On the importance of heavy fields during inflation

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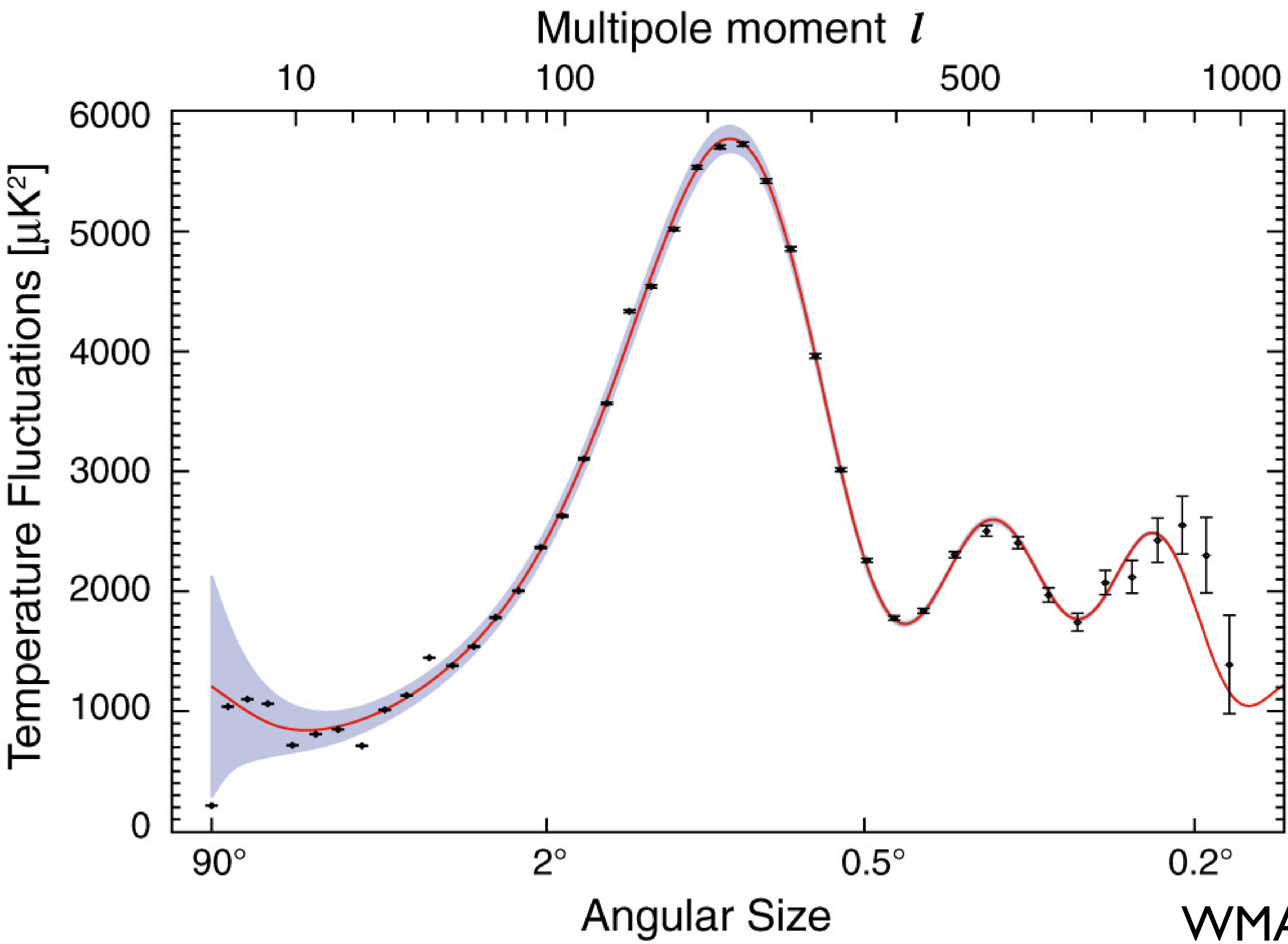
(based on arXiv:1201.4848)

Outline

- Motivation
- Inflation as an EFT
- Two Field Inflation
- Sudden curves
- Conclusions

Motivation

Cosmic inflation persits as the most compelling mechanism explaining the origin of primordial curvature perturbations.



WMAP collaboration (2010)

Motivation

- Cosmic inflation persists as the most compelling mechanism explaining the origin fo primordial curvature perturbations.
- The fact that inflation is formulated within a field theoretical framework makes it particularly compelling to test our ideas about fundamental theories
- These theories predicts the existence of a large number of degrees of fredom (scalar potentials)

Motivation

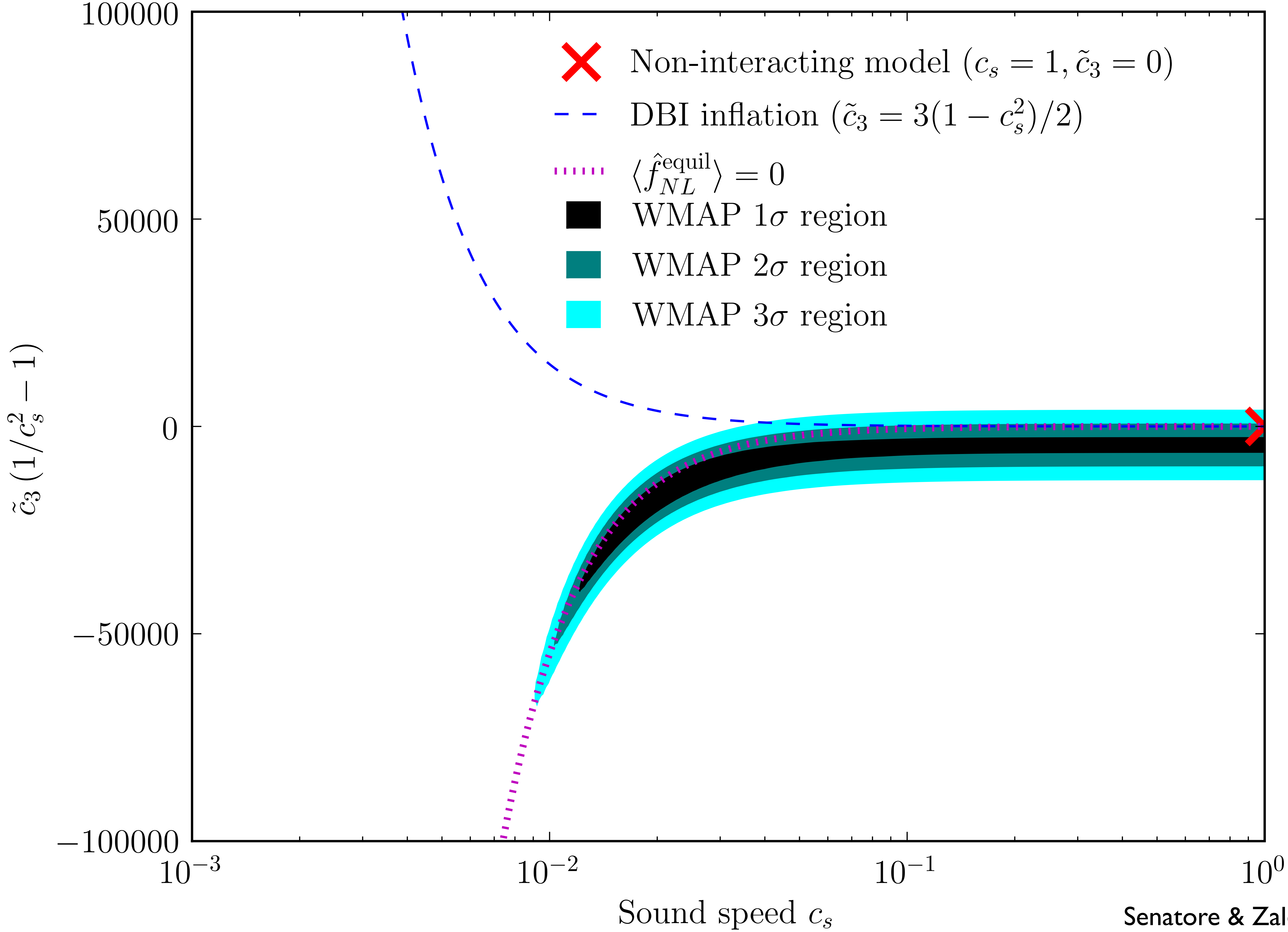
- For Example, inflation can be written as an EFT for a Goldstone Boson π

$$S_\pi = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm Pl}^2 R - M_{\rm Pl}^2\dot{H}\left(\dot{\pi}^2 - \frac{(\partial_i\pi)^2}{a^2}\right) + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\dot{\pi}^3 + ...\right]$$

Cheung et al. 2008

- Where $\pi \sim \delta t \sim \dfrac{\delta\phi}{\dot{\phi}}$ characterizes fluctuations in the clock measuring time during inflation

Inflation as an EFT



Senatore & Zaldarriaga (2009).

Inflation as an EFT

- So our question now should be. When this approach turns invalid.
- We have two possible naive answers
- When the theory becomes strong coupled?

  Avgoustadis et al(2012), Cremononi et al(2010), Green & Baumann (2011)
- When other degrees of fredom becomes relevant? Achucarro et al(2012), Xi & Chiu et al(2011)

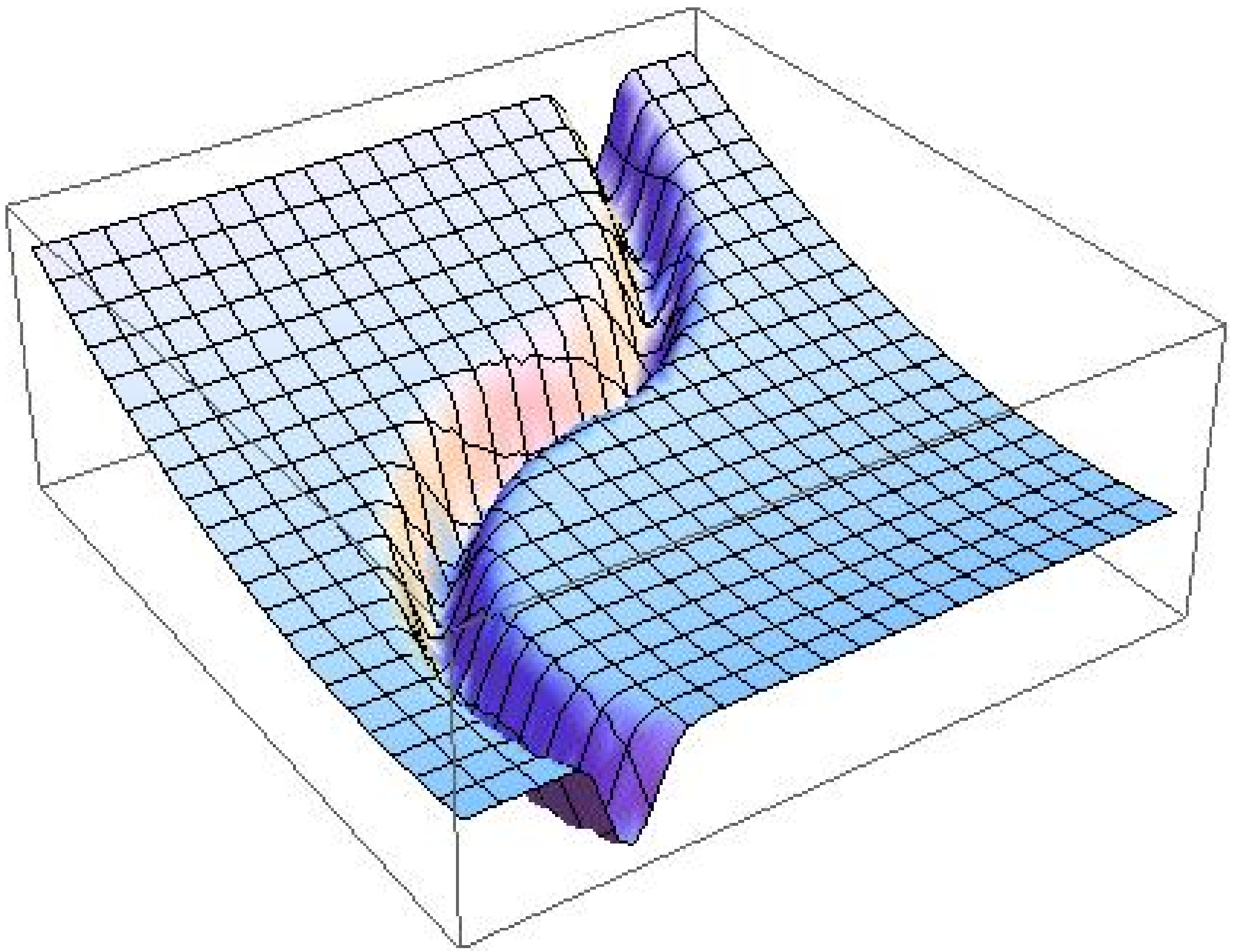
Inflation as an EFT

- According to the standard lore, other degrees of fredoms becomes relevant if they are comparable to the scale of inflation H
- If other degrees of fredoms are very massive they can be integrated out

Motivation

Inflations needs a light scalar field, but the others degrees of fredoms can be very massive $M \gg H$

Common lore: If heavy degrees of freedom are sufficiently massive, then we can ignore them



Motivation

Inflations needs a light scalar field, but the others degrees of fredoms can be very massive $M \gg H$

