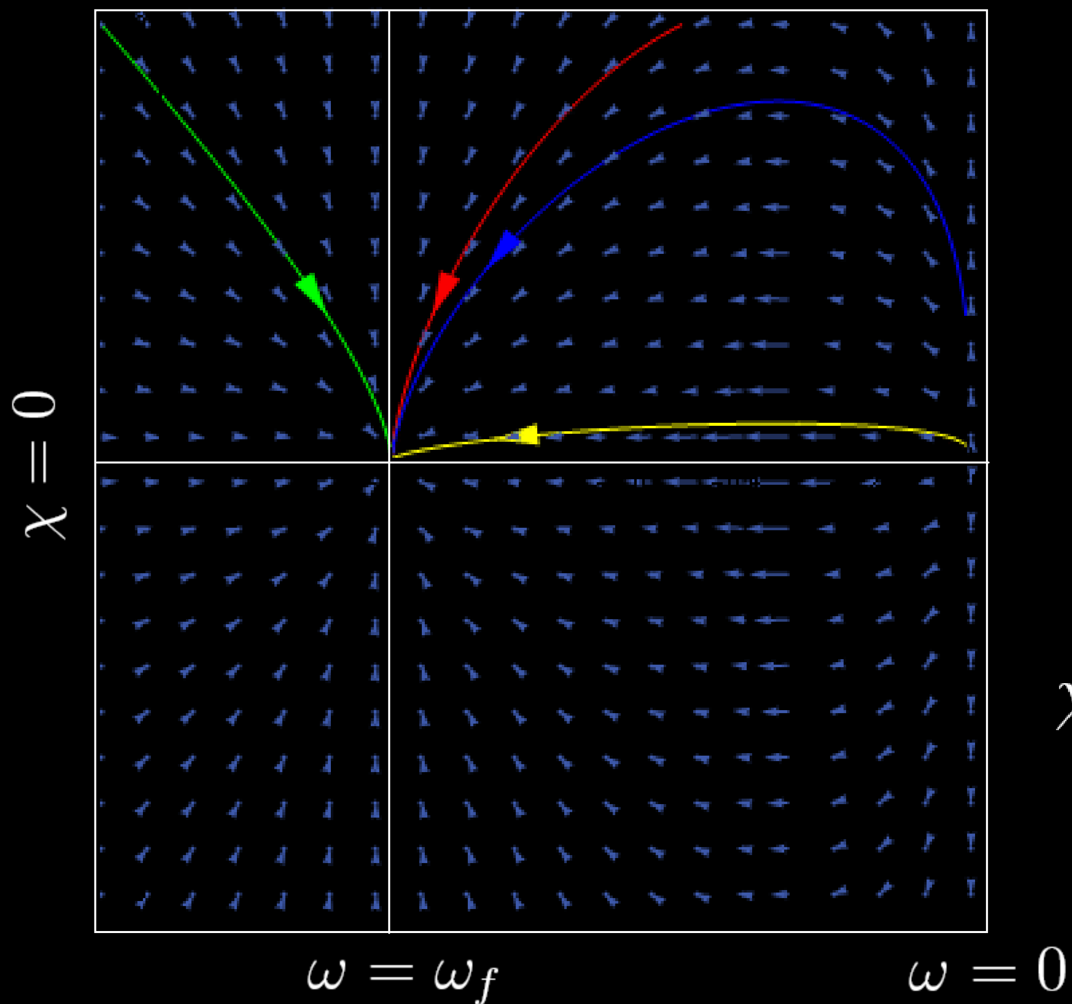
Dynamical System Analysis
Stability of Critical Points

We study the dynamics in the vicinity of the critical points by calculating the eigenvalues of the matrix:

$$\begin{pmatrix} \delta\omega' \\ \delta\chi' \end{pmatrix} = \begin{pmatrix} 3\left(1+2\omega_c - \frac{3\sqrt{\frac{\omega_c}{\omega_f}}(1+\omega_f)}{2\sqrt{1-k\chi_c}}\right) & -\frac{3k\omega_c\sqrt{\frac{\omega_c}{\omega_f}}(1+\omega_f)}{2(1-k\chi_c)^{3/2}} \\ 3\chi_c & 1+3\omega_c \end{pmatrix} \begin{pmatrix} \delta\omega \\ \delta\chi \end{pmatrix}$$