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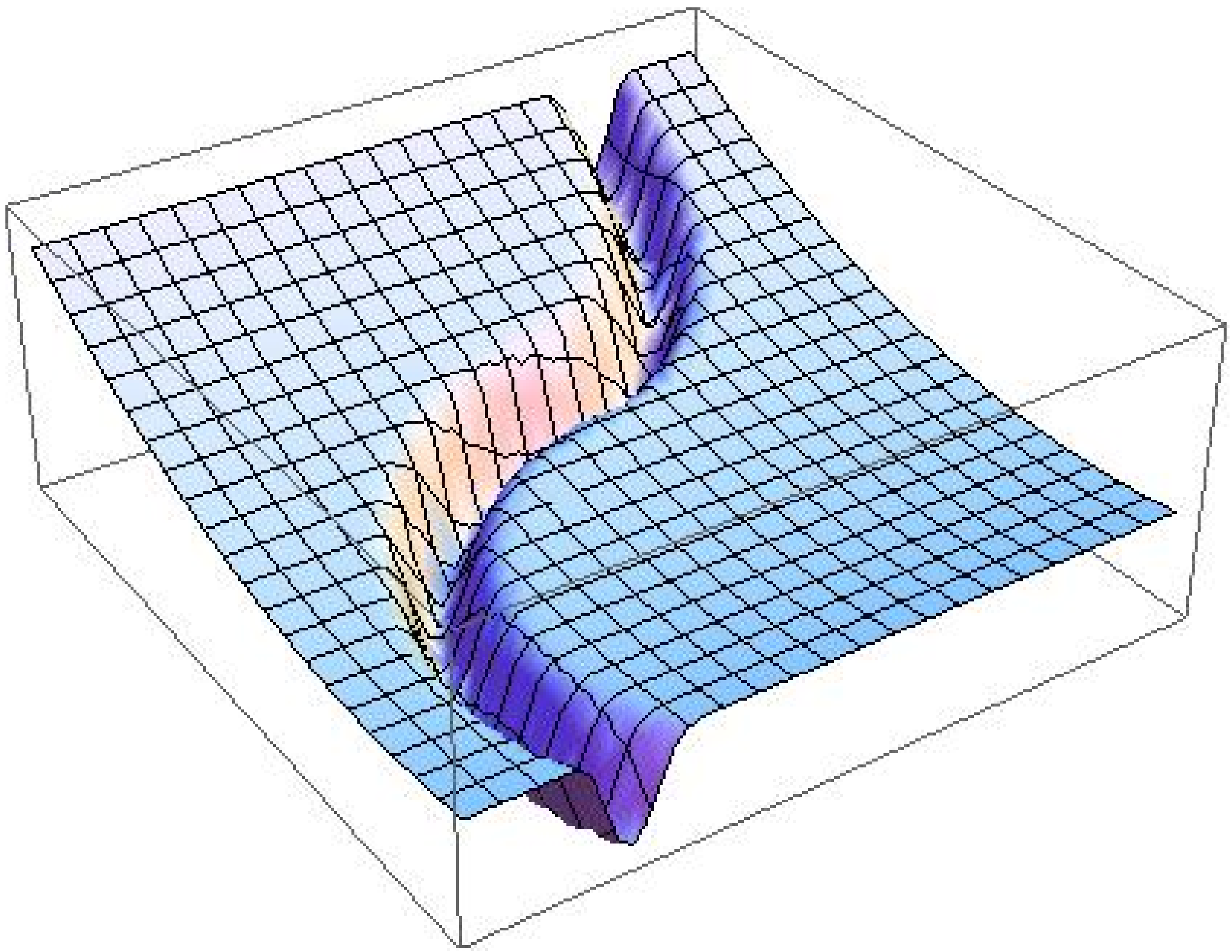
But it has been recently showed that heavy degrees of fredoms can imprints signatures on the primordial power spectrum (*1010.3693*)



Inflation

- Let us recall some basic facts about standard single field inflation
  - Inflation is driven by a single scalar field (called inflaton).
  - We need that the slow roll conditions keep satisfied

$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1$$

$$\eta = -\frac{\ddot{\phi}}{H\phi} \ll 1$$

  - The scale of Inflation is H and the quantum perturbations produced during inflation keep frozen when they reach the horizon
  - Inflation predicts an almost invariant of scale power spectrum

$$\Delta_{\mathrm{s}}^2(k) \approx \left. \frac{1}{24\pi^2} \frac{V}{M_{\mathrm{pl}}^4} \frac{1}{\epsilon_{\mathrm{v}}} \right|_{k=aH} \qquad n_{\mathrm{s}} - 1 = 2\eta_{\mathrm{v}}^{\star} - 6\epsilon_{\mathrm{v}}^{\star}$$

Two field inflation

Let us start by define the action for multifield inflation

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}g^{\mu\nu}\gamma_{ab}\partial_\mu\phi^a\partial_\nu\phi^b - V(\phi)\right]$$

We consider isotropic and homogeneus solutions given by

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

And get the following equation of motion

$$\frac{D}{dt}\dot{\phi}_0^a + 3H\dot{\phi}_0^a + V^a = 0$$

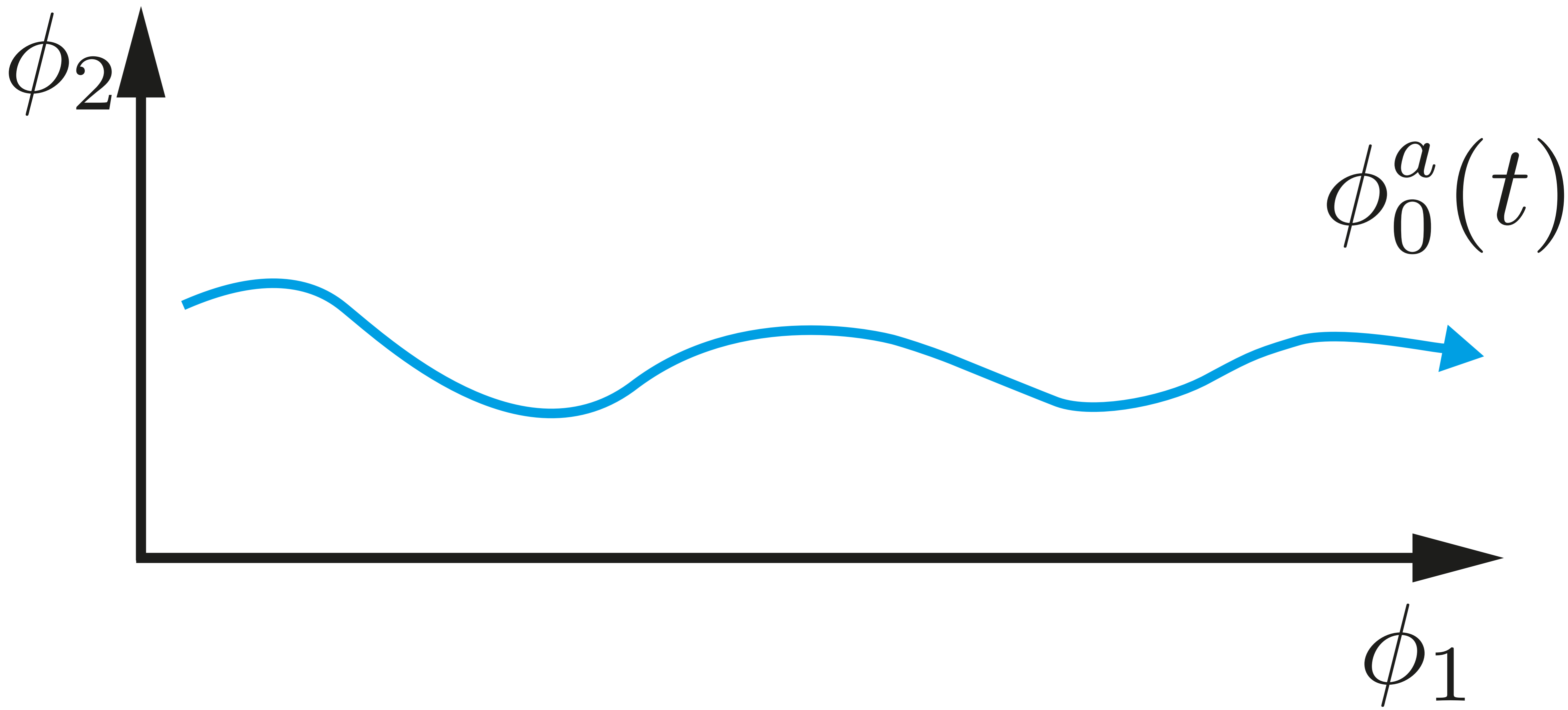
$$3H^2 = \frac{1}{2}\dot{\phi}_0^2 + V$$

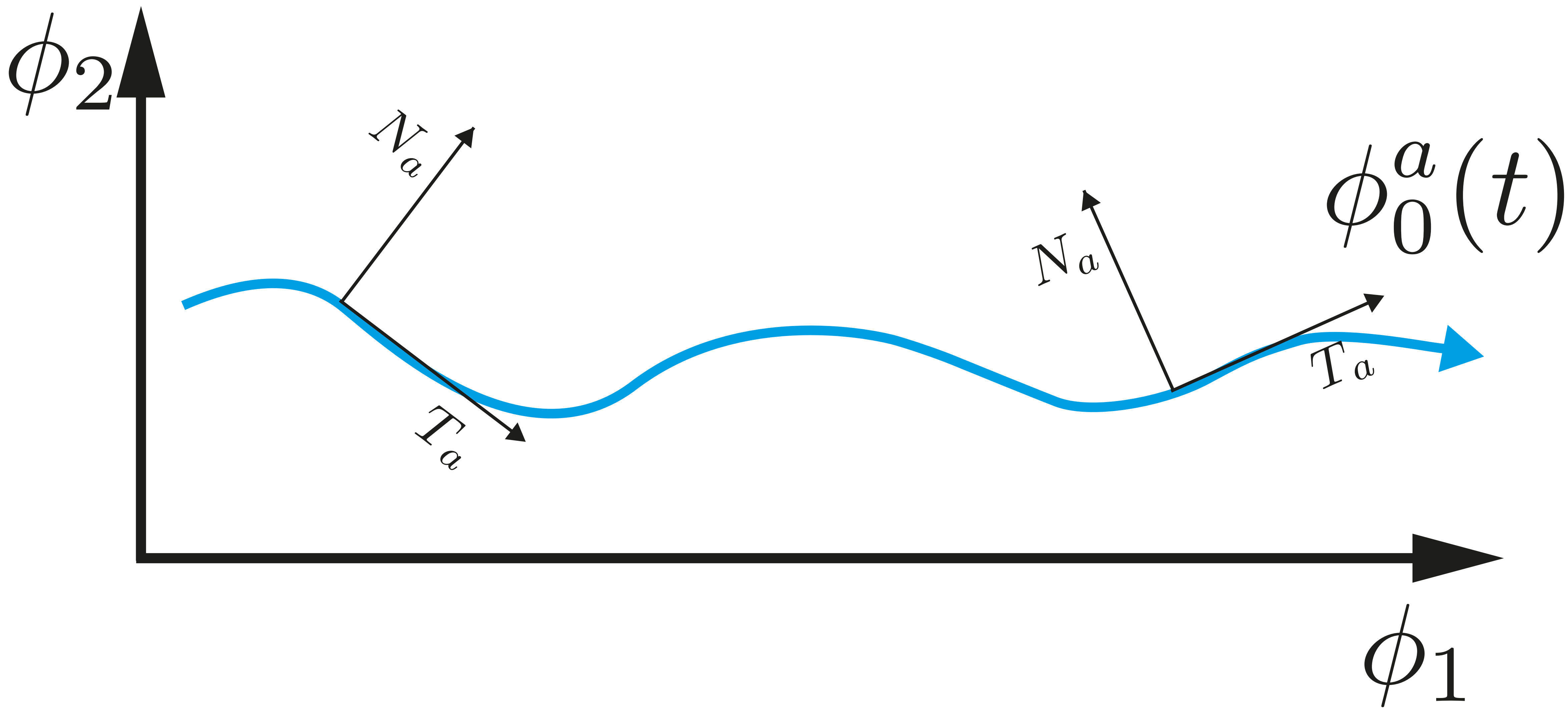
where

$$\frac{D}{dt}X^a = \dot{X}^a + \Gamma^a_{bc}\dot{\phi}_0^b X^c$$

$$H \equiv \frac{\dot{a}}{a} \qquad V^a = \gamma^{ab}\partial_b V$$

Kinematical Frame





We define a set of orthogonal unitary vectors

$$T^a \equiv \dot{\phi}/\dot{\phi}_0$$

$$N_a \equiv \epsilon_{ab} T^b$$

The equation of motion

Where $V_\phi = T_a V^a$

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_\phi = 0$$

We define the so-called slow roll parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

$$\eta^a \equiv \frac{1}{H\dot{\phi}_0}\frac{D\dot{\phi}_0^a}{dt}$$

We can decompose η^a as

$$\longrightarrow$$

$$\eta_\parallel = -\frac{\ddot{\phi}_0}{H\dot{\phi}_0}$$

$$\eta_\perp = -\frac{V_N}{\dot{\phi}_0 H}$$

The equation of motion

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We can decompose η^a as

$$\eta_{\parallel} = -\frac{\ddot{\phi}_0}{H\dot{\phi}_0}$$

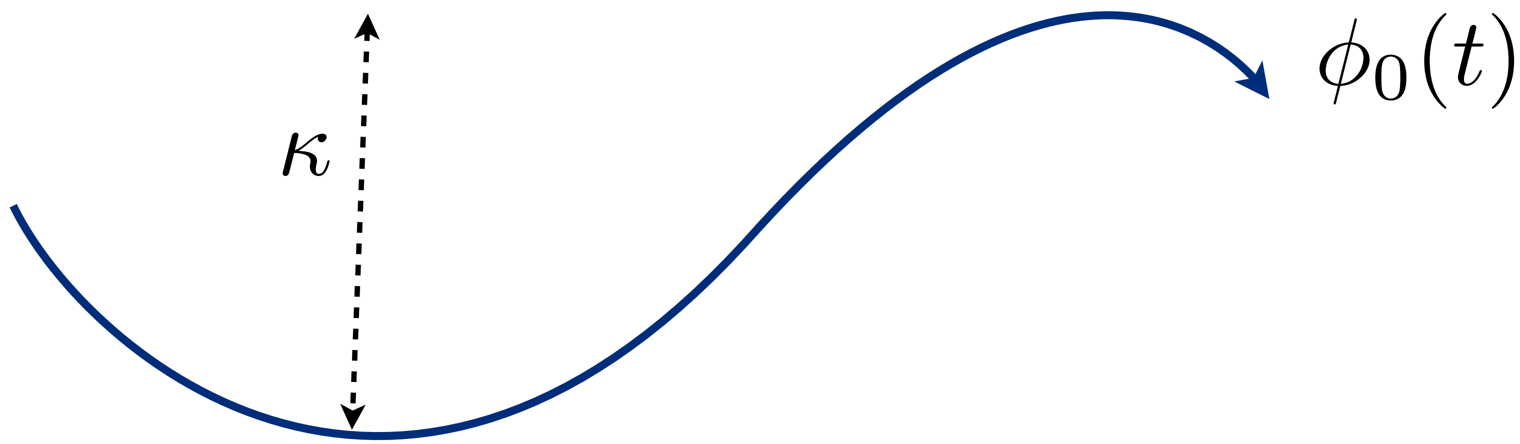
$$\eta_{\perp} = -\frac{V_N}{\dot{\phi}_0 H}$$

$\eta_{\perp}$ shows us how fast the heavy directions is changing with respect to the light direction

It will be useful to define

$$\dot{\theta} \equiv -H\eta_{\perp}$$

$$\kappa^{-1} \equiv |\dot{\theta}|/\dot{\phi}_0$$



Finally let us rememember that in standard single field inflations

$$\epsilon \ll 1$$

$$|\eta_{\parallel}| \ll 1$$

Perturbation Theory

Now, we consider the scalar perturbations from the homogeneus and isotropic background.

$$\phi^a(t,\mathbf{x}) = \phi_0^a(t) + \delta\phi^a(t,\mathbf{x})$$

Instead of working with $\delta\phi^a(t,\mathbf{x})$, it is more convenient to consider the following gauge inviante fields defined as

$$v^T = aT_a\delta\phi^a + a\frac{\dot{\phi}}{H}\psi$$

$$v^N = aN_a\delta\phi^a$$

It is more useful to change to curvature and isocurvature fields

$$\mathcal{R} = \frac{H}{a\dot{\phi}} v^T$$

$$\mathcal{S} = \frac{H}{a\dot{\phi}} v^N$$

For simplicity we will define:

$$\mathcal{F} = \frac{\dot{\phi}}{H}\mathcal{S}$$

Then we get:

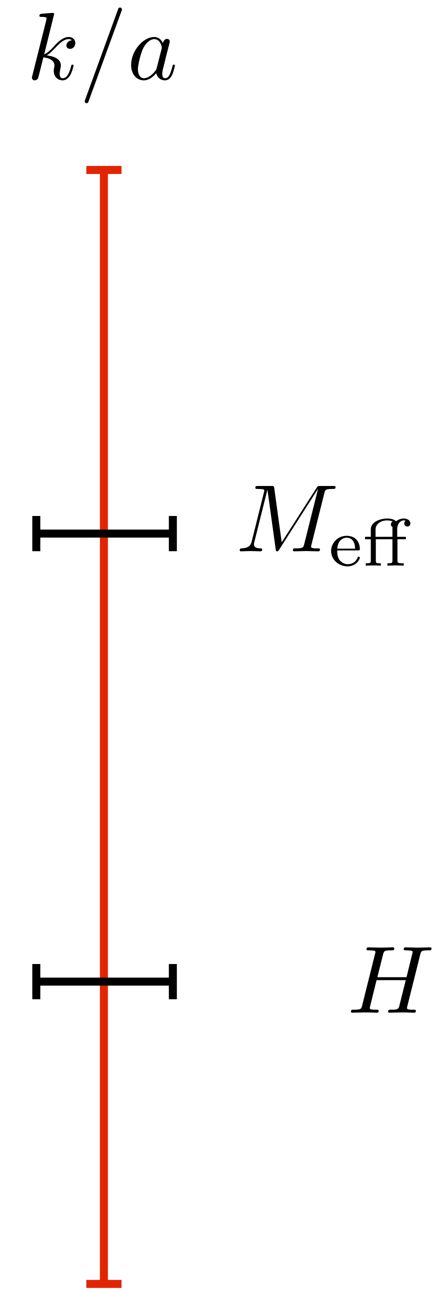
$$S_{\rm tot} = \frac{1}{2}\int d^4x a^3 \left[\frac{\dot{\phi}_0^2}{H^2}\dot{\mathcal{R}}^2 - \frac{\dot{\phi}_0^2}{H^2}\frac{(\nabla\mathcal{R})^2}{a^2} + \dot{\mathcal{F}}^2 - \frac{(\nabla\mathcal{F})^2}{a^2} + 4\dot{\phi}_0\eta_\perp\dot{\mathcal{R}}\mathcal{F} - 4M_{\rm eff}^2\mathcal{F}^2\right]$$

where $M_{\rm eff}^2 = V_{NN} + H^2\epsilon\mathbb{R} - \dot{\theta}^2$

Let us analyze this equation

$$\ddot{\mathcal{R}} + (3 + 2\epsilon - 2\eta_{\parallel})H\dot{\mathcal{R}} - \frac{\nabla^2\mathcal{R}}{a^2} = -2\frac{H^2}{\dot{\phi}_0}\eta_{\perp}\left[\dot{\mathcal{F}} + (3 - \eta_{\parallel} - \chi_{\perp})H\mathcal{F}\right]$$

$$\ddot{F} + 3H\dot{\mathcal{F}} - \frac{\nabla^2\mathcal{F}}{a^2} + M_{\rm eff}^2\mathcal{F} = 2\dot{\phi}_0\eta_{\perp}\dot{\mathcal{R}}$$



For constant $\eta_{\perp}$, we have the following solutions.

$$\mathcal{R} \sim \mathcal{R}_+ e^{-i\omega_+ t} + \mathcal{R}_- e^{-i\omega_- t},$$

$$\mathcal{F} \sim \mathcal{F}_+ e^{-i\omega_+ t} + \mathcal{F}_- e^{-i\omega_- t},$$

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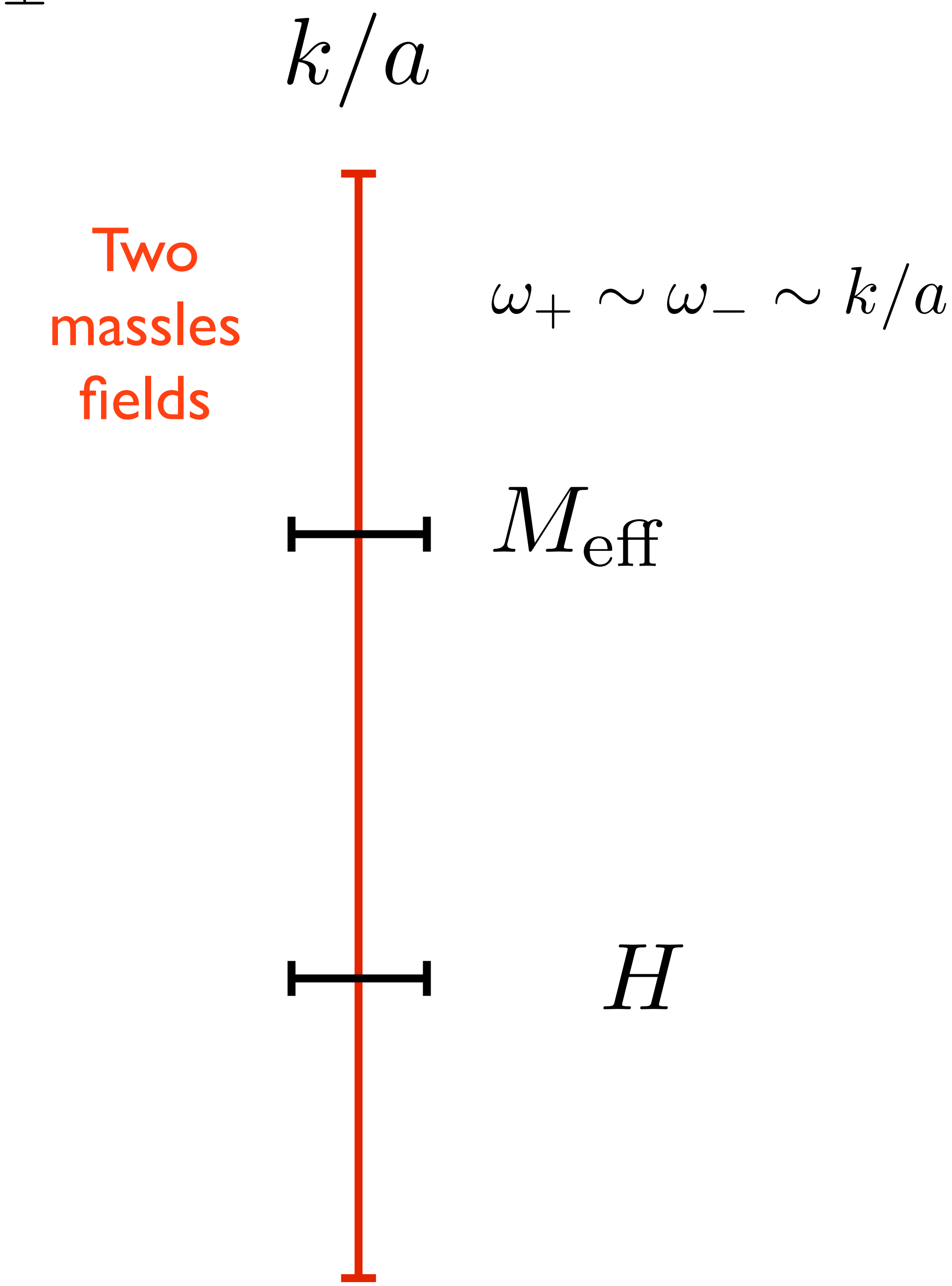

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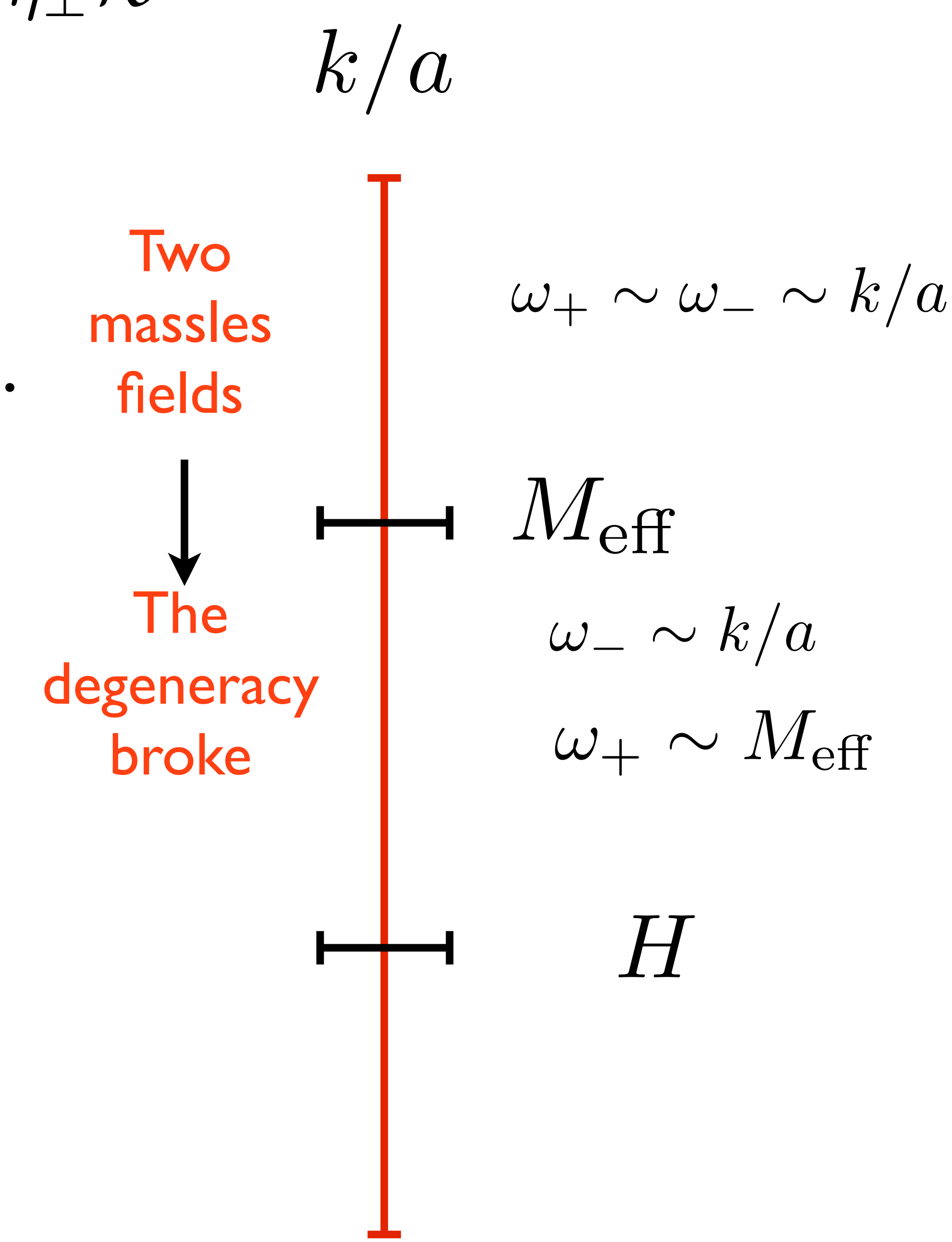