$(0,0)$	➡	Both are always positives	Unstable
$(0,\omega_f)$	➡	Both are negatives if $\omega_f < -\frac{1}{3}$	Attractor
$\left(-\frac{1}{3}, \frac{10+\frac{3}{\omega_f}+3\omega_f}{4k}\right)$	➡	$\omega_f < 0 \ \& \ \omega_f > -\frac{1}{3}$	Attractor
		$\omega_f < -\frac{1}{3} \ \& \ \omega_f > -1$	Saddle
		$\omega_f < -1$	Unstable

Dynamical System Analysis



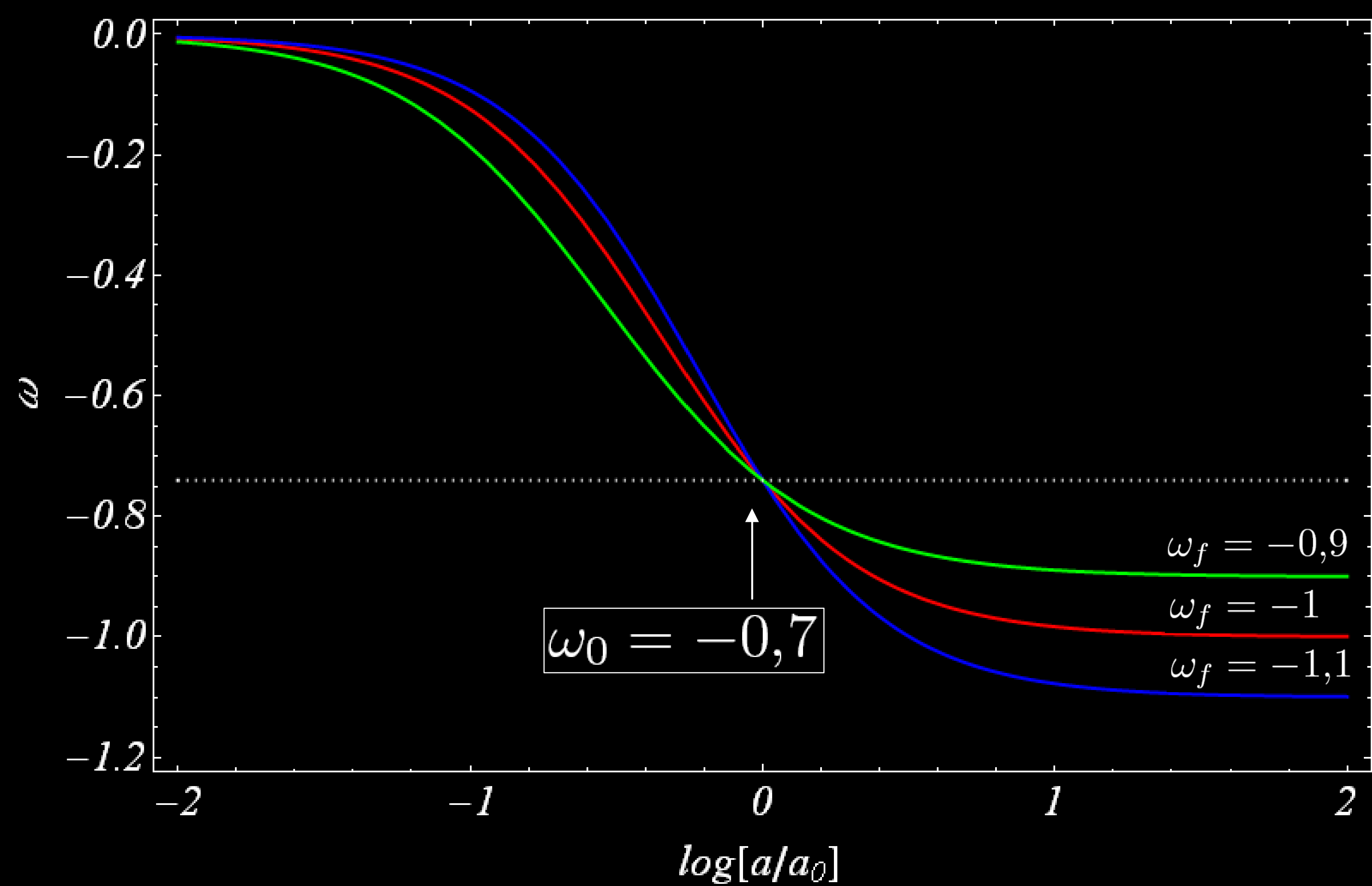
Physically interesting Critical Points

$$(\chi_c, \omega_c) = (0, 0)$$
$$(\chi_c, \omega_c) = (0, \omega_f)$$

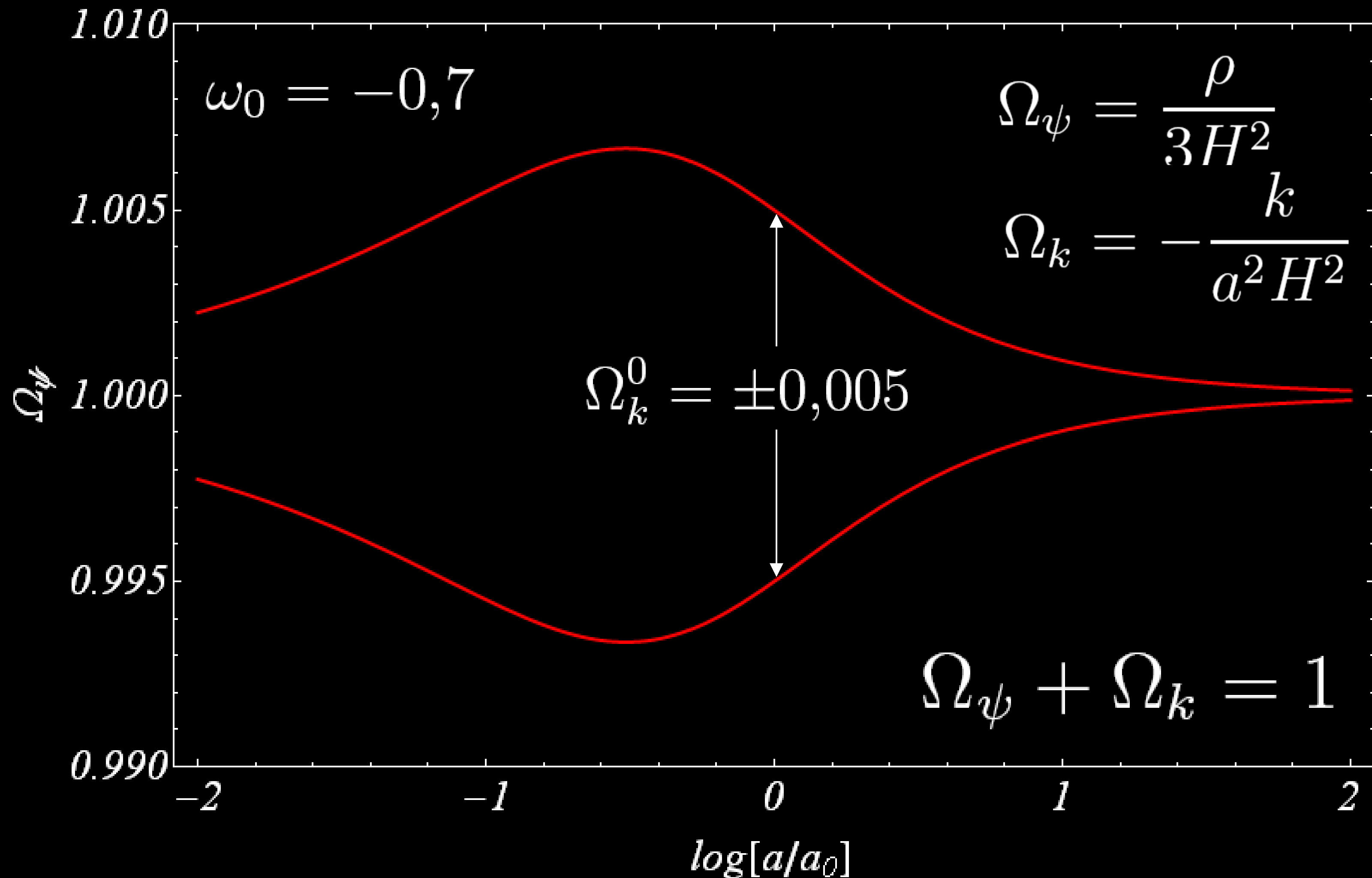
$$\chi_c \rightarrow 0 \Rightarrow a \xrightarrow[\text{Future}]{} \infty$$