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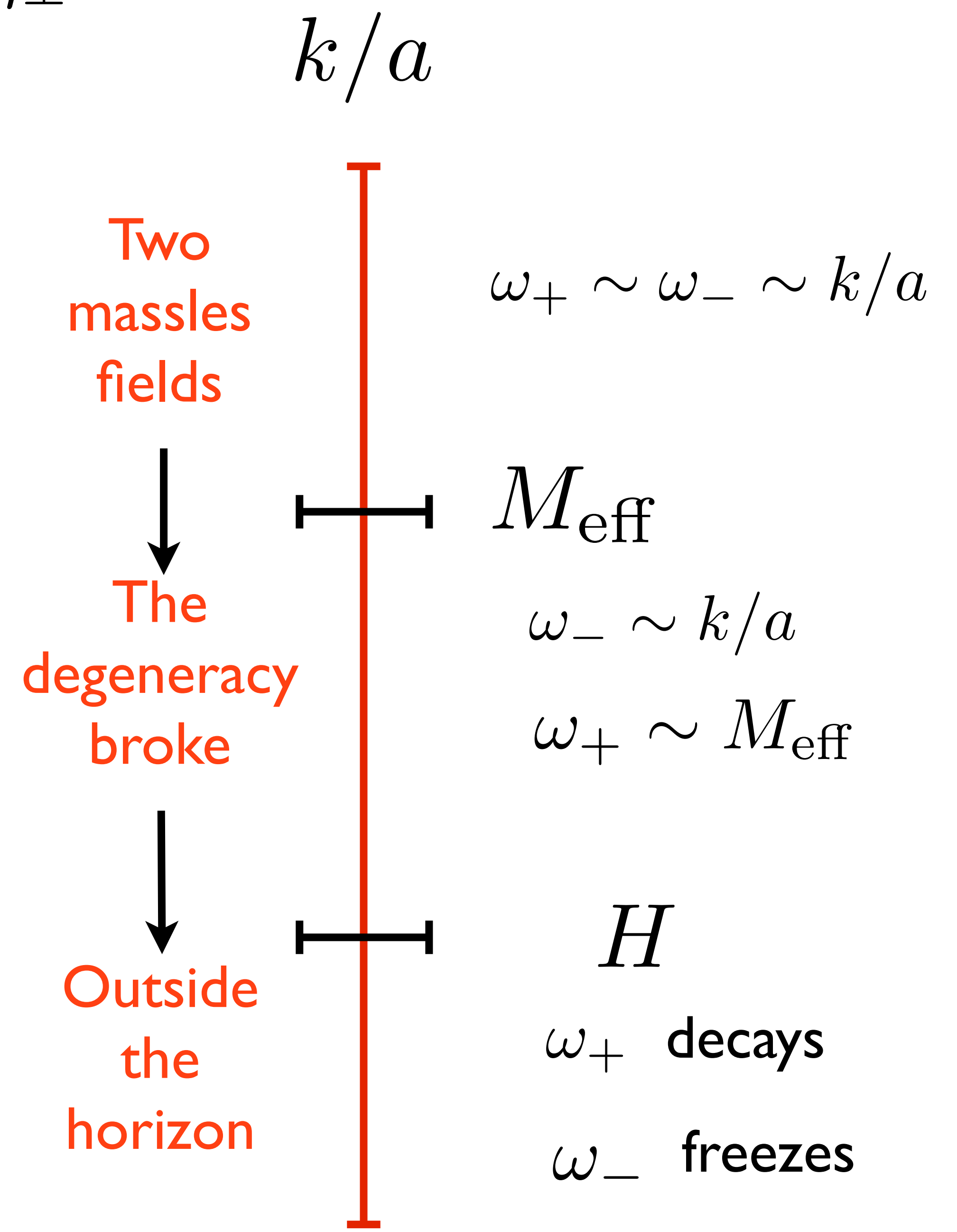


Effective field theory

$$\ddot{\mathcal{R}} + (3 + 2\epsilon - 2\eta_{\parallel})H\dot{\mathcal{R}} - \frac{\nabla^2\mathcal{R}}{a^2} = -2\frac{H^2}{\dot{\phi}_0}\eta_{\perp}\left[\dot{\mathcal{F}} + (3 - \eta_{\parallel} - \chi_{\perp})H\mathcal{F}\right]$$

$$\ddot{F} + 3H\dot{\mathcal{F}} - \frac{\nabla^2\mathcal{F}}{a^2} + M_{\text{eff}}^2\mathcal{F} = 2\dot{\phi}_0\eta_{\perp}\dot{\mathcal{R}}$$

We can integrate out

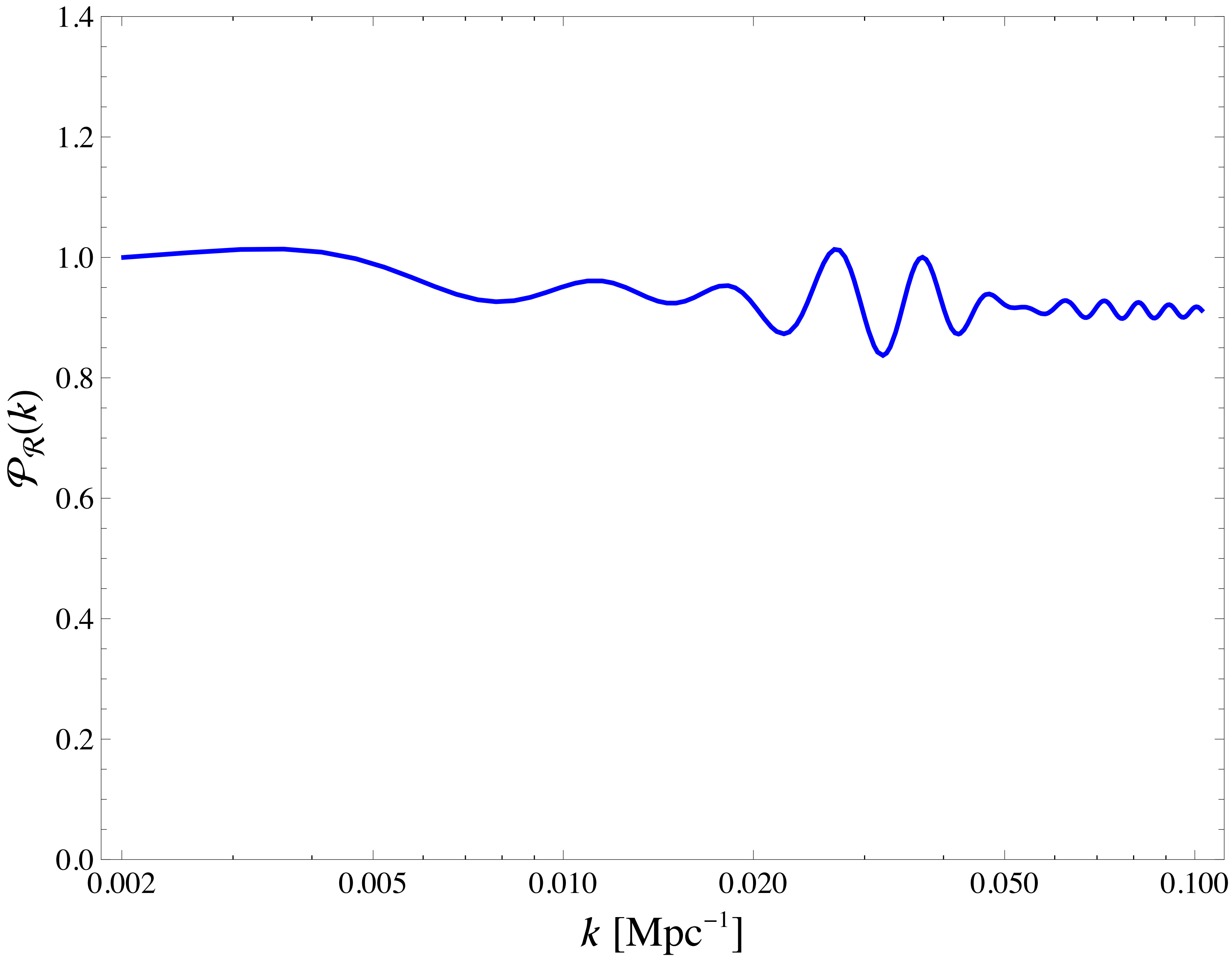
$$\mathcal{F}_{\mathcal{R}} = \frac{2\dot{\phi}_0\eta_{\perp}\dot{\mathcal{R}}}{(k^2/a^2 + M_{\text{eff}}^2)}$$

Effective Field Theory

$$S_{\text{eff}} = \int d^4x \, \frac{a^3 \epsilon}{c_s^2} \left[\dot{\mathcal{R}}^2 - c_s^2 \left(\frac{\nabla \mathcal{R}}{a} \right)^2 \right]$$

Where $$c_s^{-2} = 1 + \frac{4H^2 \eta_\perp^2}{k^2/a^2 + M_{\text{eff}}^2}$$

Effective Field Theory



Fast turns can lead to imprints on the primordial power spectrum

Achucarro et al. (2010)

Power spectra

From the observational pont, the main quantities of interest are the *n*-point correlation functions

We will calculate the 2-point correlations functions given by:

$$\mathcal{P}_{\mathcal{R}}(k,\tau) = \frac{k^3}{2\pi^2}\sum_{\alpha}\mathcal{R}_{\alpha}(k,\tau)\mathcal{R}^*_{\alpha}(k,\tau)$$

Turning Trajectories

The validity of the EFT will depend on

$$|\ddot{\mathcal{F}}_{\mathcal{R}}| \ll M_{\rm eff}^2 |\mathcal{F}_{\mathcal{R}}|$$

That could be written

$$\left|\frac{d^2}{dt^2}\left(\frac{2\dot{\phi}_0\eta_\perp}{(k^2/a^2+M_{\rm eff}^2)}\right)\right| \ll M_{\rm eff}^2\left|\frac{2\dot{\phi}_0\eta_\perp}{(k^2/a^2+M_{\rm eff}^2)}\right| .$$

Then $$\left|\frac{d^2}{dt^2}\dot{\theta}\right| \ll M_{\rm eff}^2\left|\dot{\theta}\right|$$

Turning trajectories

So, the condition was that:

$$\left| \frac{d}{dt} \ln \dot{\theta} \right| \ll M_{\rm eff}$$

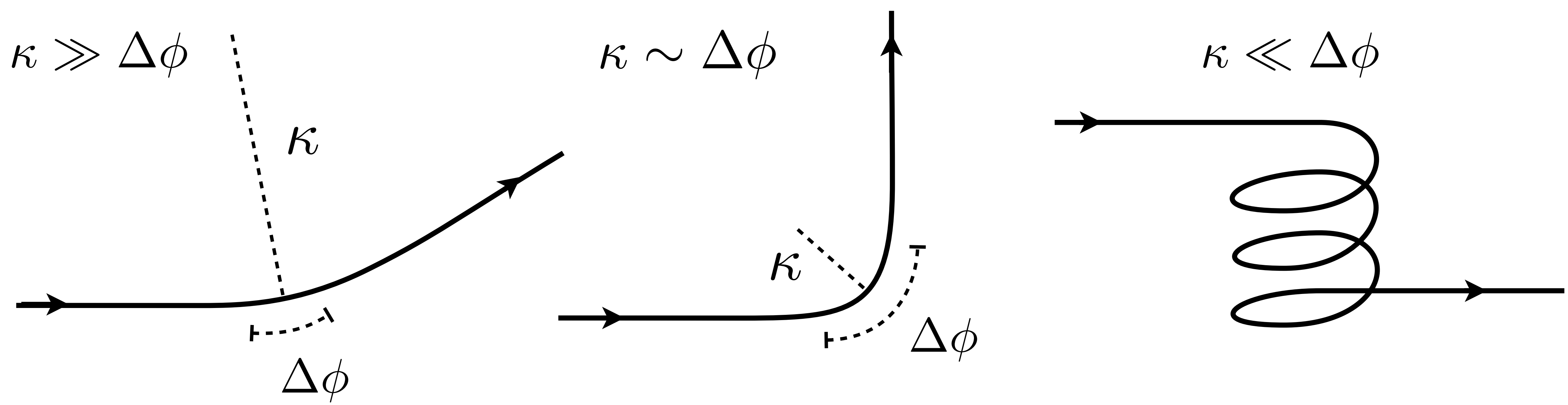
Now, we are going to see what happens when we have trajectories with turns

Let us define the parameters:

$$\Delta\phi \equiv \kappa |\Delta\theta|$$

Turning trajectories

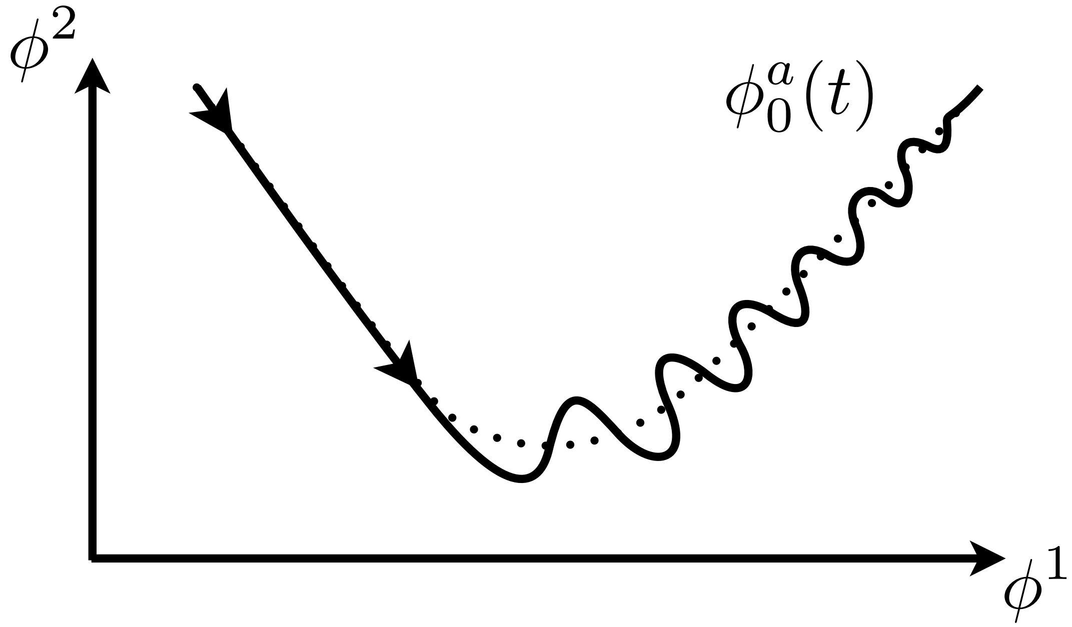
We can have the following cases:



Turning trajectories

We define the time scales asocciated with the turns as

$$T_\perp \equiv \frac{\Delta\phi}{\dot{\phi}}$$



$$T_M \equiv \frac{1}{M_{\rm eff}}$$

As $M_{\rm eff} \gg H$, the adiabaticity condition translate as:

$$T_\perp \gg T_M$$

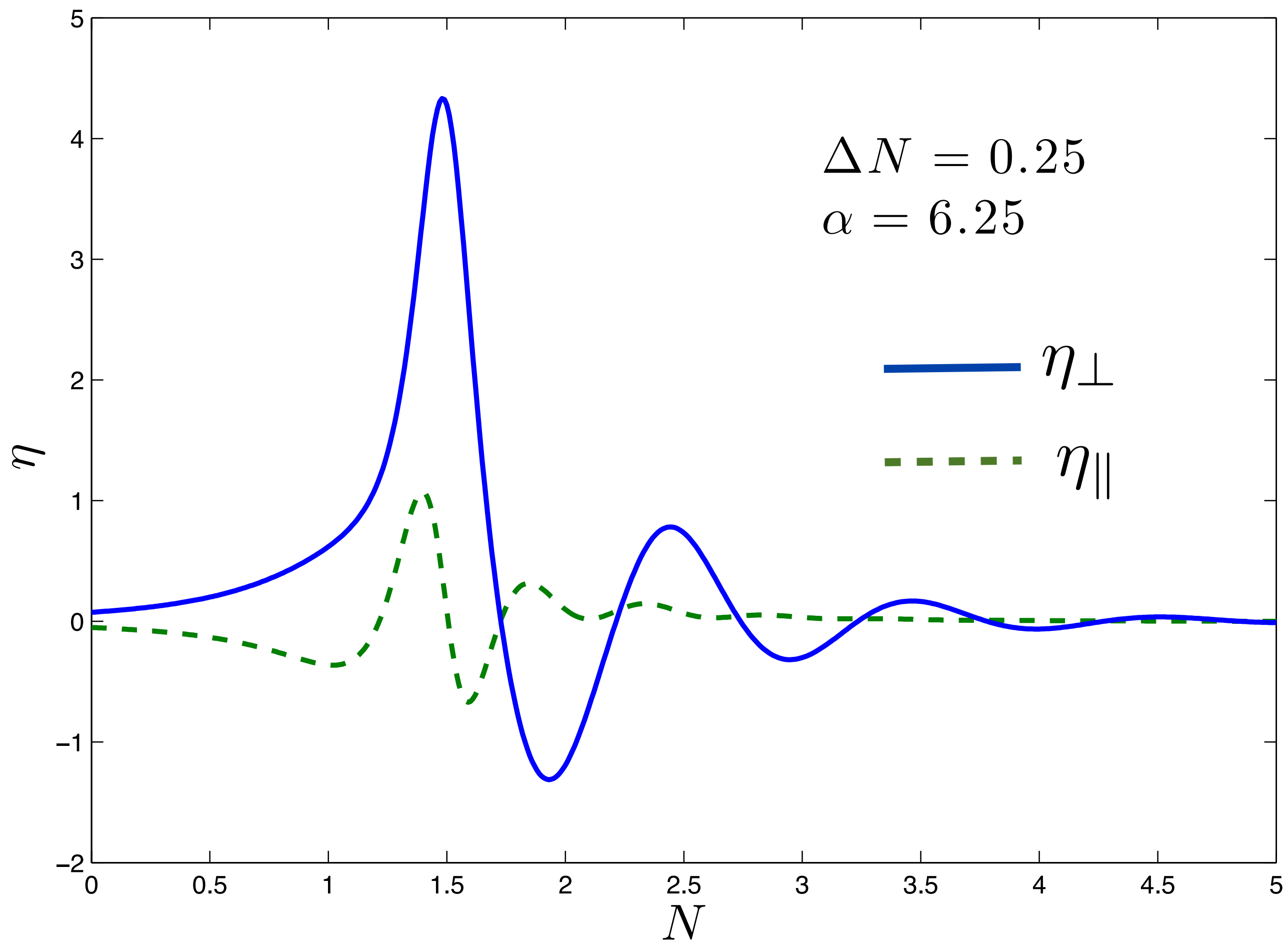
Turning trajectories

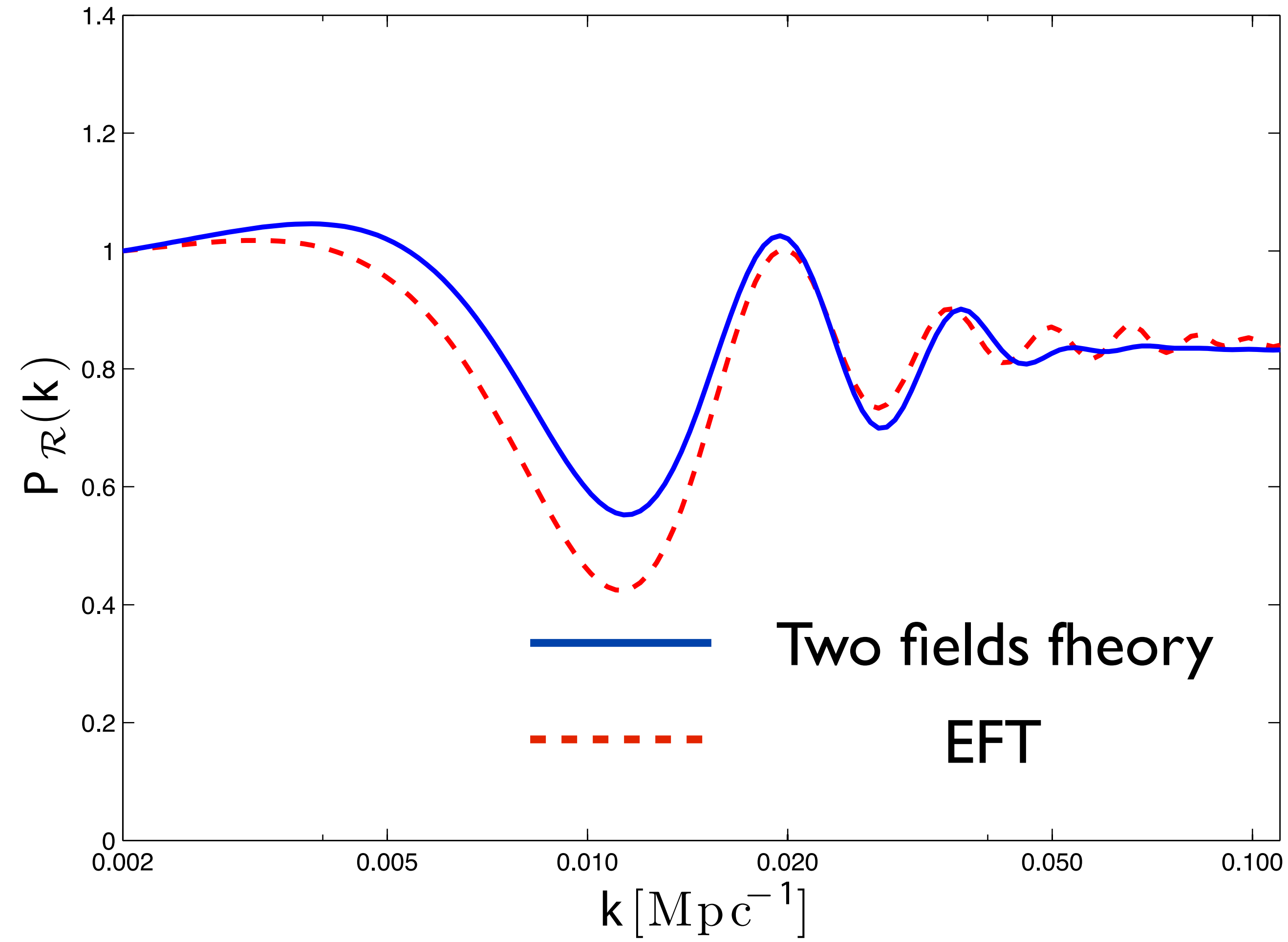
We build toy models to study numerically this turns

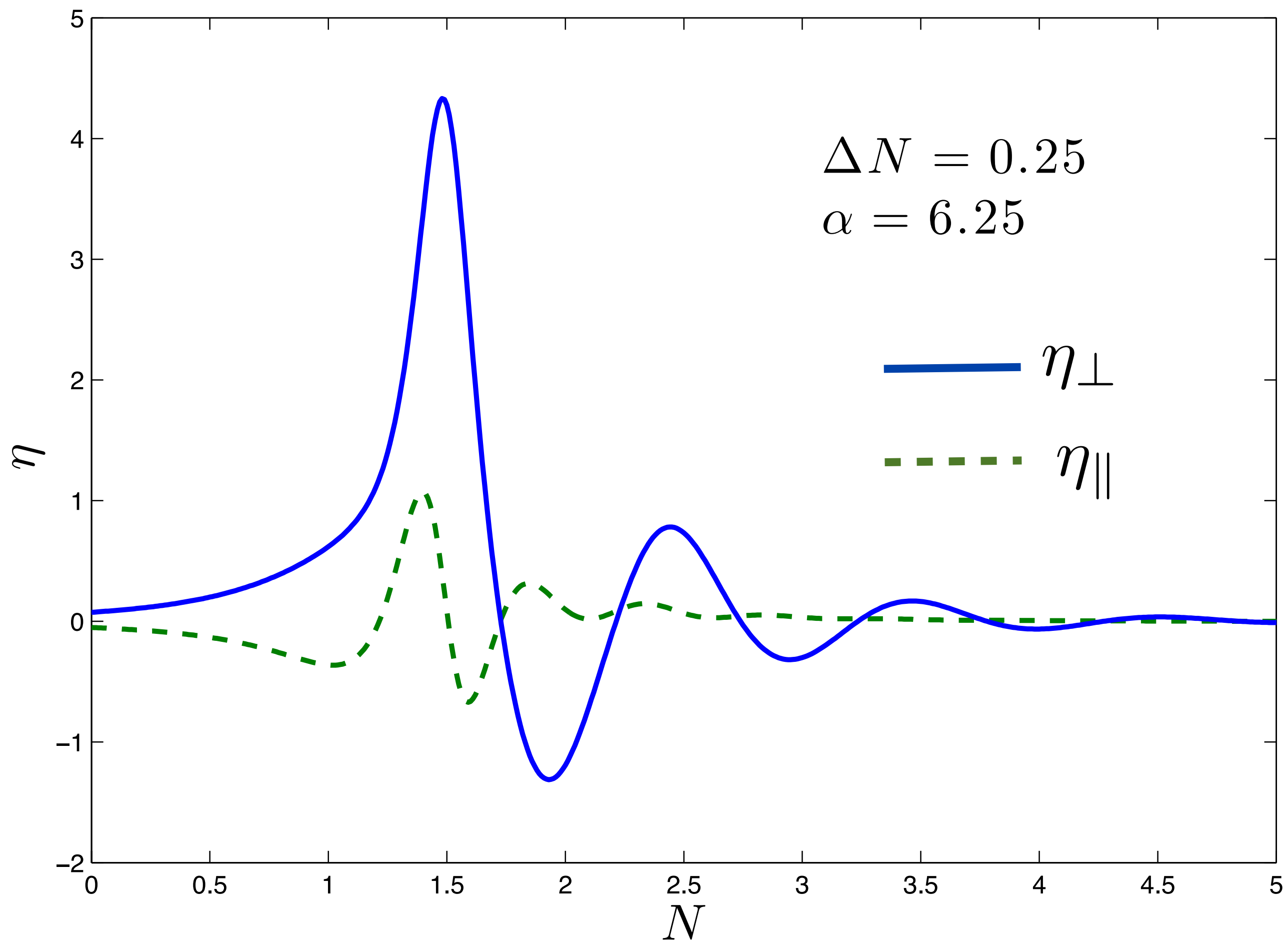
Model 1: Sudden turns in canonical models

$$V(\chi,\psi) = V_0 + V_\phi(\chi - \psi) + \frac{M^2}{2}\frac{(\chi\psi - a^2)^2}{(\chi+\psi)^2} + \cdots$$

Defining: $\alpha \equiv \frac{T_\perp^2}{T_M^2} = \frac{M^2 a^2}{2\epsilon H^2}$