$$\text{or } \rho \xrightarrow[\text{Past}]{} \infty$$

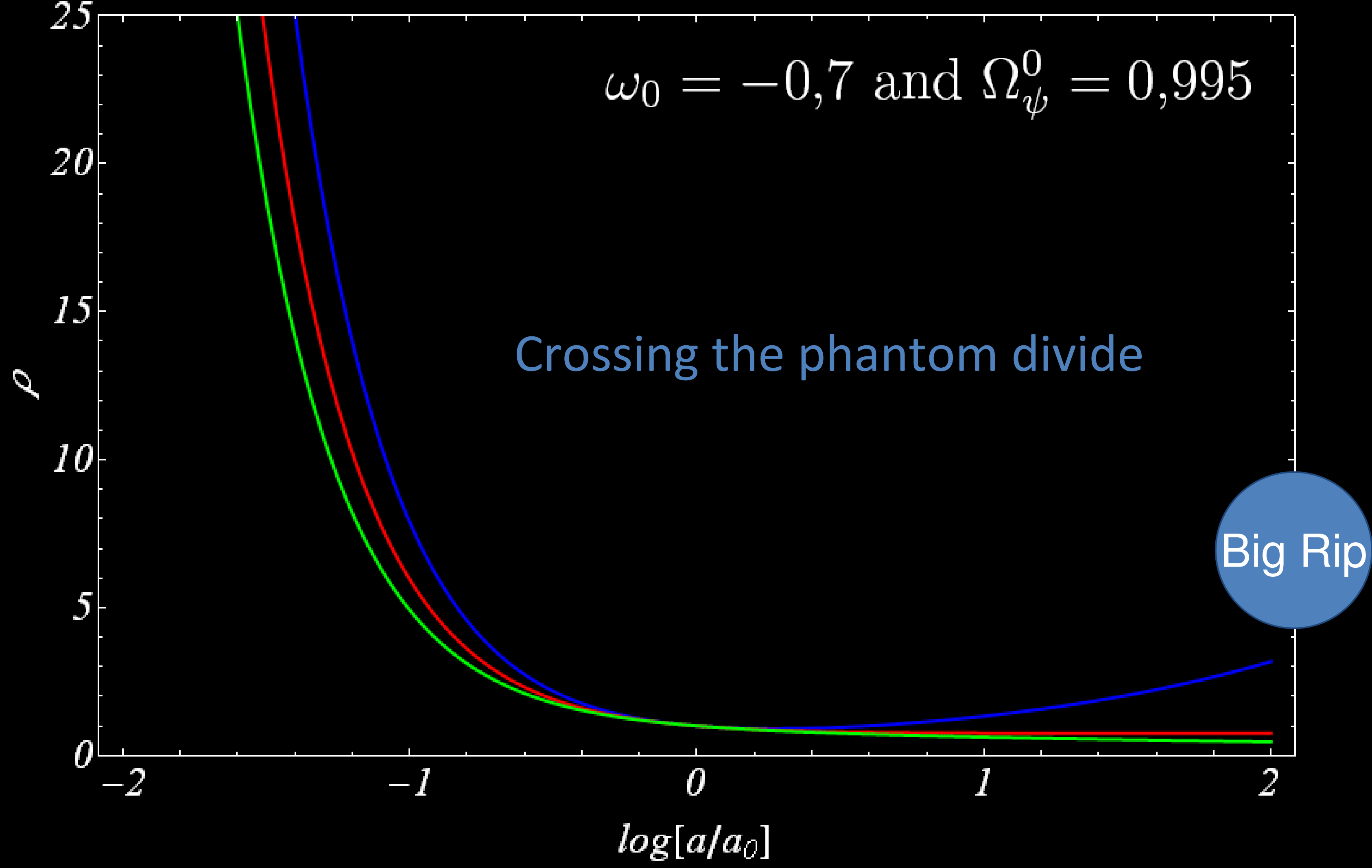
Numerical Results



Numerical Results



Numerical Results



Bayesian Analysis

We numerically solve the set of equations for 3 free parameters:

$$\boxed{\begin{aligned} \omega'(N) &= 3\left(1+\omega(N)-\sqrt{\frac{\omega(N)}{\omega_f}}\frac{1+\omega_f}{\sqrt{1+\Omega_k^0\xi(N)}}\right) \\ \xi'(N) &= \xi(N)(1+3\omega(N)) \end{aligned}}$$

$$\omega(0) = \omega_0$$

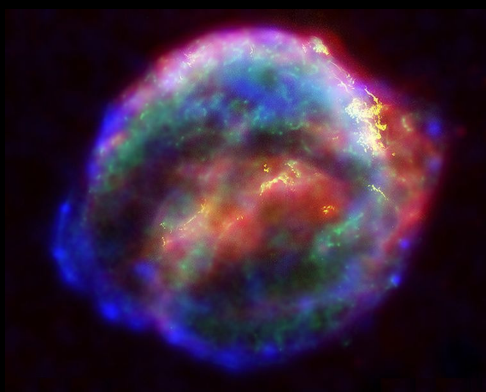
$$\xi(0) = 1-\Omega_k^0$$

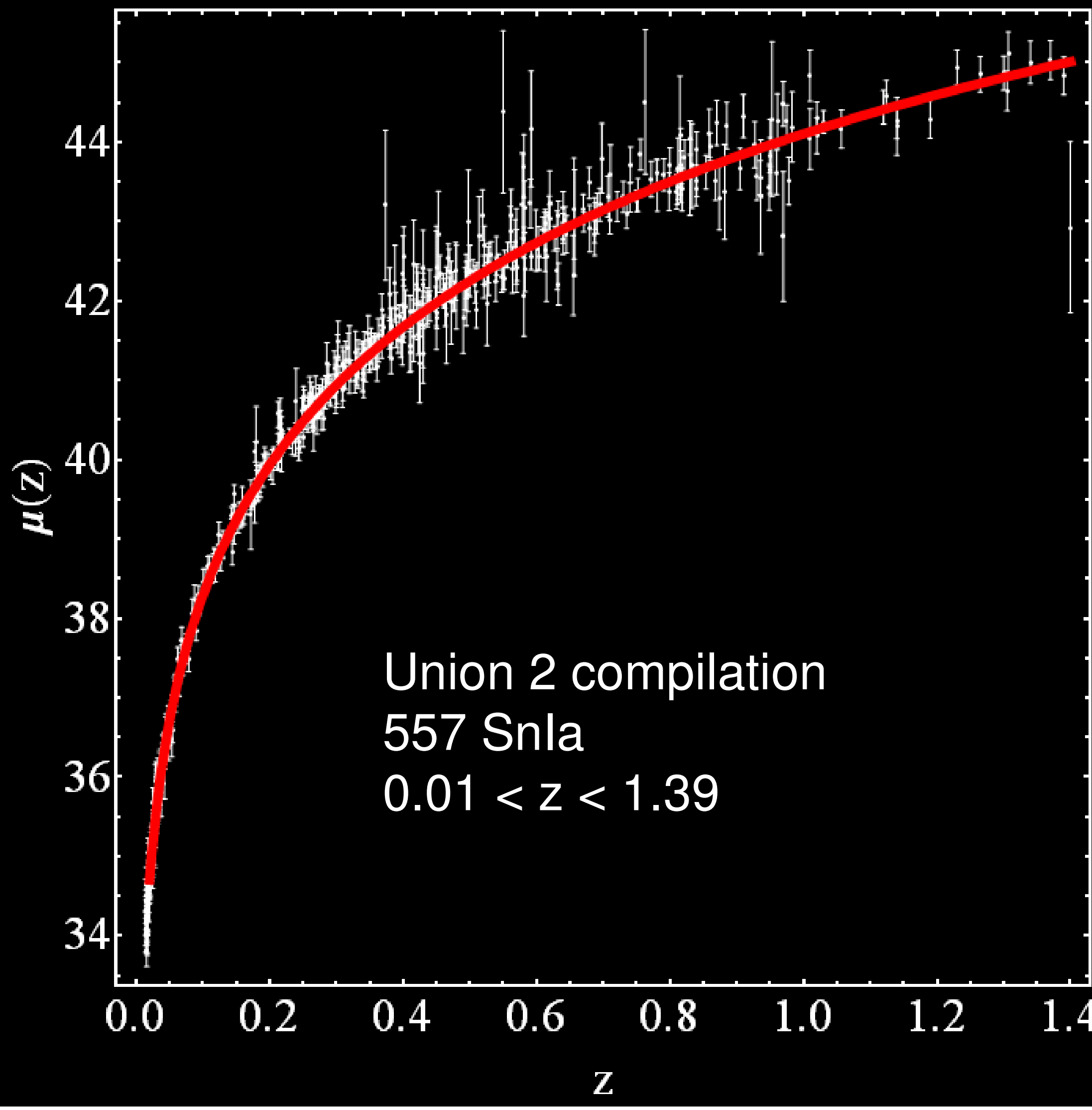
$N = \log[a/a_0]$ and $\xi \leftrightarrow \chi$