$\Delta N = 0.25$
$\alpha = 6.25$
$\eta_\perp$
$\eta_\parallel$
η
N


Two fields fheory
EFT
$P_{\mathcal{R}}(k)$
$k\,[\mathrm{Mpc}^{-1}]$



$\Delta N = 0.25$
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$\eta_\perp$
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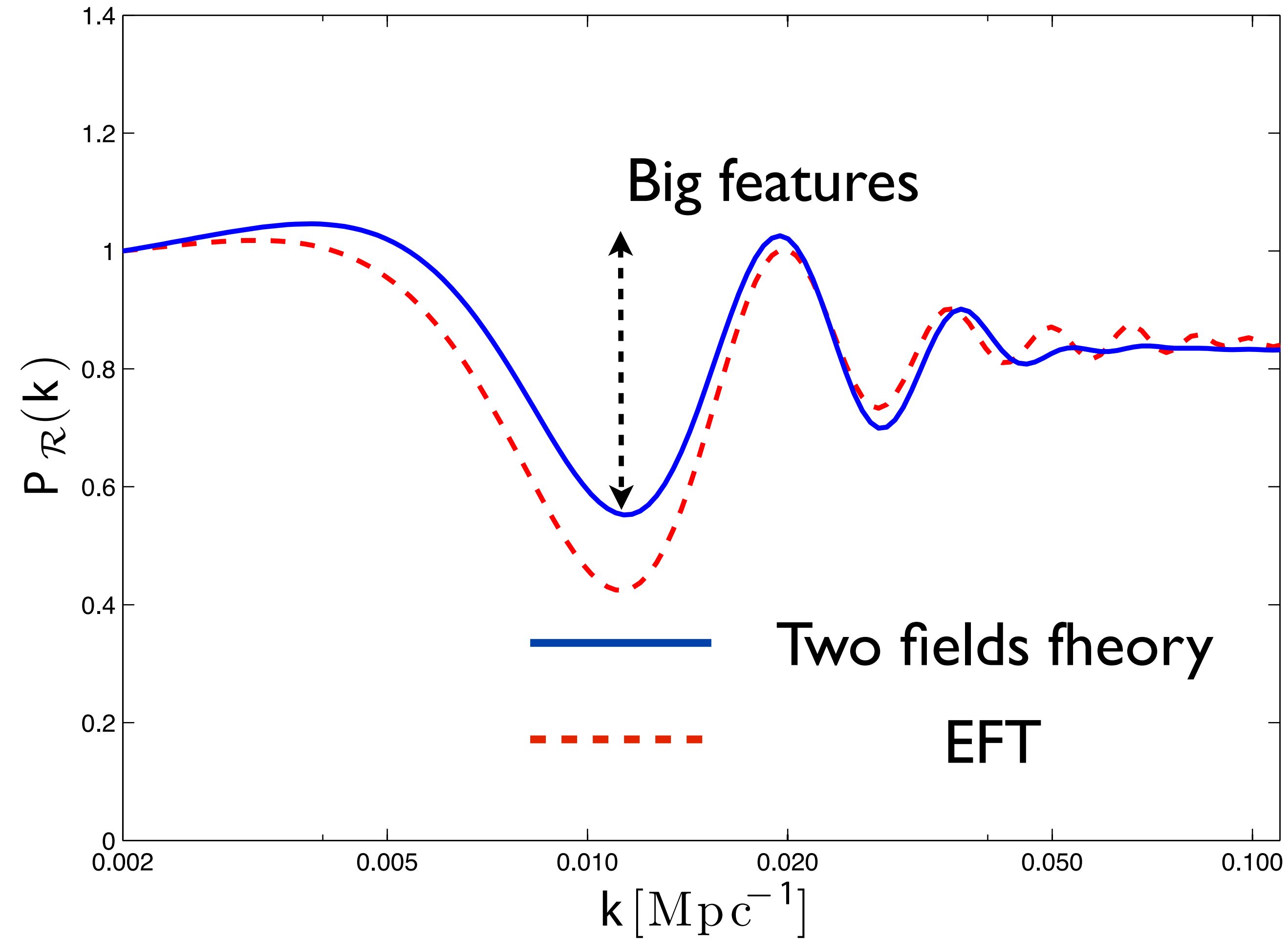
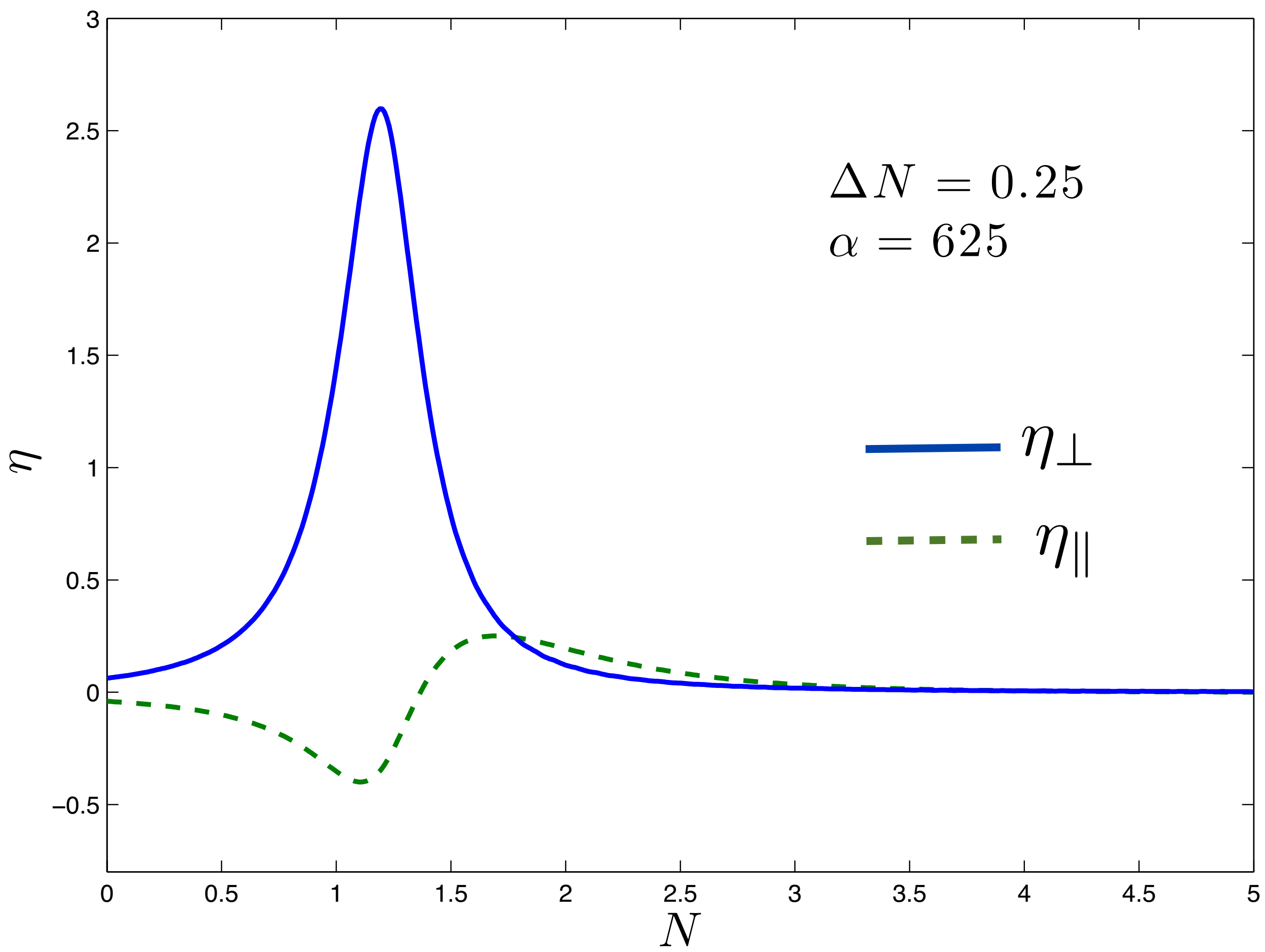
η
N



Big features
Two fields fheory
EFT
$P_{\mathcal{R}}(k)$
$k\,[\mathrm{Mpc}^{-1}]$

$$T_M \sim T_\perp$$



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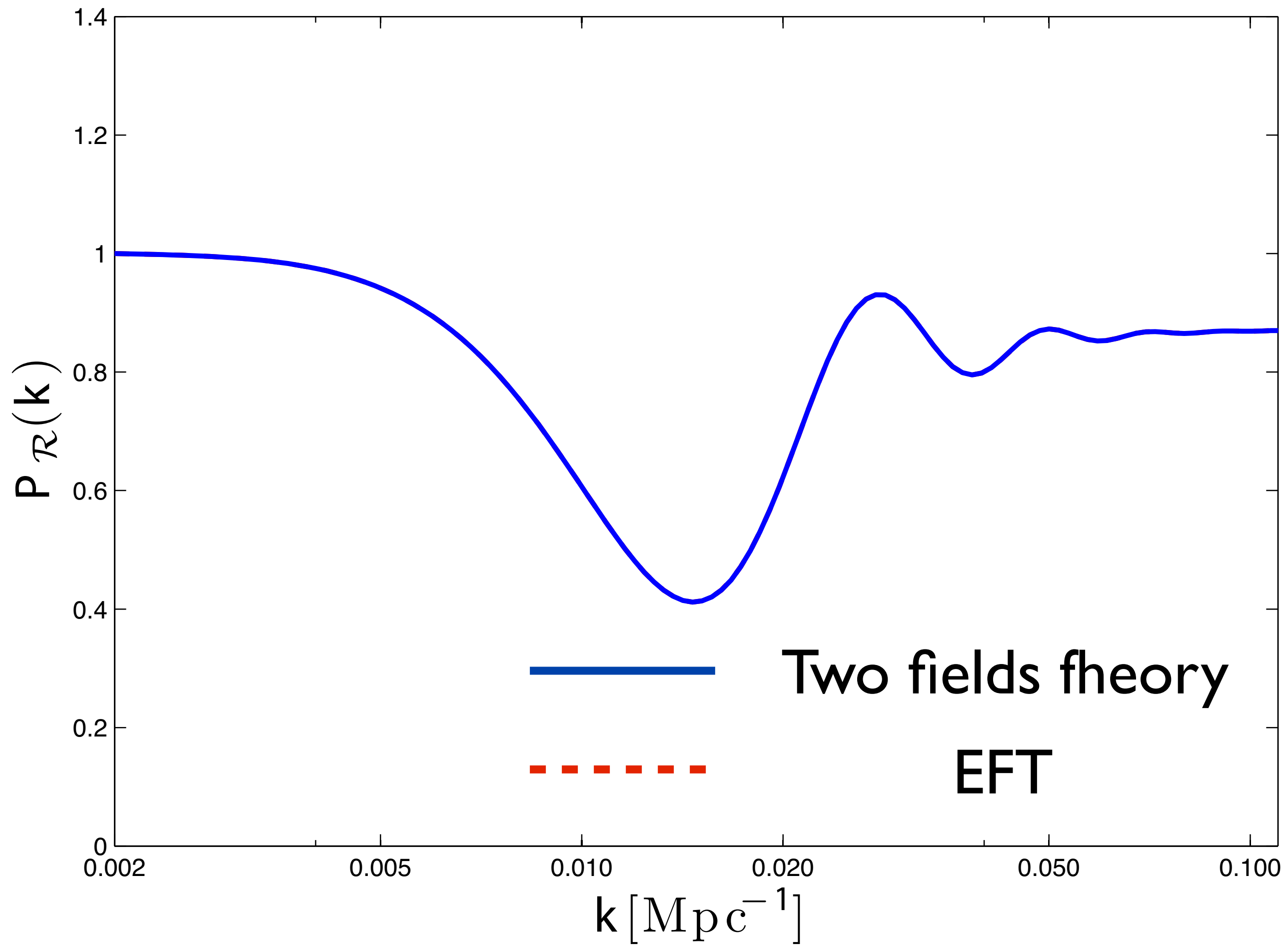
$\alpha = 625$
$\eta_\perp$
$\eta_\parallel$
η
N



Two fields fheory
EFT
$P_{\mathcal{R}}(k)$
k [Mpc^{-1}]

$$T_M \ll T_\perp$$

Model 2: Sudden turns induced by the metric

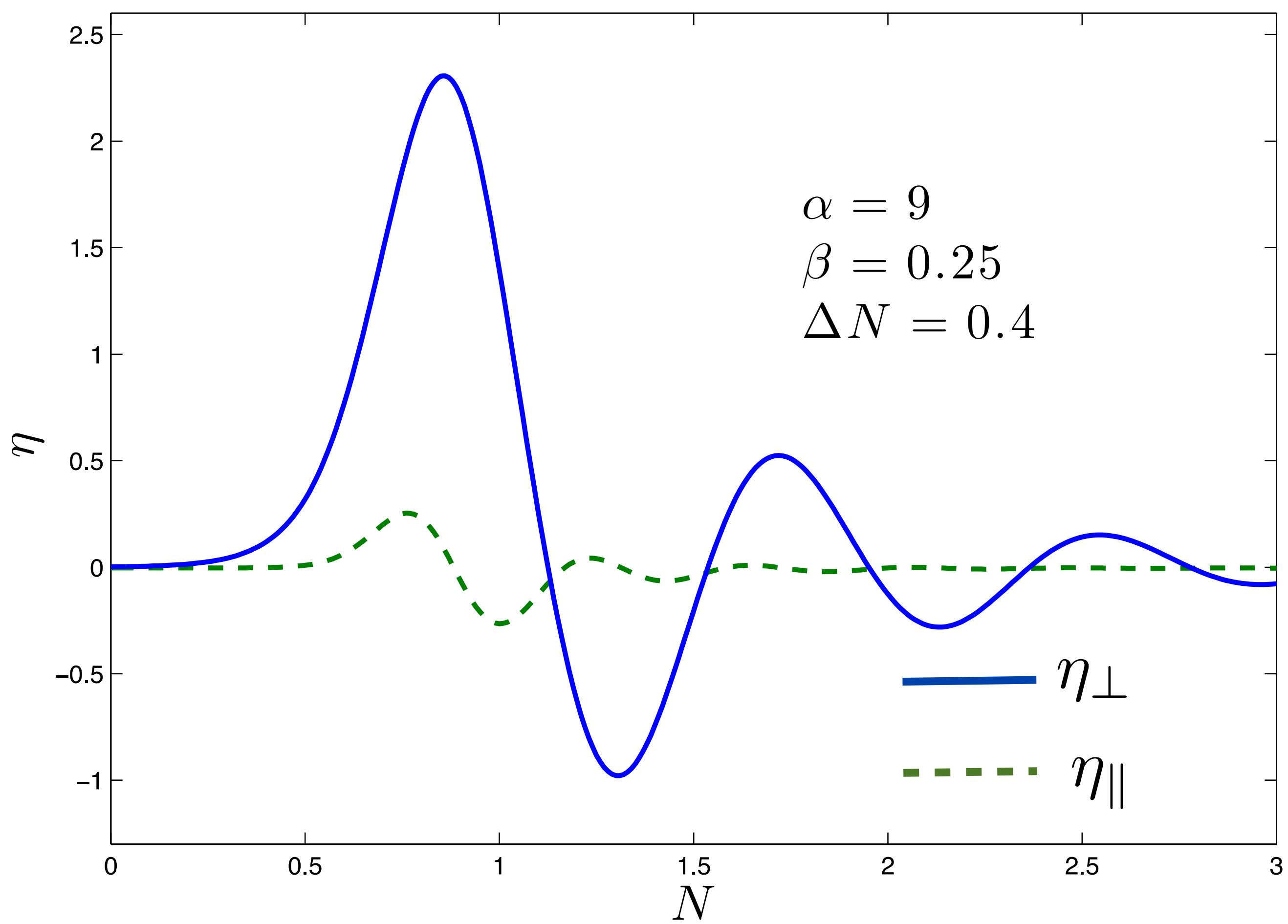
$$V(\chi,\psi) = V_0 + V_\phi\,\chi + \frac{M_\psi^2}{2}\psi^2$$

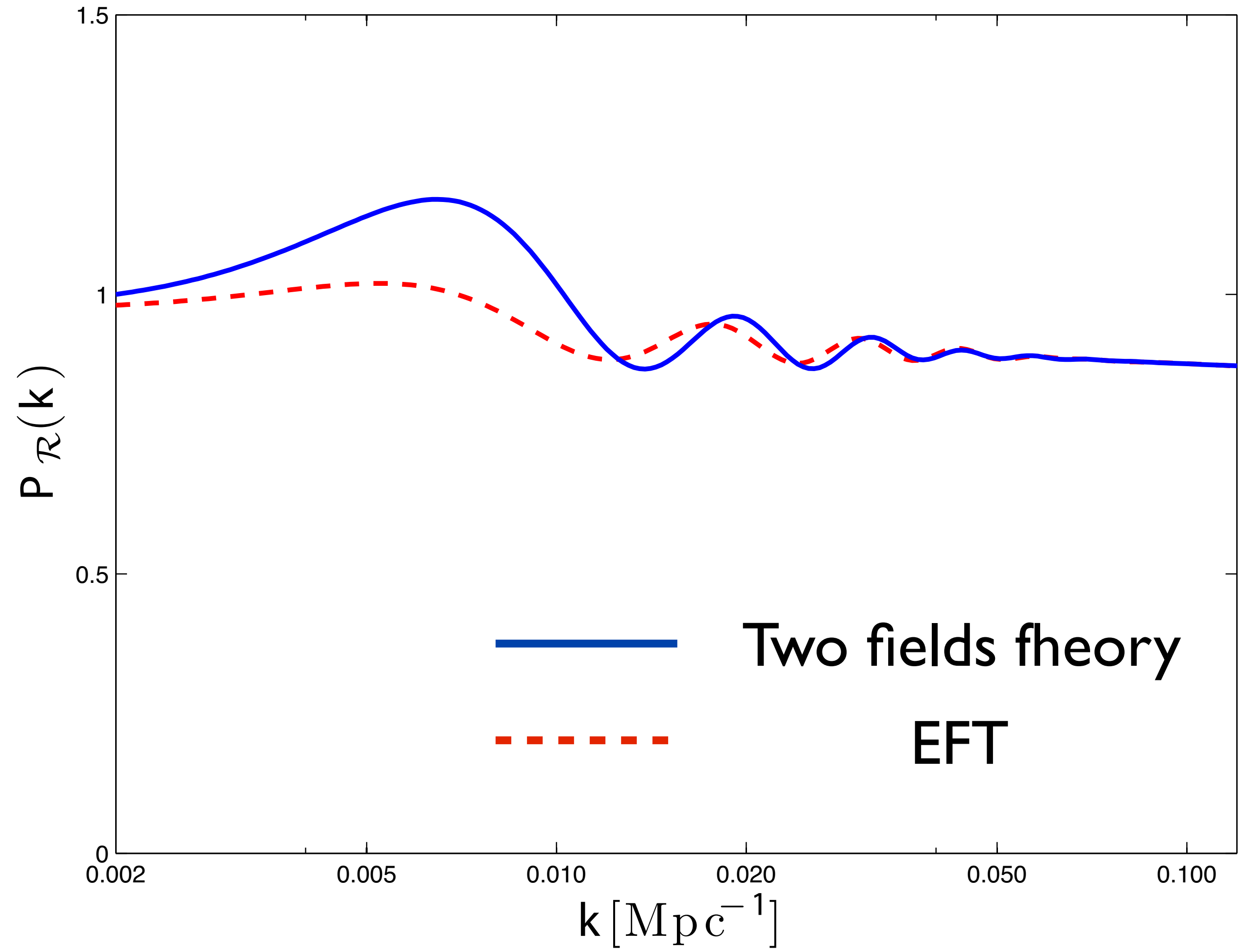
$$\gamma_{ab} = \begin{pmatrix} 1 & \Gamma(\chi) \\ \Gamma(\chi) & 1+\Gamma^2(\chi) \end{pmatrix},$$

$$\Gamma(\chi) = \frac{\Gamma_0}{2}\left(1+\tanh\left[2(\chi-\chi_0)/\Delta\chi\right]\right)$$

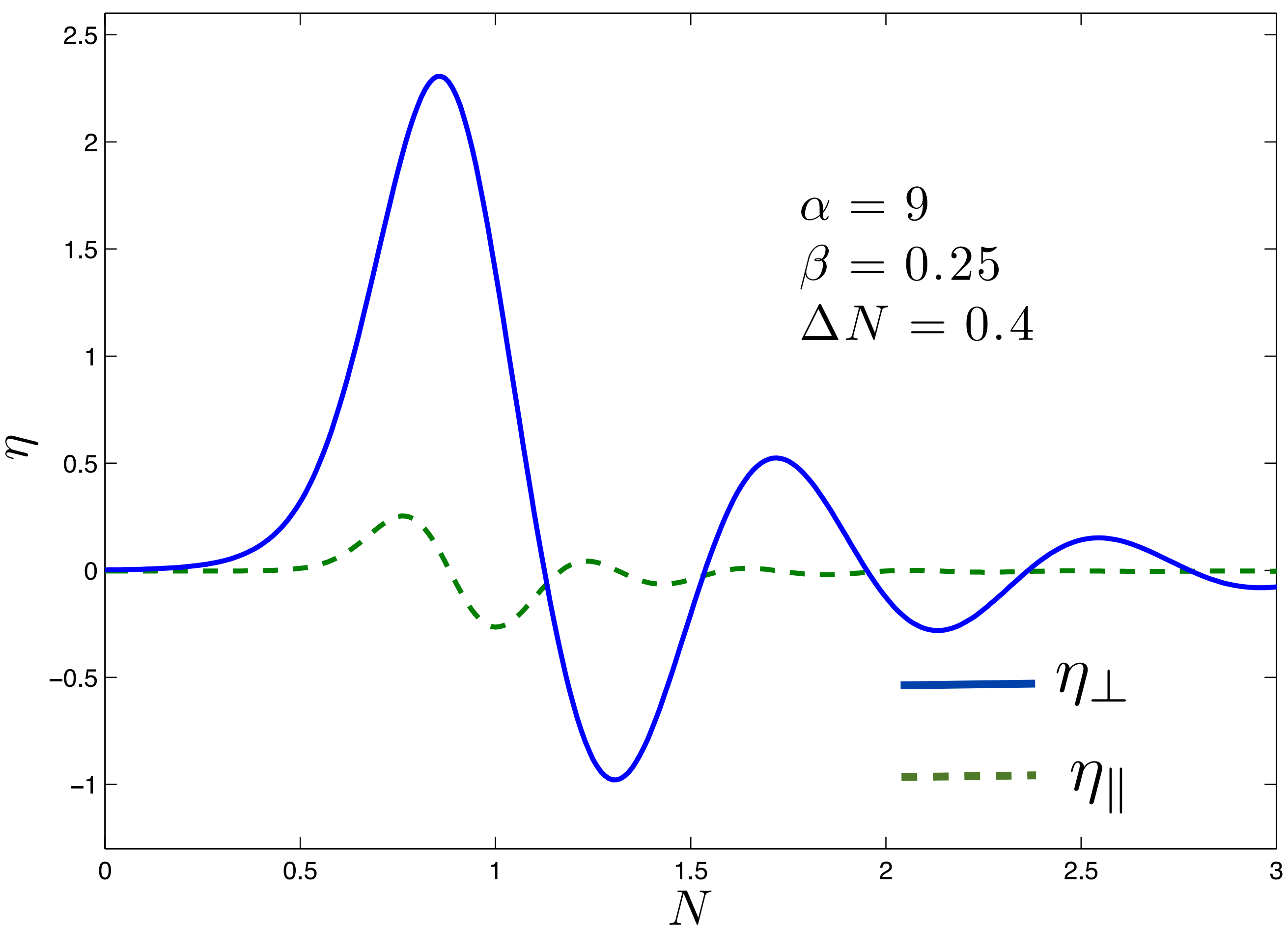
We define

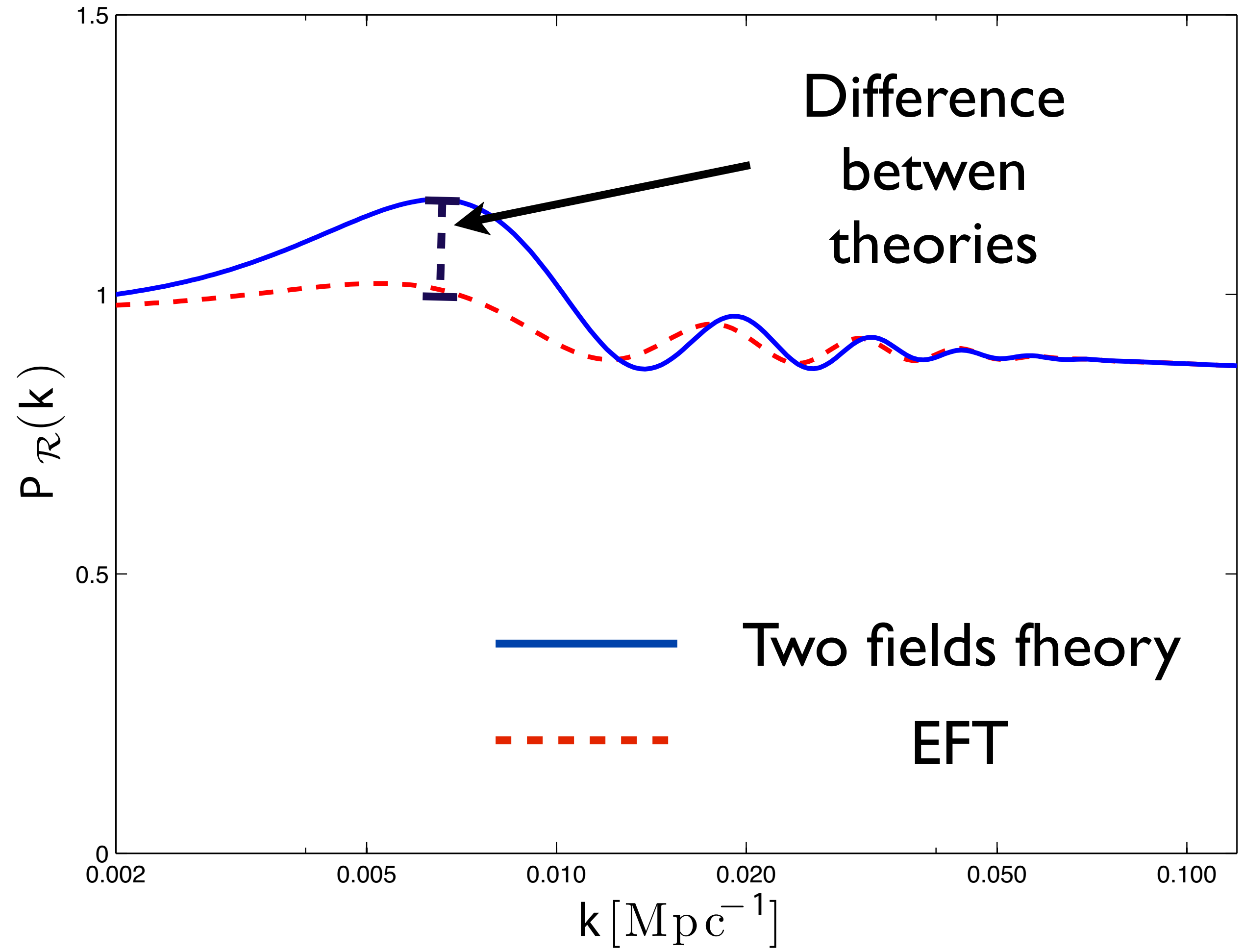
$$\alpha \equiv \frac{T_\perp^2}{T_M^2} = \frac{M_\psi^2 \Delta\chi^2}{2\epsilon H^2} \qquad \beta \equiv \frac{8\epsilon H^2 \Gamma_0^2}{\Delta\chi^2 M_\psi^2}$$