We compute the χ^2 function associated and minimize it:

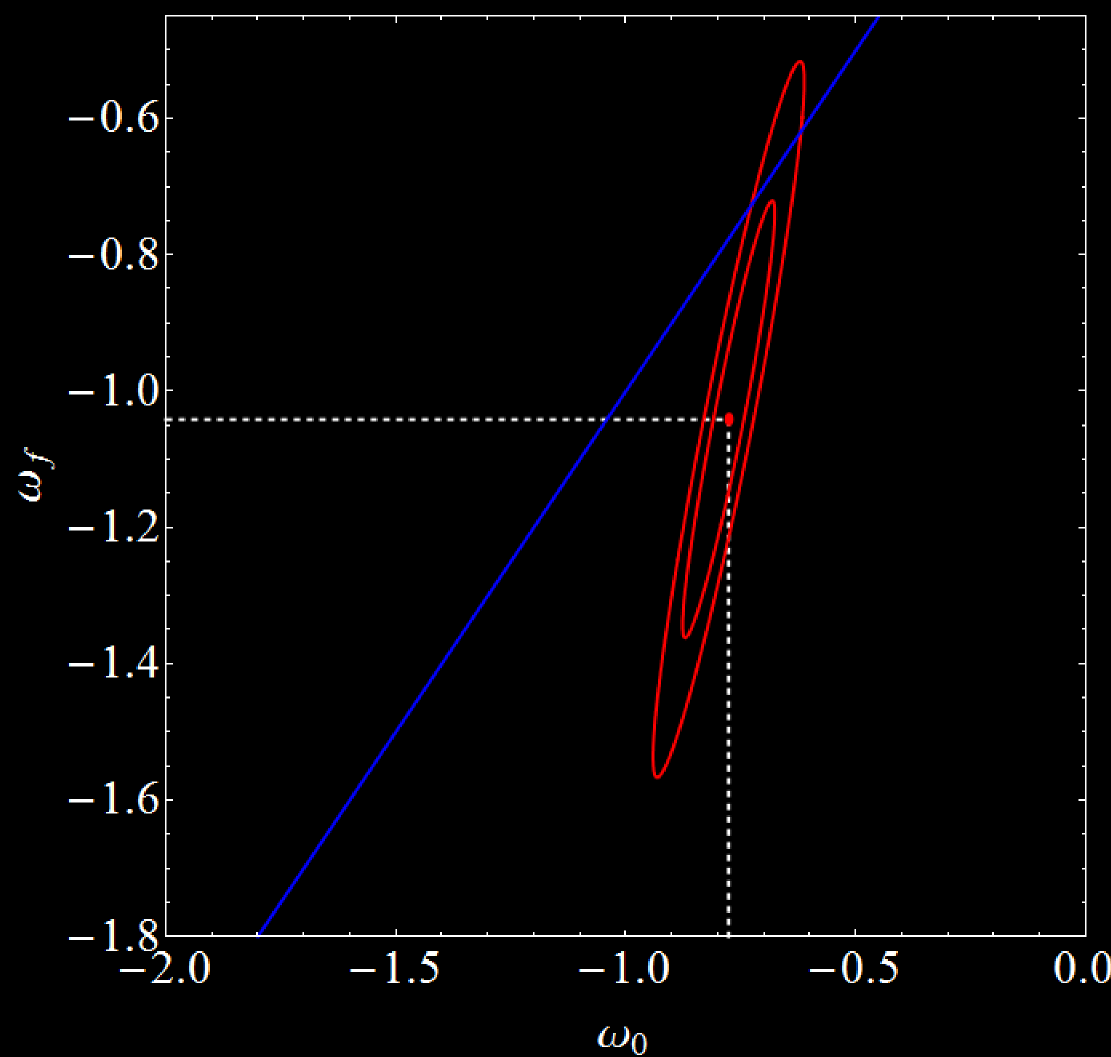
$$\chi^2 = \sum_i^{ndat}\left(\frac{f[\omega_0, \omega_f, \Omega_k^0] - f_0}{\sigma_f}\right)^2$$