$\alpha = 9$
$\beta = 0.25$
$\Delta N = 0.4$
$\eta_\perp$
$\eta_\parallel$
η
N

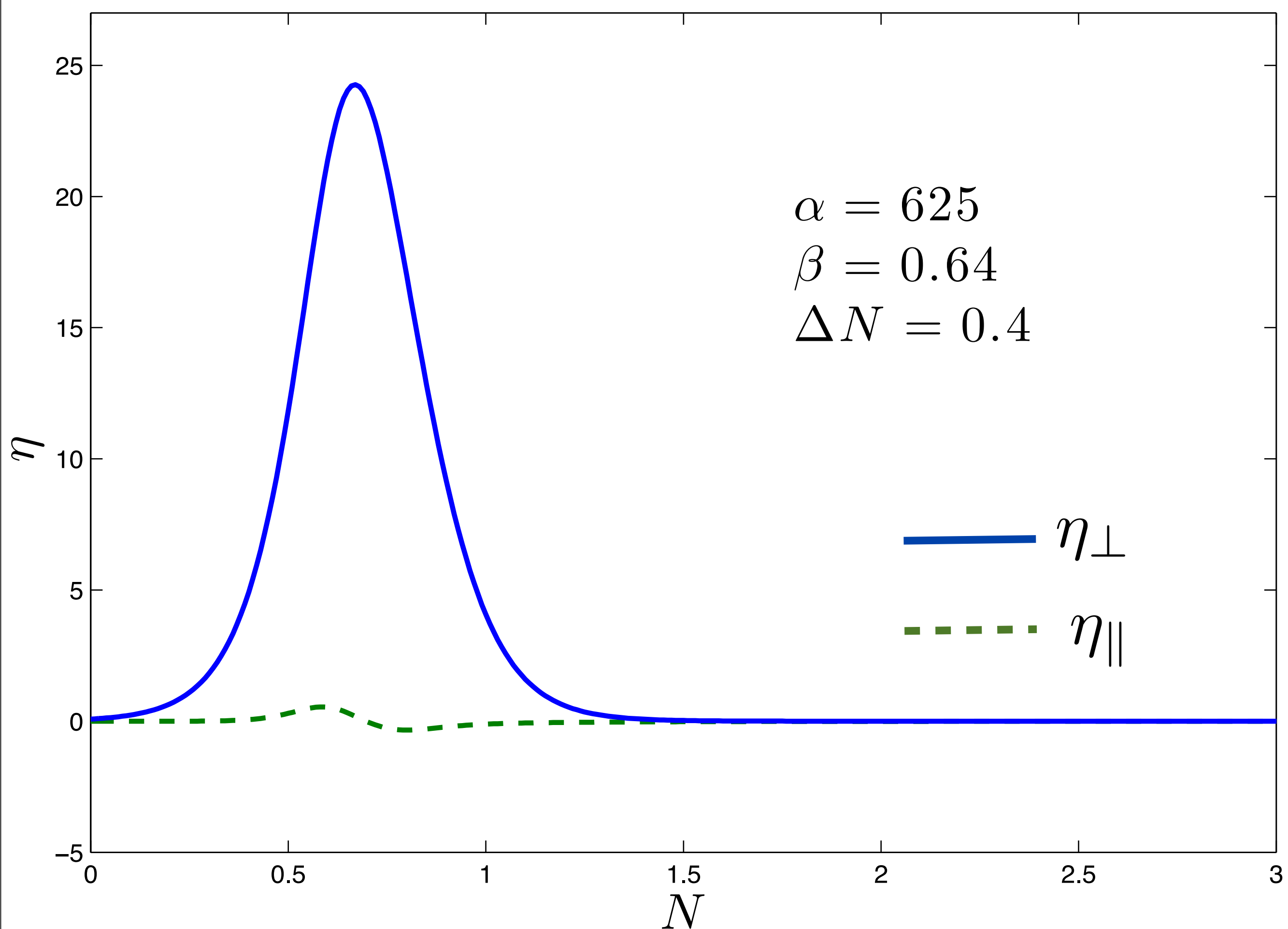

Two fields fheory
EFT
$P_{\mathcal{R}}(k)$
$k\,[\mathrm{Mpc}^{-1}]$

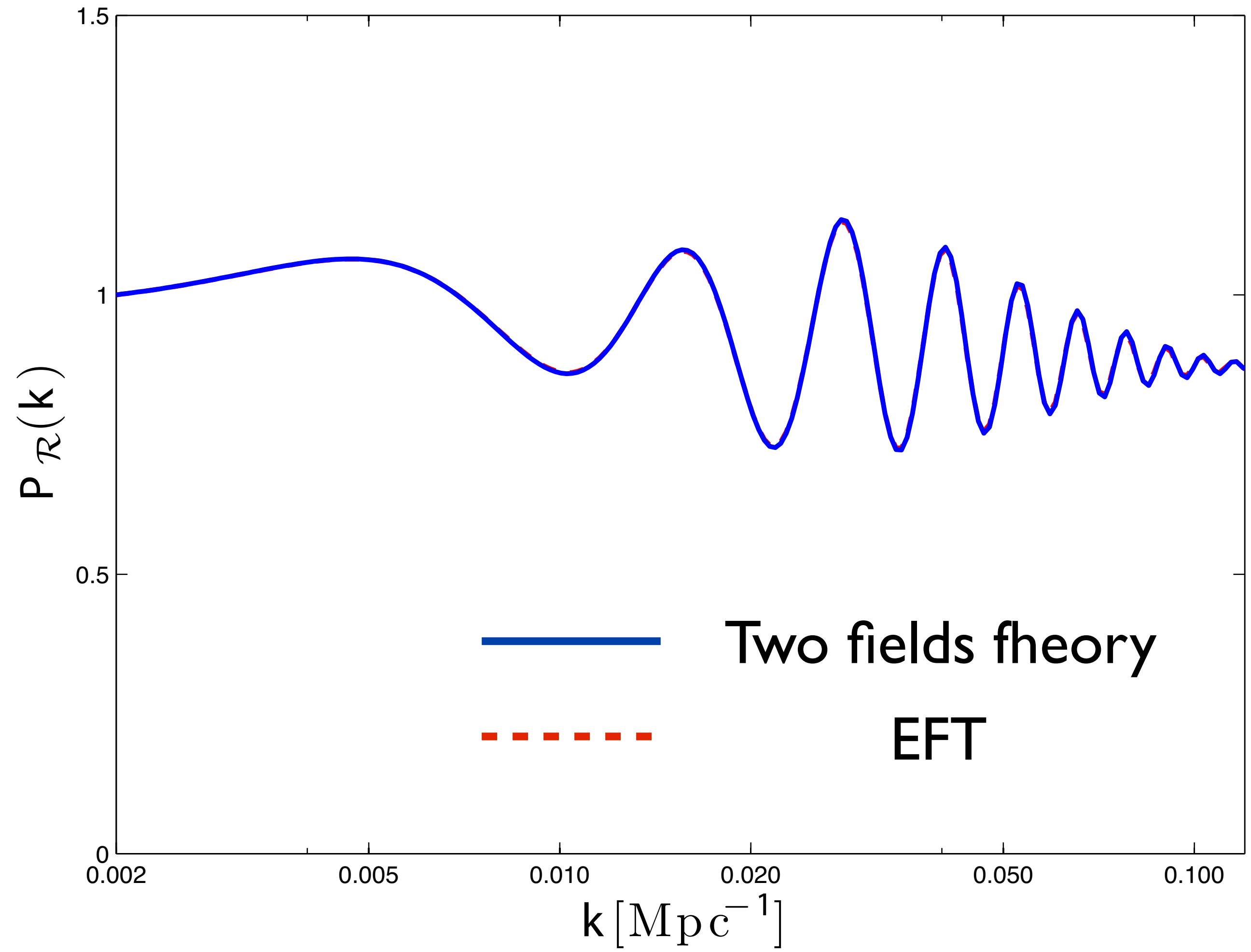
$$T_M \sim T_\perp$$


2.5
2
1.5
1
0.5
0
−0.5
−1
η
0
0.5
1
1.5
2
2.5
3
N
$\alpha = 9$
$\beta = 0.25$
$\Delta N = 0.4$
$\eta_\perp$
$\eta_\parallel$


1.5
1
0.5
0
$P_{\mathcal{R}}(k)$
0.002
0.005
0.010
0.020
0.050
0.100
$k\,[\mathrm{Mpc}^{-1}]$
Difference
betwen
theories
Two fields fheory
EFT

$$T_M \sim T_\perp$$

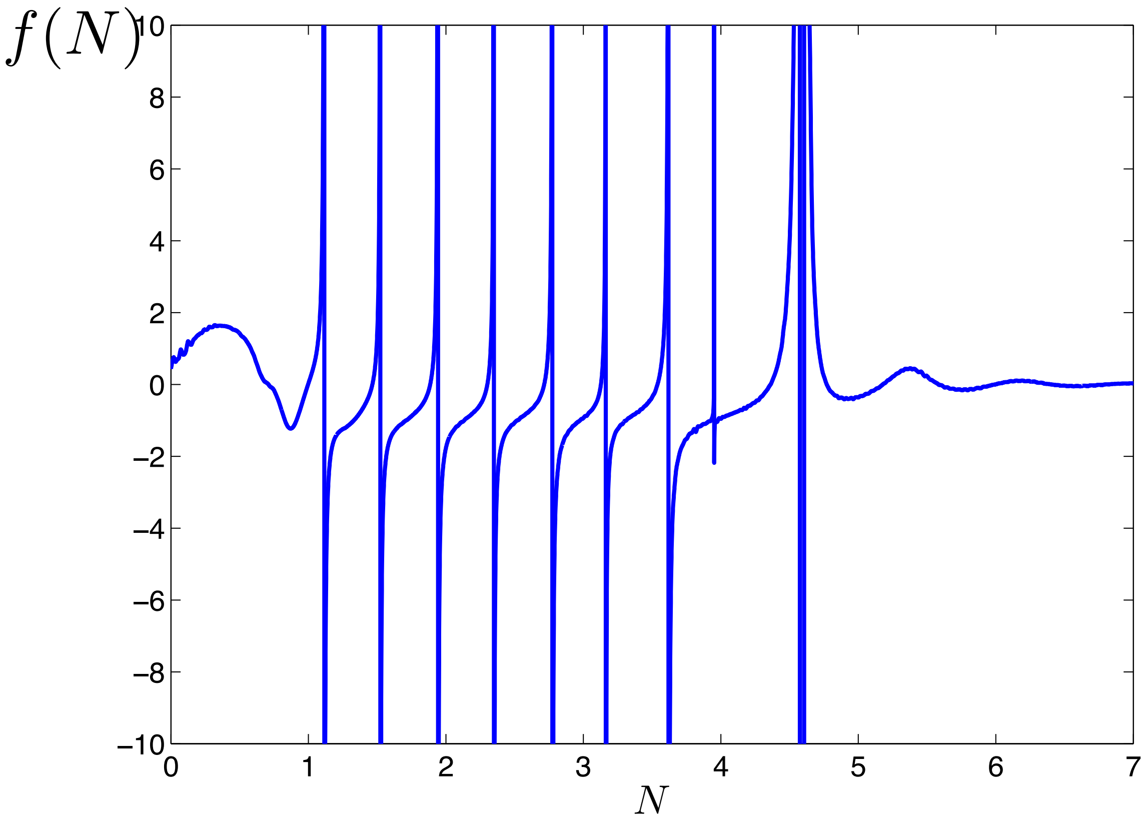




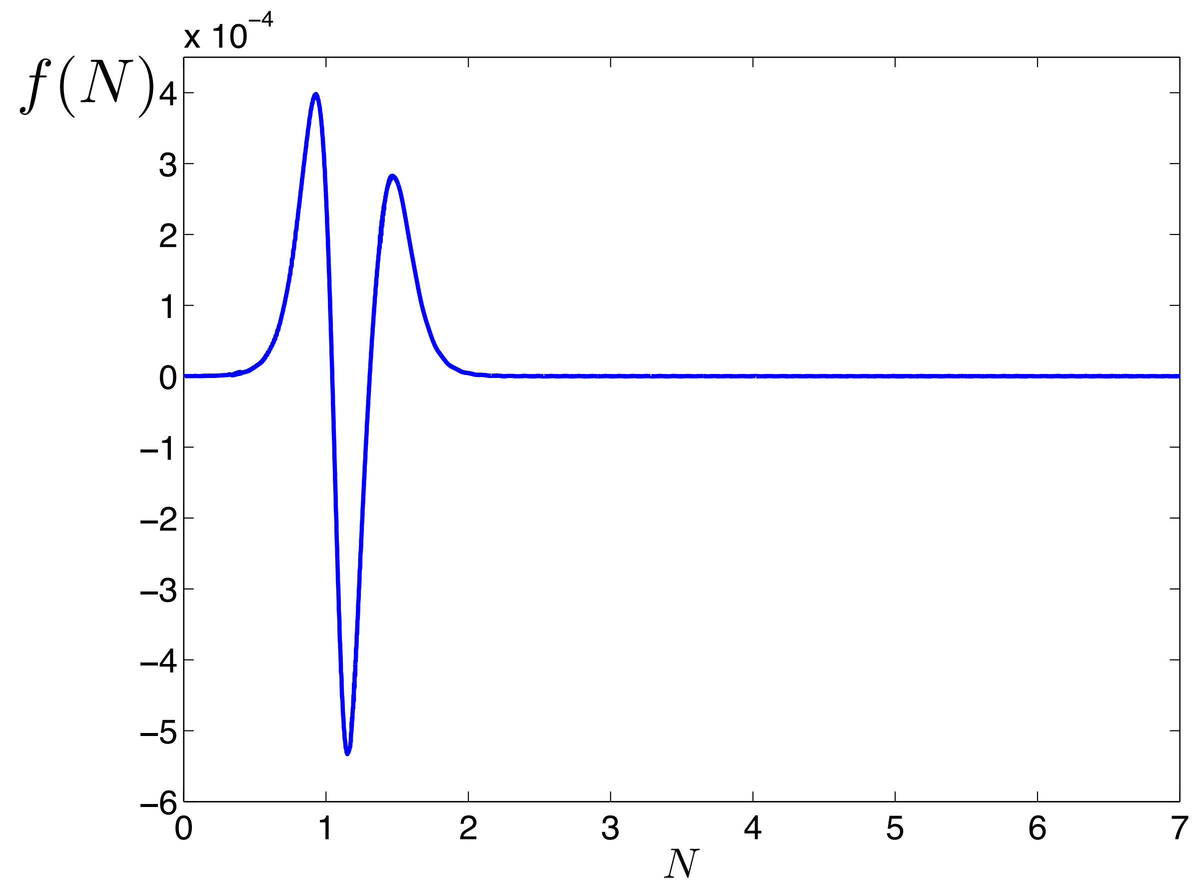
$$T_M \ll T_\perp$$

large features and an excellent agreement between both power spectra

Finally we define $f(t) = \frac{H^2}{M_{\text{eff}}^2} \frac{1}{\dot{\theta}} \frac{d^2}{dt^2} \dot{\theta}$



$T_M \sim T_\perp$



$T_M \ll T_\perp$

Conclusions

Fast turns produce features in the primordial spectrum

Features might offer a direct insight on heavy physics

Heavy phyiscs can be encoded in an EFT with a modified speed of sound when $\left| \frac{d}{dt} \ln \dot{\theta} \right| \ll M_{\rm eff}$

Future data will put strong constraints on primordial features, improving our understanding of the role of UV physics in the early universe