Supernova Ia Data

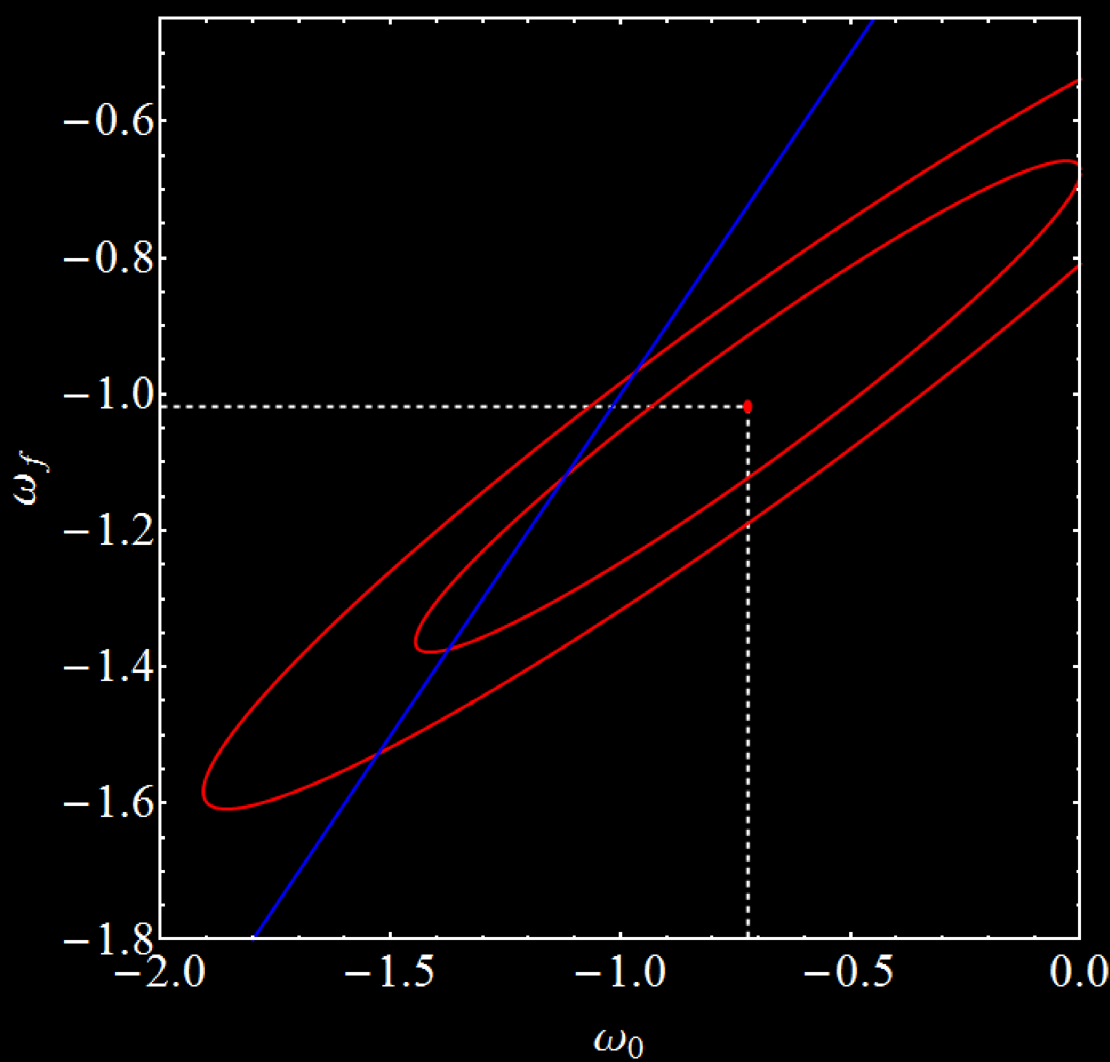




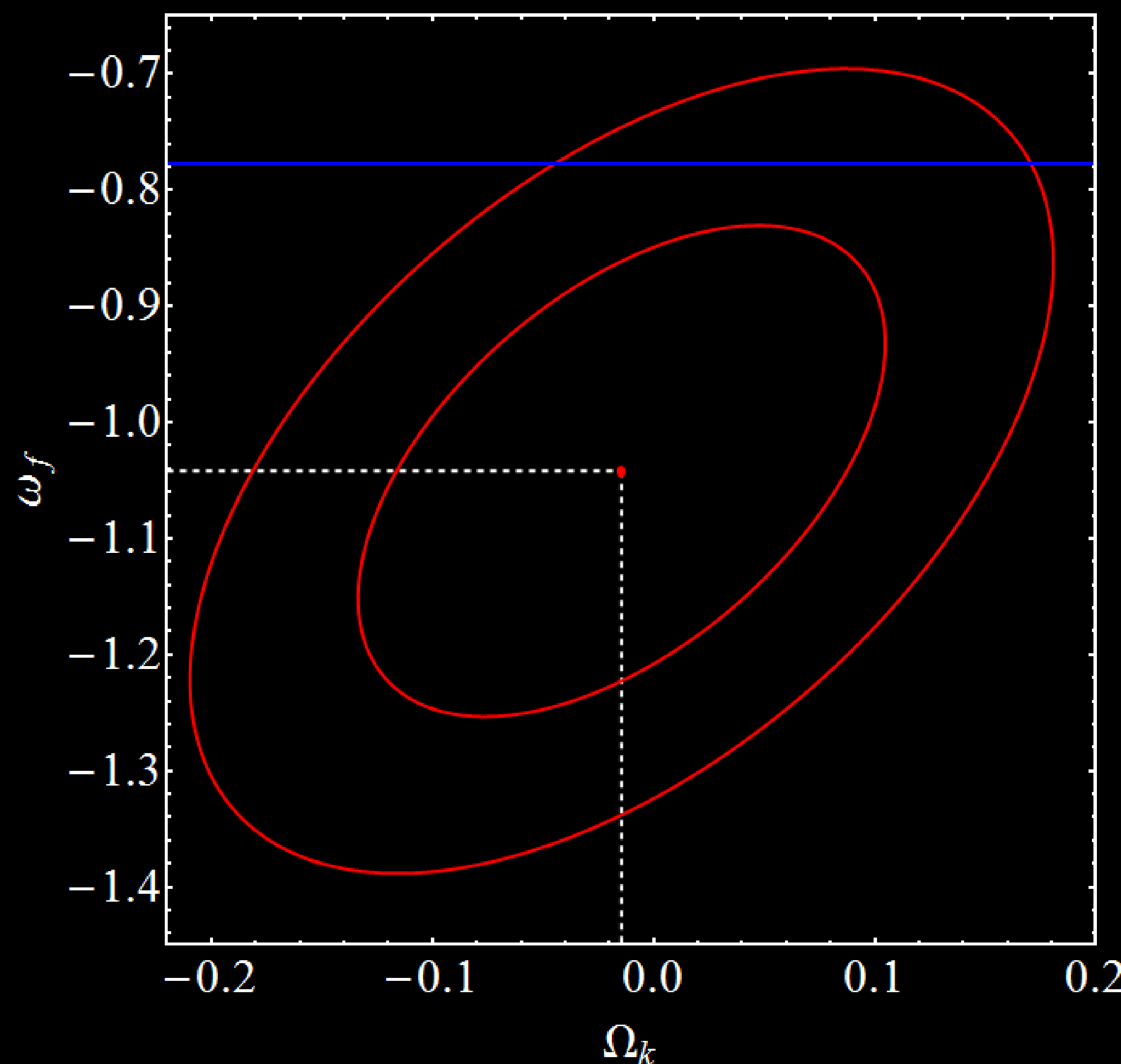
SnIa data without curvature



H(z) data without curvature



SnIa data with curvature



Results and Perspectives

- The dynamical system analysis of the model is independent of the curvature of spatial sections
- We have used SnIa data to constraint the model with and without curvature. The analysis shows that current SnIa data does not rule out spatial curvature.
- The data seems to slightly favored a closed universe
- Perspectives: It is possible to use this kind of model to describe early universe?
- Persepectives: To study structure formation in detail